IS/LM Model

The IS/LM Model is a simple framework that ties together goods and financial markets, holding fiscal and monetary policy \((G, T, M)\) constant. In this note we will represent the two markets mathematically, compute equilibrium values of real income and the return to a bond, and analyze various fiscal and monetary policies by altering \((G, T, M)\).

Real output \(Y\) goes to: consumption \(C\), investment \(I\), the government via taxes \(T\), and once scaled by a price index \(P\) it also can be invested in government bonds. Think of \(I\) as “private sector” investment, and bonds as “public sector” investment.

It is assumed that you have studied class notes and chapters 3-5 of the textbook. The following focuses on specific mathematical models of the IS/LM framework.

GOODS MARKETS: DEMAND

The goods market is characterized by total (aggregate) real demand \(Z\), total (aggregate) output \(Y\). Demand is

\[
Z = C(Y, T) + I(Y, i) + G
\]

\(G\) denotes real government expenditure (roads, schools, military aircraft, Medicaid).

\(C(Y,T)\) is real private consumption as a function of real income \(Y\) and total taxes \(T\) (sales tax, income tax, land taxes, etc.).

\(I(Y,i)\) denotes real investment as a function of output \(Y\) and the return to a bond \(i\). Notice that “investment” is not in an asset (e.g. a government bond). It is real investment in the goods that are required to expand production (pipes, brick, chairs, consultation, land).

GOODS MARKETS: A MODEL OF DEMAND

Consider the following model of demand:

\[
Z = c_0 + c_1(Y - T) + \frac{c_2}{1 + i}Y + G
\]

where implicitly we are in fact modeling consumption and investment:

\[
C(Y, T) = c_0 + c_1(Y - T) \quad \text{and} \quad I(Y, i) = \frac{c_2}{1 + i}Y
\]

The rate of return \(i\) is a fraction. The parameters \(0 < c_1, c_2 < 1\) represent fractions of real income (or real disposable income \(Y - T\)) that go to consumption or investment.

We will not allow for debt in this model, other than government debt in the form of bonds. Since private citizens can consume \(C\), invest \(I\), or buy bonds (see below), the total fraction \(c_1 + c_2\) going to \(C\) and \(I\) will in general be less than 1:

\[
0 < c_1 + c_2 < 1.
\]
Notice
\[ \frac{\partial}{\partial Y} I(Y, i) > 0 \quad \text{and} \quad \frac{\partial}{\partial i} I(Y, i) < 0 \]

As real income \( Y \) rises there is more wealth to invest in output expansion. As the rate of return \( i \) to a government bond increases, however, \textit{ceteris parabus} people will place more wealth into bonds (public sector investment), hold less money (see below), and place less wealth into private sector investment.

**GOODS MARKETS: THE IS CURVE**

The IS ("investment-savings") curve is the total relationship between \textit{equilibrium} output \( Y \) and the return to bonds \( i \).

In the goods market \textit{equilibrium} total output \( Y \) equals total demand \( Z \), hence

\[ Y = Z = C(Y, T) + I(Y, i) + G \]

\[ \Rightarrow Y = C(Y, T) + I(Y, i) + G \]

Consider the previous model. In equilibrium

**Equilibrium**: \( Y = Z = c_0 + c_1(Y - T) + \frac{c_2}{1+i}Y + G \)

**Solution**: \( Y = \left( \frac{1}{1 - c_1 - \frac{c_2}{1+i}} \right) \times \left( c_0 + G - c_1T \right) \)

where

**Multiplier** = \( \frac{1}{1 - c_1 - \frac{c_2}{1+i}} \)

The relationship between \( Y \) and \( i \) is the IS curve. In general, it is more accurate to write

\[ Y^{IS}(i) = \left( \frac{1}{1 - c_1 - \frac{c_2}{1+i}} \right) \times \left( c_0 + G - c_1T \right) \]

**Observations**:

1. As people consume and or invest more (high \( c_1 \) and or \( c_2 \)), the multiplier is larger, resulting in larger total output \( Y \).

2. As the rate of return to a government bond \( i \) increases, the multiplier decreases leading to less total output \( Y \).
EXAMPLE 1

Let 
\[ c_1 = .70 \quad c_2 = .25 \quad c_0 = 0 \quad G = 100 \quad T = 100 \]

Then the IS curve is 
\[ Y^{IS}(i) = \left( \frac{1}{1 - .7 - .25} \right) \times 100 \times .3 = 30 \times \left( \frac{1+i}{.3i + .05} \right) \]

We plot \( i \) on the Y-axis, and output on the X-axis. That means we have to solve for \( i \) as a function of \( Y \):

\[
Y = 30 \times \left( \frac{1+i}{.3i + .05} \right) \\
\Rightarrow Y (.3i + .05) = 30 \times (1+i) \\
\Rightarrow .3iY + .05Y = 30 + 30i \\
\Rightarrow .3iY - 30i = 30 - .05Y \\
\Rightarrow i = \frac{30 - .05Y}{.3Y - 30}
\]

A plot of \( i \) as a function of \( Y \) follows:

If \( c_1 \) is .75, then \textit{ceteris paribus}

\[ Y^{IS}(i) = 120(1+i) / i \]

hence

\[ i = 120 / (Y - 120) \]
For each bond return $i$, total output $Y$ is larger. An apparently small increase the propensity to consume augments total output substantially.

**FINANCIAL MARKETS: MONEY AND BONDS**

We assume private citizens hold some wealth in currency, and some in government bonds. There are no other "investment" opportunities. Clearly, in the real world there are equity shares (stocks), mutual funds (bundles of stocks), hedge funds, certificates of deposits, private savings accounts, etc. Here, we assume money is either in your pocket or lent to the government.

**DEMAND FOR MONEY**

The demand for money is a fraction of nominal income $P \times Y$ ($P$ is a price index) based on the return $i$ to a bond:

$$M^d = P \times Y \times L(i) \quad \text{where} \quad 0 \leq L(i) \leq 1$$

As the bond return $i$ rises, ceteris paribus people invest more in bonds and hold less currency:

$$\frac{\partial L(i)}{\partial i} < 0$$

Real money demand is

$$\frac{M^d}{P} = Y \times L(i)$$
DEMAND FOR MONEY: A SIMPLE MODEL

Suppose \( L(i) \) had a simple form:

\[
L(i) = \frac{L}{1 + i} \quad \text{where} \quad 0 \leq L \leq 1
\]

then

\[
\frac{M^d}{P} = Y \times \frac{L}{1 + i}
\]

EQUILIBRIUM IN FINANCIAL MARKETS

The central bank supplies \( M \) currency units (e.g. dollars). This is fixed. In equilibrium the real money supply \( M/P \) is equal to the real money demand:

\[
\frac{M^*}{P} = \frac{M^d}{P} \Rightarrow \frac{M}{P} = Y \times \frac{L}{1 + i}
\]

FINANCIAL MARKETS: THE LM CURVE

The LM curve is the total relationship between the equilibrium output \( Y \) and the interest rate \( i \) within the market for money. Use the simple model above to solve for \( Y \) as a function of \( i \):

\[
Y^{LM}(i) = \frac{M(1 + i)}{P \times L}
\]

Since we will plot \( i \) as a function of \( Y \), reverse the solution:

\[
i = \frac{P \times Y \times L - M}{M}
\]

Clearly as \( Y \) increases \( i \) increases: more real output in equilibrium generates more demand, requiring more money to make the purchases. Money is removed from the bond market, driving the bond rate up to induce private citizens to still invest in bonds.

As the price level rises, ceteris paribus, more money is required to make purchases. Hence \( i \) increase ceteris parabus (the LM curve shifts).

EXAMPLE 2

If \( P = 5, M = 1000 \) and \( L = .80 \) then

\[
Y^{LM}(i) = \frac{1000(1 + i)}{5 \times .8} = 250(1 + i)
\]

and
\[ i = \frac{5 \times Y \times 0.8 - 1000}{1000} = -1 + 0.004Y \]

**IS/LM EQUILIBRIUM**

The equilibrium rate of return on bonds must clear the goods market. People must be willing to hold just enough bonds, such that the remaining money goes into consumption and investment thus generating output which is bought, at a price level \(P\), with whatever money was not put into bonds. There must not be one cent left unaccountable for, not one good not used for consumption of investment of government spending.

Put succinctly, in equilibrium there is a bond return \(i^*\) such that output implied by the goods market and output implied by the financial markets is the same:

\[ i^* \] solves \[ Y^{IS}(i^*) = Y^{LM}(i^*) \]

**EXAMPLE 3**

Combine the above two examples, where

\[ Y^{IS}(i) = 30 \times \left( \frac{1 + i}{0.3i + 0.05} \right) \]

and

\[ Y^{LM}(i) = 250(1 + i) \]

to deduce
\[ Y^{IS}(i) = Y^{LM}(i) \]
\[ 30 \times \left( \frac{1 + i}{.3i + .05} \right) = 250(1 + i) \]
\[ \left( \frac{3}{25} - .05 \right) \times \frac{1}{.3} = 0.23333 = i^* \]

Therefore, at roughly a return of \( i = 23.33\% \) each market clears. In equilibrium, output is

\[ Y^* = 250(1 + .2333) = 308.33 \]

Graphically:

![Graph of IS/LM Model](image)

**EXAMPLE 4**

Suppose **consumer confidence drops** severely such that the propensity to consume falls to \( 65\% \). We now have

\[ c_1 = .65 \quad c_2 = .25 \quad c_0 = 0 \quad G = 100 \quad T = 100 \]

The IS curve is

\[ Y^{IS}(i) = \left( \frac{1}{1 - .65 - .25 - \frac{1}{1+i}} \right) \times 100 \times .35 = 35 \left( \frac{1+i}{.35i+.10} \right) \]

or

\[ i = \frac{35 - .1Y}{.35Y - 35} \]
The IS and LM plots are now

![IS/LM Model](image)

In equilibrium the rate \( i \) and output \( Y \) fall.

The government could respond with a **policy mix**: lower \( T \) and/or increase \( G \) (*expansionary fiscal policy*), and buy bonds (*expansionary monetary policy*). The fiscal policy would increase \( Y \) at any \( i \), and therefore shift IS right, increasing \( Y \) and \( I \) in equilibrium.

Loweaxes spurs spending, hence real income, which implies people need to hold more cash to cover their expenditures, so the bond rate increases to competitively induce people to still invest in bonds.

The monetary policy would put money into the economy, lowering \( i \), giving incentive for investment \( I \) to increase, and therefore output to increase. The black dots denote the original equilibrium, and the equilibrium after consumer consider falls and the government responds with expansionary fiscal and monetary policies. Output increases, and interest rates remain low.

This may heat up the economy, though, as spending and private sector investment increase. Based on the shifts plotted below, it may be the case that the government overshot its expansionary monetary policy, and the central bank may be instructed to start selling bonds. Also, as the money supply increases due to expansionary policy, holding everything else constant we should see prices rise eventually: real money changes should be zero in the medium to long run. So, only use the graph below to interpret what is predicted to happen in the (very) short run.
Notice the IS/LM model as stated is not entirely correct. It over simplifies the connection between the bond market and government spending and taxation. We do not link $T$ and $G$ to the bond market, but the only way $G$ can increase above $T$ (and therefore increase $Y$ at any $i = IS$ shift right) is for the government to sell bonds. This would remove money from the economy, reduce investment and therefore deduce output, and increases interest rates. Thus, in order for IS to shift right (more $G$, and/or less $T$) LM must shift left (more bonds, less money for spending), both driving interest rates up.