Final Examination – Answer Key
Economics 440: Public Finance
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1.

a. **False/Uncertain**: Zero tax revenues occurs if Q = 0. If Q = 0 before *and* after the tax, then EB = 0. Otherwise, EB > 0.

b. **False/Uncertain**: The Laffer curve requires IE ≠ SE. It is possible to obtain “quadratic” tax revenues with SE > IE, or SE < IE for high net wages and SE > IE for low net wages.

c. **True**: A net saver who faces a tax on returns will consumer more in the first period and save less (SE), and consume less in both periods (IE). If IE > SE then consumption in the first period falls, hence savings increases.

d. **True:** In general the net present value of the depreciation allowances increases when accelerated depreciation is allowed.

e. **False**: If firm *A* or *B* has well defined property rights then either *A* pays *B* to be able to produce, or *B* pays *A* to not produce as much. If A has property rights: B will pay A to produce less since at any $Q > Q^*$ (the socially efficient level) the Marginal Damage (MD) to A is greater than the lost marginal profit ($\Delta\pi$). This follows since at $Q > Q^*$ it must be MSC = MPC + MD > MB hence MD > MB - MPC = $\Delta\pi$. Therefore B pays A to produce less, until $Q = Q^*$. If B has property rights then it pays to produce from Q = 0 to Q^* because at any $Q < Q^*$ it must be MPC + MD < MB hence MD < $\Delta\pi$. The gain to A is greater than the damage to B, so they bargain and A pays B to be able to produce.

f. **True/Uncertain**: Ignoring special cases, in general

$$EB = \frac{1}{2} \times \frac{PQ\tau^2}{\frac{1}{\varepsilon_d} + \frac{1}{\varepsilon_s}}$$

Therefore, holding either elasticity constant (say demand), then as supply elasticity falls to zero the denominator grows to infinity, hence EB goes to zero:

$$EB = \frac{1}{2} \times \frac{PQ\tau^2}{\underbrace{\frac{1}{\varepsilon_d} + \frac{1}{\varepsilon_s \downarrow 0}}_{\to \infty}} \to 0.$$

The same is true if both fall. This is the "*true*" part.

The catch: if we one rises and one falls we do not know what happens to EB. This is the ""*uncertain*" part.

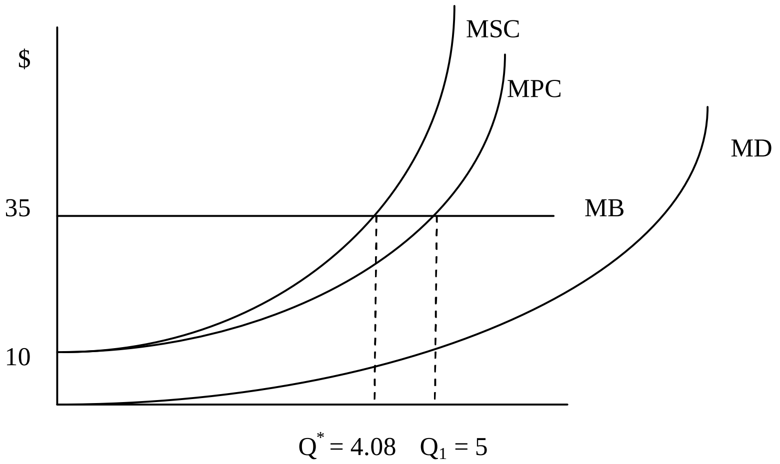
2.

a. Solve for the firm’s optimal level of output.

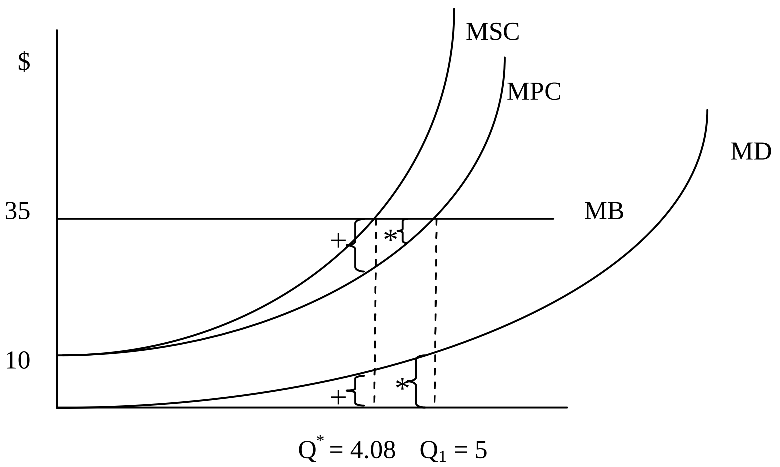
$$MC = MB \quad \Rightarrow \quad 10 + Q^2 = 35$$
$$Q^2 = 25 \quad \Rightarrow \quad Q = \sqrt{25} = 5$$

b. MSC = MPC + MD = 10 + Q^2 + .5Q^2 = 10+ 1.5Q^2.

$$MSC = MB \quad \Rightarrow \quad 10 + Q^2 + .5Q^2 = 35$$
$$1.5Q^2 = 25 \quad \Rightarrow \quad Q = \sqrt{25/1.5} = 4.08$$



c. The argument is given in #1.e. We need only graph it here. Firm *F* has property rights. At $Q = Q_1 - 1$ the firm loses a very small $\Delta\pi < MD$: we know it is small because at Q_1 be profit maximizing $\Delta\pi = MB - MPC = 0$. So, at values below Q_1, but close to Q_1, $\Delta\pi < MD$ as shown at *'s.



So, *F* needs at least $\Delta\pi$ and the community is willing to pay no more than MD. Since $MD > \Delta\pi$ they bargain, and Q drops.

At each $Q^* < Q < Q_1$ this is repeated because $\Delta\pi < MD$ is always true. We know this because at Q^* by construction $MPC + MD = MB$ hence $MD = MB - MPC = \Delta\pi$. Below Q^* there is profit to be earned hence $MPC + MD < MB$ or $MD < MB - MPC = \Delta\pi$. In other words, the damage to the community is too small, and the profits lost too great, for an agreement to be met. The community only pay for Q to drop to Q^*. At the +'s by construction $MD = \Delta\pi$.

d. By definition, the Pigouvian tax level, T, is the amount of marginal damage at the socially efficient output level:

$$T = MD(q_*) = .5q_*^2 = .5 \times 4.08^2 = \$8.32$$

3.

a. We want to solve

$$\max_s = (50000 - s)^{1/5} + \frac{1}{3}(10000 + s(1+.075))^{1/5}$$

The FOC with respect to *s* is

$$-\frac{1}{5}(50000 - s)^{-4/5} + \frac{1}{15}(10000 + s(1.075))^{-4/5} \times 1.075 = 0$$

and we solve

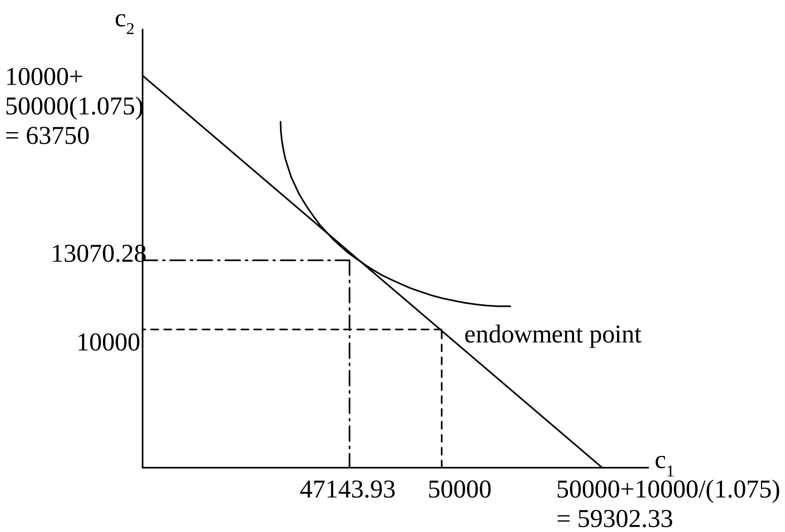
$$\frac{1}{3}(10000 + s(1.075))^{-4/5} \times 1.075 = (50000 - s)^{-4/5}$$
$$\frac{1}{3}(50000 - s)^{4/5} \times 1.075 = (10000 + s(1.075))^{4/5}$$
$$\left(\frac{1}{3}\right)^{5/4} \times 1.075^{5/4}(50000 - s) = 10000 + s(1.075)$$
$$0.277242 \times 500000 - 10000 = (0.277242 + 1.075)s$$

$$s = \frac{0.277242 \times 500000 - 10000}{0.277242 + 1.075} = 2856.07$$

Thus, the optimal choice is *savings s* = \$2856.07 > 0. This gives

$$c_1 = 50000 - 2856.07 = 47143.93$$
$$c_2 = 10000 + 2856.07 \times (1.075) = 13070.28$$

b.



c. We now want to solve

$$\max_s = (50000 - s)^{1/5} + \frac{1}{3}(10000 + s \times [1 + .075 \times (1 - .25)])^{1/5}$$
$$= (50000 - s)^{1/5} + \frac{1}{3}(10000 + s \times 1.05625)^{1/5}$$

The FOC is

$$-\frac{1}{5}(50000 - s)^{-4/5} + \frac{1}{15}(10000 + s \times 1.05625)^{-4/5} \times 1.05625 = 0$$

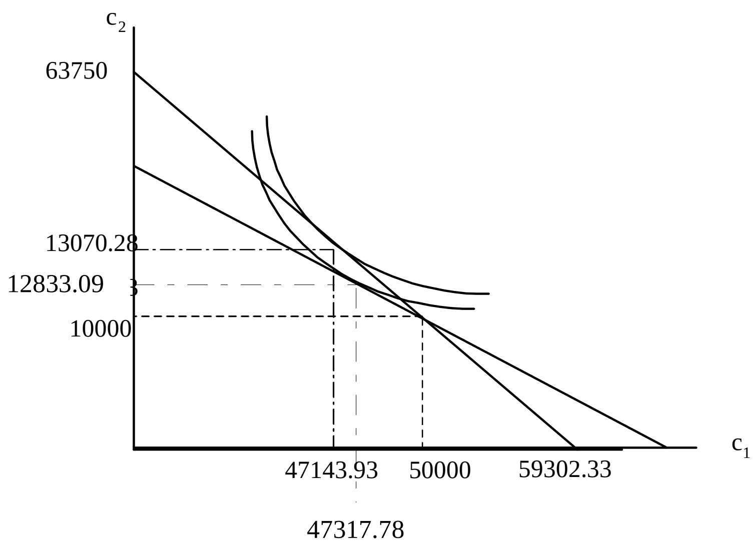
and we solve

$$\frac{1}{3}(10000 + s \times 1.05625)^{-4/5} \times 1.05625 = (50000 - s)^{-4/5}$$
$$\frac{1}{3}(50000 - s)^{4/5} \times 1.05625 = (10000 + s \times 1.05625)^{4/5}$$
$$\left(\frac{1}{3}\right)^{5/4} \times 1.05625^{5/4}(50000 - s) = 10000 + s \times 1.05625$$
$$0.271211 \times 500000 - 10000 = (0.271211 + 1.05625)s$$

$$s = \frac{0.271211 \times 500000 - 10000}{0.271211 + 1.05625} = 2682.22$$

Thus, savings shrinks to s = \$2682.22. The optimal post-tax consumptions levels are

$$c_1 = 50000 - 2682.22 = 47317.78$$
$$c_2 = 10000 + 2682.22 \times (1 + .075 \times (1 - .25)) = 12833.09$$



d. The *substitution effect* for a net saver, whose returns are taxed, is a reduction in savings, an increase in first period consumption, and a decrease in second period consumption. The *income effect* is a decrease in both periods' consumption levels. Above, the optimal choice is a net *increase* in *first period* consumption, therefore less savings: *the substitution effect dominates*.

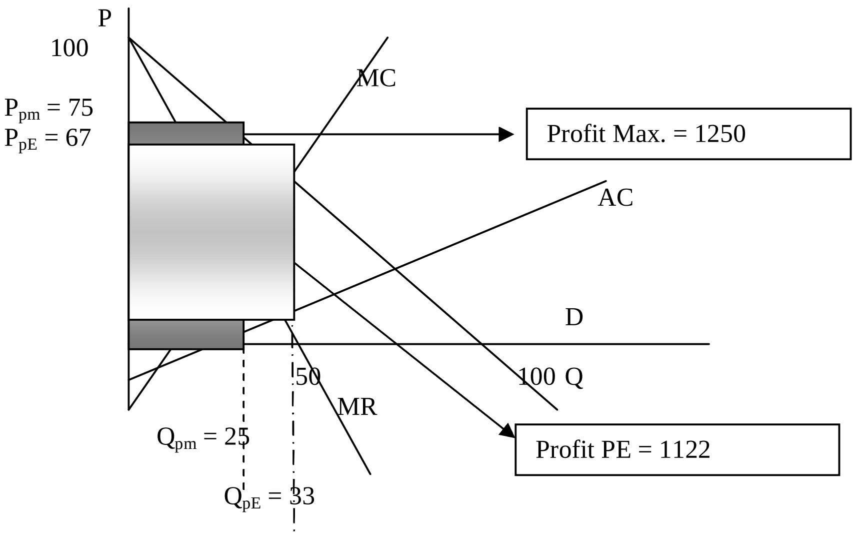
4. Recall Q = 100 - P and TC = Q^2.

a. First, MR, AC and MC are derived as follows:

$$R = Q \times P = Q(100 - Q) = 100Q - Q^2 \Rightarrow MR = 100 - 2Q$$

$$AC = TC / Q = Q$$

$$MC = \frac{\partial}{\partial Q} TC = \frac{\partial}{\partial Q} Q^2 = 2Q$$



b. We solve for profit, and take the derivative:

$$\pi = QP - TC = Q100 - Q^2 - Q^2 = Q100 - 2Q^2$$

$$\text{Marginal Profit} = \frac{\partial}{\partial Q} \pi = 100 - 4Q = 0 \Rightarrow Q_{pm} = 25$$

$$P_{pm} = 100 - Q_{pm} = 100 - 25 = 75$$

So, profit is:

$$\pi = Q(P - AC) = 25 \times 100 - 2 \times 25^2 = 1250$$

c. In this case we set WTP = MC:

$$WTP = 100 - Q = 2Q \Rightarrow Q_{PE} = 33 \Rightarrow P_{PE} = 67$$

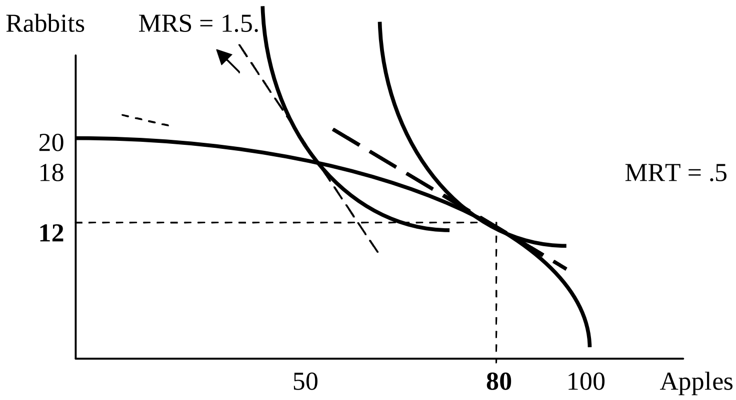
$$\pi = Q(P - AC) = 33 \times 100 - 2 \times 33^2 = 1122$$

This is not the profit max. output level, so of course it is less: 1250-1122 = 128.

d. If there 100 current uses, the fee is 128/100 = $1.28. But this is a de facto tax, hence demand will drop vertically by exactly 1.28. This implies a. EB in general; b. equilibrium Q falls, so the number of users falls. Together, it is not obvious the fee is right, or efficient.

5. The maximum number of apples available is 100, and the maximum number of rabbits is 20. Pat's present production level is 50 apples and 18 rabbit, which is on the PPF. At Pat's production level MRT = .5 (i.e. 1/2), and Pat's MRS = 1.5 (i.e. 3/2).

a.



b. The condition for efficiency in a production economy is $MRT_{x,y} = MRS_{x,y}$.

This is clearly not satisfied here. At Pat's present level (50,18) we have $MRS_{x,y} = 3/2 > 1/2 = MRT_{x,y}$. Literally, Pat is willing to give up 3 rabbits only if s/he is receives (produces) 2 apples and still have the same level of utility. However, Pat only needs to forgo 1 rabbit in order to produce 2 apples. In other words, Pat would really like more apples, but they are very cheap to make.

An efficient move would be to produce more apples and fewer rabbits, say (80, 12).

6. The *First Fundamental Theorem of Welfare* states that, if produces and consumers freely trade in a perfectly competitive market and a markets exists for all commodities, then a Pareto Efficient allocation of resources will occur.

It is important for students of economics to understand that the concluding statement means $MRT_{x,y} = MRS^{(A)}_{x,y} = MRS^{(B)}_{x,y} = P_x/P_y$ for all goods (X,Y), for all people (A,B): the technological tradeoff equals the psychological trade-off equals the market trade-off. Fairly profound, it you really think about it. It is, therefore, an elegant mathematical representation of Adam Smith's heraldic "invisible hand". As it turns out, on a good day it likely overwhelming fails to hold, and it not even obvious it should hold[1].

The sufficient conditions require perfect competition and a market to exist for all commodities. We need only consider environment "goods", like clean air, or welfare concerns, like "insurance against poverty" to generate examples of commodities for which markets do not exist.

Likewise, we need only consider natural monopolies (e.g. Microsoft nearly monopolizes the operating system market through shear [mostly legal] competitive behavior); natural resource monopolies (e.g. OPEC working together to increase the price of oil; De Beers and control of the diamond market); public utilities which require a legal monopoly status to ensure a minimum production/sales level to overcome massive average costs; and monopolistically competitive firms which rely on marketing, advertisement, and consumer loyalty to protect their market niche. We need only consider one of those examples to find a case of a non-competitive market. They are everywhere.