Consumer and Producer Theory

Students of Public Finance must master the theory of the consumer: the majority of our models will focus on the consumer. The theory of the producer will used at times and is therefore also presented here. Students who have not had intermediate micro-theory recently, or have taken such a course recently but the course did not contain much mathematical modeling/theory, should study this document.

1. CONSUMPTION THEORY: THEORY OF THE CONSUMER

We assume there are two goods X and Y with givens prices P_x and P_y. The consumer has income I. The consumer's budget constraint in a simple world simply describes possible expenditure now, or in some fixed period of time (e.g. this week). But there is no past and no future, hence no savings, no debt, no bequests, etc.[1]

1.1 THE BUDGET CONSTRAINT

Imagine you have I = \$500 and must decide on how many X = apples and Y = shoes to buy, where P_x = 2/pound P_y = 10. The **budget constraint** is

$$\text{Budget Constraint : } I = XP_x + YP_x$$
$$\text{X Expenditure : } XP_x \geq 0 \quad X \geq 0$$
$$\text{Y Expenditure : } YP_y \geq 0 \quad Y \geq 0$$

For example

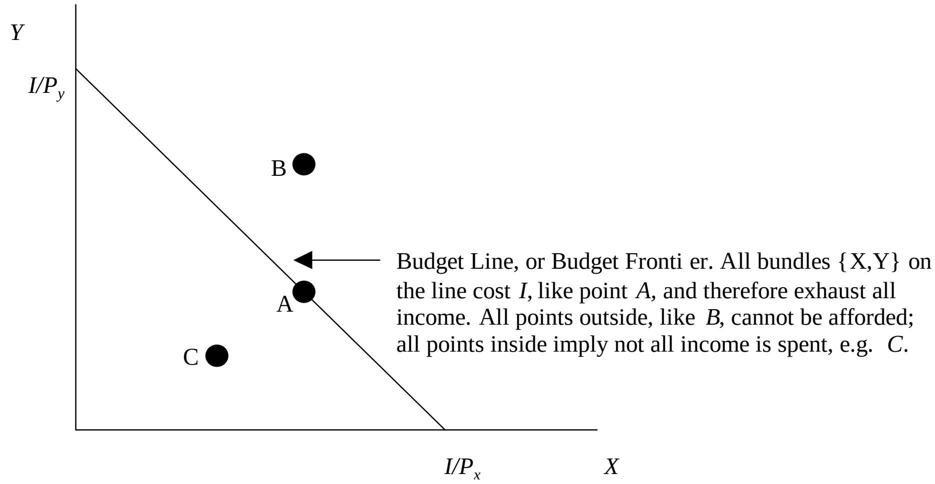
$$500 = X \times 2 + Y \times 10$$

The constraint implies a **budget line**:

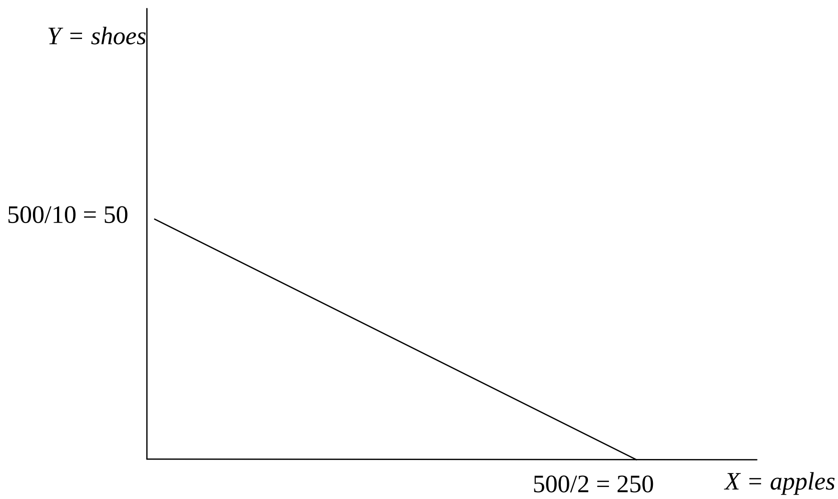
$$Y = \frac{I}{P_y} - X\,\frac{P_x}{P_y}$$
$$\text{Maximum number of Y units affordable} = I / P_y$$
$$\text{Maximum number of X units affordable} = I/P_x$$
$$\text{Slope} = \text{Market Tradeoff} = P_x / P_y$$

which can be plotted on a Euclidean X,Y-plane:

[1] The two goods can be anything. In the future (but not in this document) we will assume X is simply a consumption good, and Y is leisure time. The less Y one has the more time spent working so the more X. In this scenario the price of consumption is 1, and the price of leisure is the wage that would have been earned if the person worked.



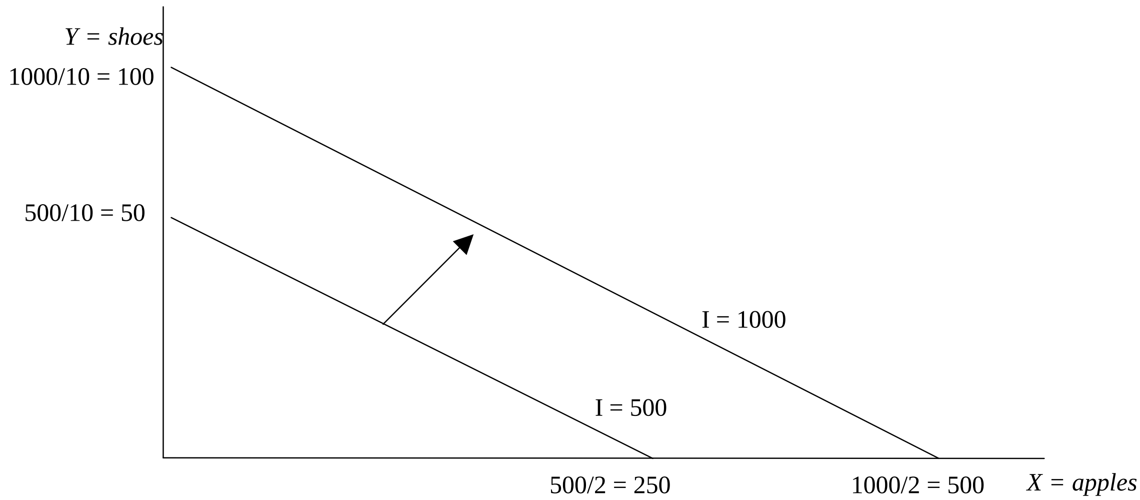
For example



Notice the budget line articulates **consumption possibilities**: all points on or inside the line are possible; all outside are not.

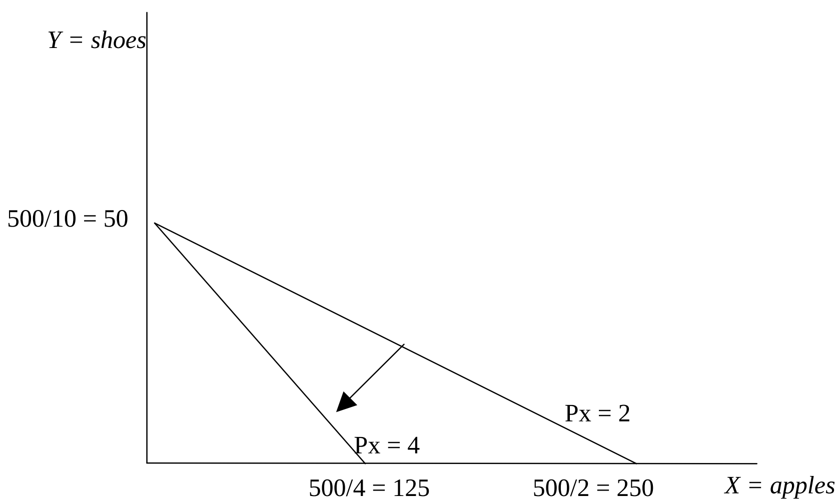
1.2 INCOME SHIFTS

As income increases or decreases, consumption possibilities increase or decrease for any price level. If income increases to $1000 then



1.3 PRICE SHIFTS

As the price of a good increases or deceases the market tradeoff changes: the budget line slope changes and therefore consumption possibilities change. If I = 500 and the price of X increases to 4 then



1.4 UTILITY

We assume individuals gain some sort of utility, or personal welfare, when they consumer goods. We assume more consumption leads to more personal welfare, but there are decreasing marginal returns. For example, if you are very hungry the first apple gives you a lot of satisfaction, the second gives you less satisfaction since you are no longer very hungry, etc.

The **utility function** is written as $U(X,Y)$. For example, the **Cobb-Douglas** utility function is

$$U(X,Y) = X^{\alpha}Y^{\beta} \quad 0 \le \alpha, \beta \le 1 \quad \alpha + \beta = 1$$

The parameters α and β are **preference parameters** that reflect a consumers overall predisposition toward a good. Here, if Frank has α = 3/4 and Susan has α = 2/3, Frank will literally prefer X more than Susan, holding everything else constant (i.e. *ceteris parabus*). If Frank and Susan were otherwise identical (i.e. same income level), Frank would consume more X then Susan.

In Susan's case if α = 2/3 and β = 1/3, then

$$U(X,Y) = X^{2/3}Y^{1/3}$$

Marginal utility, or MU, denotes the change in utility when the consumption level is changed. We do this for infinitesimal changes, hence MU is the partial derivative of $U(X,Y)$:

$$MU_x = \frac{\partial}{\partial X}U(X,Y) \qquad MU_y = \frac{\partial}{\partial Y}U(X,Y)$$

Recall "**more is better**", but there are "**decreasing marginal returns**". This implies we must have

$$MU_x = \frac{\partial}{\partial X}U(X,Y) > 0 \text{ and } MU_x \text{ gets smaller as } X \text{ increase}$$
$$MU_y = \frac{\partial}{\partial Y}U(X,Y) > 0 \text{ and } MU_y \text{ gets smaller as } Y \text{ increase}$$

1.5 COBB-DOUGLAS UTILITY FUNCTION

Recall Susan's preference structure for X = apples and Y = shoes is

$$U(X,Y) = X^{2/3}Y^{1/3}$$

The marginal utility functions are

$$MU_x = \frac{\partial}{\partial X}U(X,Y) = \frac{2}{3}X^{2/3-1}Y^{1/3} = \frac{2}{3}X^{-1/3}Y^{1/3} = \frac{2}{3}\left(\frac{Y}{X}\right)^{1/3}$$
$$MU_y = \frac{\partial}{\partial Y}U(X,Y) = \frac{1}{3}X^{2/3}Y^{1/3-1} = \frac{1}{3}X^{2/3}Y^{-2/3} = \frac{1}{3}\left(\frac{X}{Y}\right)^{2/3}$$

Clearly

$$MU_x = \frac{2}{3}\left(\frac{Y}{X}\right)^{1/3} > 0 \text{ and } MU_x \downarrow \text{as } X \uparrow$$
$$MU_y = \frac{1}{3}\left(\frac{X}{Y}\right)^{2/3} > 0 \text{ and } MU_y \downarrow \text{as } Y \uparrow$$

So, the Cobb-Douglas utility function satisfies our basic requirements about people.

1.6 UTILITY MAXIMIZATION - OPTIMAL CONSUMPTION CHOICE

We assume people choose their consumption bundles by maximization the utility they obtain from consumption, subject to their budget constraint:

$$\max_{x,y} U(X,Y) \text{ s.t. } I = XP_x + YP_y$$

The following skips several steps simply to present the solution. The utility maximizing bundle $\{X^*, Y^*\}$ must satisfy two conditions. First, it must be affordable and therefore satisfy the **budget constraint**:

$$I = X^* P_x + Y^* P_y$$

Second, it must satisfy the **optimality condition**:

$$\frac{MU_x}{MU_y} = \frac{P_x}{P_y}$$

With these two equations we solve for $\{X^*, Y^*\}$.

1.7. COBB-DOUGLAS REVISITED

Suppose Susan has income I = 500, the price of X = apples is 2, the price of Y = shoes is 10, and her utility function is

$$U(X,Y) = X^{2/3} Y^{1/3}$$

Then her optimal consumption choice solves

$$\boxed{\begin{array}{l} \text{Budget Constraint}: \ 500 = X^* 2 + Y^* 10 \\ \text{Optimality Condition}: \ \dfrac{MU_x}{MU_y} = \dfrac{P_x}{P_y} \end{array}}$$

Use the marginal formulas above to solve:

$$\frac{MU_x}{MU_y} = \frac{\frac{2}{3}\left(\frac{Y}{X}\right)^{1/3}}{\frac{1}{3}\left(\frac{X}{Y}\right)^{2/3}} = 2\frac{Y}{X}$$

The optimality condition implies

$$2\frac{Y}{X} = \frac{2}{10} \Rightarrow Y = \frac{1}{10} X$$

Use the budget constraint to get

$$Y = 50 - .2X = .1X \Rightarrow X = 50/.3 = 500/3 = 166.67$$
$$\Rightarrow Y = .1X = .1 \times 167 = 16.67$$

$$\text{The Optimal Bundle } \{X^*, Y^*\} = \{166.67, 16.67\}$$

Double check affordability:

$$XP_x + YP_y = 166.67 \times 2 + 16.67 \times 10 = 333.33 + 166.67 = 500$$

So, the optimal bundle does indeed cost exactly the person's income.

1.8 COBB-DOUGLAS AGAIN: DEMAND FUNCTION

We can solve for the optimal consumption of X and Y as functions of income I and prices P. We use

$$\text{Budget Constraint}: I = X^* P_x + Y^* P_y$$
$$\text{Optimality Condition}: \frac{MU_x}{MU_y} = \frac{P_x}{P_y}$$

Solve

$$\frac{MU_x}{MU_y} = 2\frac{Y}{X} = \frac{P_x}{P_y} \Rightarrow YP_y = \frac{1}{2} XP_x$$

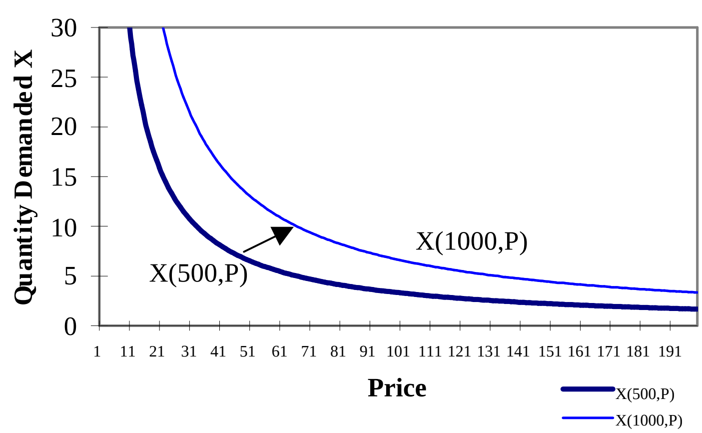
Use the budget constraint to get

$$I = X^* P_x + Y^* P_y = X^* P_x + \frac{1}{2} X^* P_x = \frac{3}{2} X^* P_x$$

$$X^* = \frac{2}{3} \frac{I}{P_x}$$

So, quantity demanded of X is decreasing in price, and increasing in income (i.e. it is a normal good). For I = 500 and 1000 we have

Demand Functions: X(I,P)



2. PRODUCER THEORY: THEORY OF THE FIRM

By comparison the theory of the firm is fairly transparent. The firm uses **inputs**, or **factors of production** (e.g. L = labor, K = capital; but also human capital, land, energy, etc…), to generate **output** *Y*. The relationships between inputs and output is the **production function**, which itself must imply some level or state of technology. In this class we write

$$Y = A \times F(K, L)$$

were F is the **production function**, and A denotes **total factor productivity**. We use A to capture an abstract sense of technology: a more technologically advanced nation will be able to produce more goods with the same number of labor and capital inputs as comparatively less advanced nations.

We assume increasing **marginal product**: more inputs must generate more output. The **Marginal Product of Labor** and **Capital** are

$$MP_L = A \times \frac{\partial}{\partial L} F(K, L) \qquad MP_K = A \times \frac{\partial}{\partial K} F(K, L)$$

We also assume decreasing marginal returns: more input leads to more output, but at a decreasing rate:

$$MP_L \downarrow \text{ as } L \uparrow \qquad MP_K \downarrow \text{ as } K \uparrow$$

2.1 COBB-DOUGLAS PRODUCTION FUNCTION

The Cobb-Douglas function was investigated as a likely expression of output inputs translate to output by Paul Douglas and Charles Cobb in (1928). Since then economists used the functional form for utility.

Consider a **short-run** production problem: capital is fixed at $K^* = 1$ and

$$F(L) = F(1, L) = AL^{2/5}$$

This means the short-run marginal product of labor is

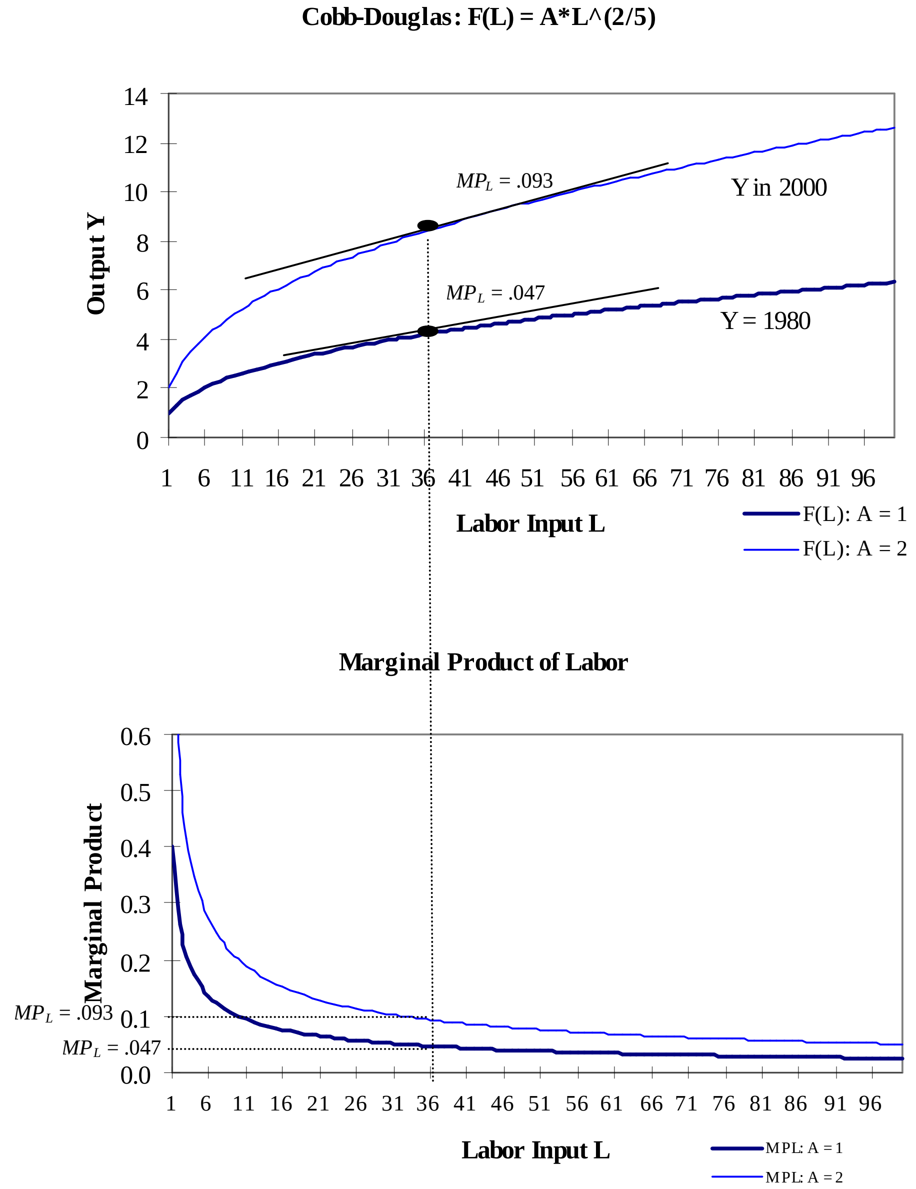
$$MP_L = \frac{\partial}{\partial L} AL^{2/5} = \frac{2A}{5} \frac{1}{L^{3/5}}$$

which is positive, and decreasing in L.

If A = 1 in 1980 and A = 2 in 2000, the marginal product of labor is

$$MP_L = \frac{2A}{5}\frac{1}{L^{3/5}} \Rightarrow A = 1 \text{ then } MP_L = \frac{2\times 1}{5}\frac{1}{36^{3/5}} = 0.046588$$

$$\Rightarrow A = 2 \text{ then } MP_L = \frac{2\times 2}{5}\frac{1}{36^{3/5}} = 0.093177$$



We can imagine the product is a table: one new unit of labor (e.g. one person working one more hour) can produce 1/20th of a table in 1980, but 1/11th of a table in 2000.

2.2 PROFIT MAXIMIZATION: OPTIMAL OUTPUT

The firm solves the following profit maximization problem. We assume the firm is small, hence a **price-taker**: price is simply given. The firm also takes the market wage *w* as given. In the short-run only labor-inputs are adjusted to optimize profit, and capital is held fixed (e.g. it is relatively easy to hire more labor inputs; but expanding production by adding an assembly line takes months or years to prepare for). This is the problem with the Classical Model: there is only a "short-run", and no capital accumulation such that the economy can grow.

Profit as a function of labor inputs, denoted $\pi(L)$, is

$$\text{Profit} = \text{Revenues} - \text{Cost}$$

$$\pi(L) = P \times A \times F(K^*, L) - wL$$

The profit maximization problem is

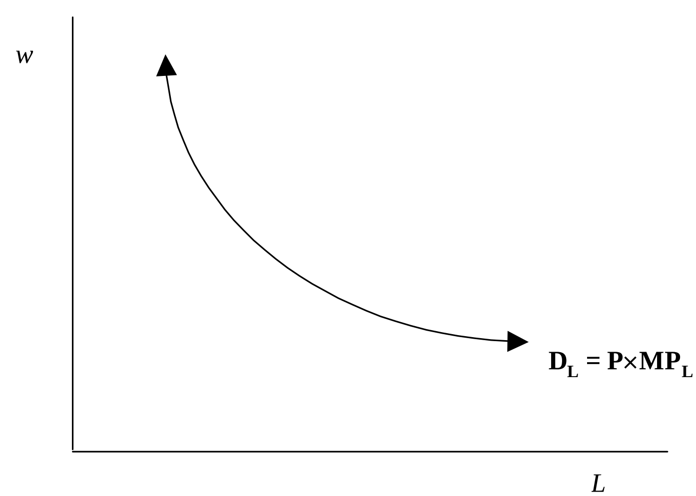
$$\boxed{\max_L \pi(L) = P \times A \times F(K^*, L) - wL}$$

The first order condition is

$$\begin{aligned} \frac{\partial}{\partial L}\pi(L) = 0 \quad &\Rightarrow \quad P \times A \times \frac{\partial}{\partial L} F(K^*, L) - w = 0 \\ &\Rightarrow \quad P \times MP_L = w \\ &\Rightarrow \quad \text{Value of Marginal Product} = \text{labor cost} \end{aligned}$$

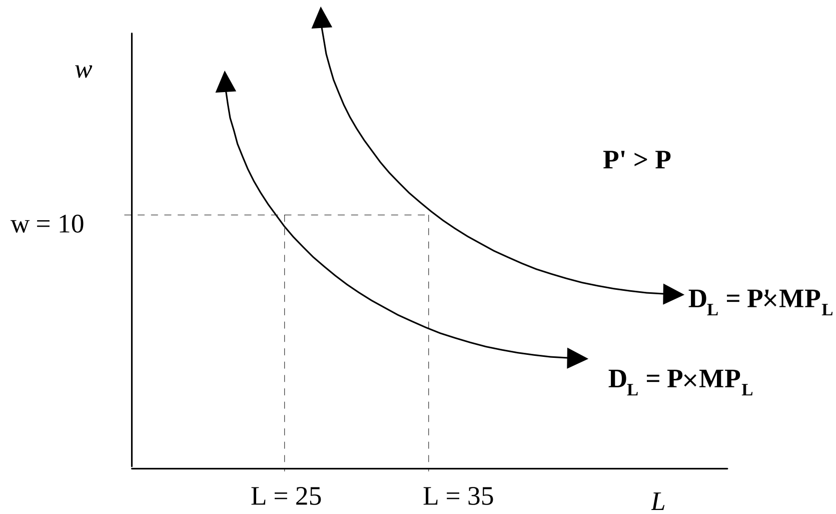
Thus, the firm uses labor to the point where the cost of an additional hour, *w*, is exactly off-set by the market value of that additional unit of labor: $P \times MP_L$.

Notice $\mathbf{P \times MP_L}$ denotes the **Labor Demand** function, based on the what it costs to employ labor. Since we assume diminishing marginal returns, the demand for labor is



So, what **shifts the demand for labor**? *First*, a high price: *ceteris paribus* an in **increase in price** makes production more profitable, hence output will be expanded which requires more labor inputs at whatever the wage is. *Second*, **more productive labor**: if people can produce output faster[2] this represents a *de facto* savings to the firm. This lower cost implies greater profits, hence output expands and again the demand for labor increases.

For example, an increase in the price implies more labor is demanded at every wage:



2.3 EXAMPLE

If the market wage and price are $w = 20$ and $P = 50$, then the firm uses labor until

$$50 \times MP_L = 20 \quad \Rightarrow \quad MP_L = 2/5 = .4$$

Since the price is so high, the value of labor is high. Recall that the Marginal Product of Labor decreases as labor units are added (diminishing marginal returns). So the resulting tiny 2/5 implies a large number of labor units: valuable labor = employ a lot of labor.

2.4 EXAMPLE

Fix capital at $K = 1$, and assume $w = 10$, $P = 50$, and

$$Y = A \times F(L) = AF \times (1, L) = 2 \times L^{2/5}$$

Then the profit maximization problem reduces

$$P \times MP_L = w$$
$$50 \times \left(2 \times \frac{2}{5}\frac{1}{L^{3/5}}\right) = 10 \quad \Rightarrow \quad 4^{5/3} = L \quad \Rightarrow \quad L = 10.07937$$

The firm's profit maximizing level of labor is 10.08 units (e.g. individuals/day, hours, etc., whatever the units are). The resulting level of output and profit are

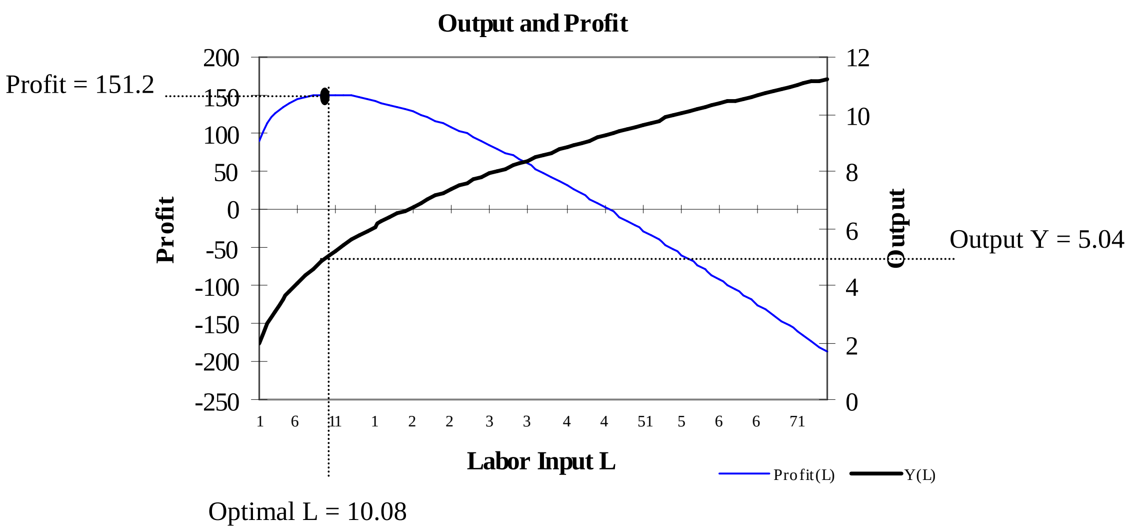
[2] That is, fewer labor hours are required to produce one unit: think of car manufacturing in 1960 versus 2000.

$$Y = A \times F(L^*) = 2 \times 10.07937^{2/5} = 5.039671$$

$$\pi = 50 \times 5.039671 - 10 \times 10.07937 = 151.1898$$

Notice we can represent and plot profit as a function of labor:

$$\pi(L) = 50 \times 2 \times L^{3/5} - 10 \times L$$



The **Demand for Labor** in this case is

$$50 \times MP_L = 50 \times \left(2 \times \frac{2}{5} \frac{1}{L^{3/5}} \right) = 40 \frac{1}{L^{3/5}}$$

which looks like