

Final Examination
Economics 871 - Time Series
Spring 2009

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May 3, 2009

Instructions: Choose 4 problems. I will not grade 5, so do not attempt all of them: I will automatically grade #1-#4 in that case. Each problem is equally weighted.

1. **Spurious Regression** Consider three random variables

$$\begin{bmatrix} \Delta y_{1,t} \\ \Delta y_{2,t} \\ \Delta y_{3,t} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \delta \end{bmatrix} + \begin{bmatrix} \epsilon_{1,t} \\ \epsilon_{2,t} \\ \epsilon_{3,t} \end{bmatrix}$$

where $\epsilon_t \stackrel{iid}{\sim} (0, W)$, for positive definite W .

- a. Characterize the limiting properties of the least square estimator $\hat{\gamma}$ of the slope parameters in

$$y_{1,t} = \gamma_1 y_{2,t} + \gamma_2 y_{3,t} + u_t. \tag{1}$$

In order to do so, give each imperative step, cite the appropriate limit theory result (by the corresponding result in Lecture Notes or a well known result by name) and declare why the result holds. Prove whether the limit distribution is affected by $\delta = 0$ (be careful).

- b. Give a brief explanation as to why (1) does not make sense as a "regression" and why $\hat{\gamma}$ approaches a random variable. Is there a dominating factor in (1)? Explain.

2. Stochastic Breaks Occasional stochastic breaks can give the appearance of permanent shocks. Consider the following model:

$$\begin{aligned} y_t &= \alpha_1 + \delta t + \epsilon_t \quad \text{it } t < t^* \\ y_t &= \alpha_2 + \delta t + \epsilon_t \quad \text{it } t \geq t^* \end{aligned}$$

where $t = t^*$ with probability p , $p \in [0, 1]$.

- a. Prove $\Delta y_t \sim I(0)$.
- b. Prove the mse of the best linear h -step ahead forecast $y_{t+h|t}^*$ diverges if $p \in (0, 1)$, and give an expression for the rate. Compare it to the mse of $y_{t+h|t}^*$ when $p = 0$.
- c. Prove the least squares estimator of δ is super- \sqrt{n} -consistent by characterizing its limit distribution (hint: there is not a deterministic constant in this model, so write as one equation with two stochastic regressors).

3. Unobserved Components Consider the following model of $y_t = \ln(\text{GDP}_t)$ decomposed into stochastic trend x_t and covariance stationary cycle c_t components:

$$\begin{aligned} y_t &= x_t + c_t & (2) \\ x_t &= \delta + x_{t-1} + \epsilon_{x,t} \\ c_t &= \phi_1 c_{t-1} + \phi_2 c_{t-2} + \epsilon_{u,t} \end{aligned}$$

where $\epsilon_{x,t}, \epsilon_{y,t}$ are mutually and serially independent, zero-mean with unit variance, and the roots of $\phi(z)$ lie outside the unit circle.

- a. Write (2) is state-space form.
- b. Characterize an operational estimator of $\theta = [\delta, \phi_1, \phi_2]$ by QML. Fully detail each step.

4. Random Volatility Consider the following GARCH process

$$\begin{aligned} y_t &= \sigma_t \epsilon_t \quad \text{where } \epsilon_t \sim (0, 1), \quad \mathfrak{S}_t := \sigma(y_\tau : \tau \leq t) \\ \sigma_t^2 &= \omega + \alpha y_{t-1}^2 + \beta \sigma_{t-1}^2, \quad \omega > 0, \alpha, \beta \geq 0. \end{aligned}$$

- a. Define $\kappa := E[\epsilon_t^4] < \infty$. Characterize the kurtosis of y_t : $E[y_t^4]/(E[y_t^2])^2$. If $E[\epsilon_t^4] = 3$ prove y_t is leptokurtic relative to the Gaussian distribution (i.e. y_t is "heavy tailed").
- b. Give parameter boundaries that ensure $E[y_t^4] < \infty$, and separately $E[y_t^2] < \infty$.
- c. Characterize the QMLE of $\theta = [\omega, \alpha, \beta]'$. Show at least $E[\epsilon_t^4] < \infty$ and $E[\epsilon_t^2 | \mathfrak{S}_{t-1}] = 1$ are required to ensure asymptotic normality. Detail the remaining required conditions thoroughly.

5. Cointegration Consider the following set of relationships:

$$\begin{aligned}y_{1,t} &= \alpha y_{2,t} + u_{1,t} \quad \text{where} \quad u_{1,t} = \delta_1 u_{1,t-1} + \epsilon_{1,t} \\y_{1,t} &= \beta y_{2,t} + u_{2,t} \quad \text{where} \quad u_{2,t} = \delta_2 u_{2,t-1} + \epsilon_{2,t}\end{aligned}$$

and $\epsilon_t \stackrel{iid}{\sim} (0, W)$, for positive definite W .

a. Determine the degree of integratedness of $y_{1,t}$ and $y_{2,t}$, state the required parameter restrictions in each case (there may be more than one set of restrictions for each $I(d)$ case).

b. Under what parameter restrictions are $\{y_{1,t}, y_{2,t}\}$ cointegrated? Characterize the cointegration vector basis in each case (there may be more than one).

c. Choose a set of parameter restrictions that ensures $\{y_{1,t}, y_{2,t}\}$ are cointegrated. Derive the MA, error correction and triangular forms.