Midterm Examination - Answer Key
Economics 440: Public Finance
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1.

a. **UNCERTAIN**: A payroll tax that is equally split between worker and employer will have the entire incidence fall on the worker if labor supply is fixed. In that case we might argue the tax is unfair. The statement, however, did *not say if the tax is split nor if labor supply is fixed*. So, we cannot declare it is unfair.

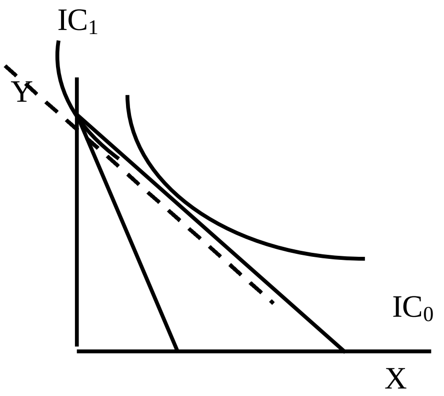
b. **FALSE**: By definition the contract curve defines all Pareto efficient allocations: any deviation causes at least one person to have a reduced utility (by the definition of "Pareto efficient").

c. **TRUE**: If $\tau_L = \tau_K = \tau$, then

$$C(L,K) = P_L(1+\tau_L) + P_K K(1+\tau_K) = [P_L L + P_K K](1+\tau)$$

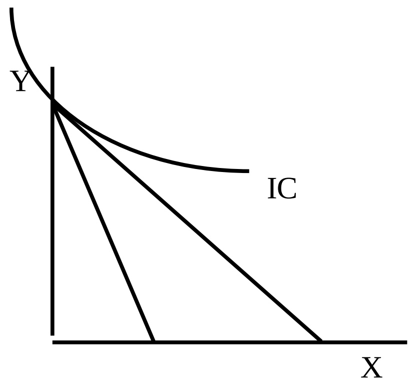
Hence costs uniformly rise for any factor mix. This is identical simply to taxing the firm's good itself that by definition uniformly raises costs for any factor mix.

d. **FALSE/UNCERTAIN**: A tax on X when pre-tax $X_0 > 0$ can lead to X falling to $X_1 = 0$, hence TaR = 0! But that clearly generates EB:



Thus, EV > 0 (the parallel distance between the solid and dotted budget lines at the pre-tax prices), TaR = 0, hence EB = EV > 0.

But, a tax that generates TaR = 0 can also lead to EB = 0 if nothing changes: X = 0 before and after a tax on X implies the same maximum expenditure on Y (since it is not being taxed). Thus nothing changes, and X = 0 means TaR = 0, therefore EB = 0:



Since nothing has changed EV = 0: a zero income change results in the same consumption change as above: nothing. Further, TaR = 0 since X = 0. Combined we get EB = EV - TaR = 0 - 0 = 0.

2. We have $U(X,Y) = .5\ln(X) + .5\ln(Y)$, income I = 60,000, prices $P_x = 20$ and $P_y = 10$. Let $\tau_x = .25$.

a.

$X = .5*60000/P_x(1+\tau)$ = 1500 before tax
= 1200 after tax

$Y = .5*60000/\ P_y$ = 3000 before and after tax

b. EV is the reduction in income required to generate an equivalent drop in income due to a tax.

EV is zero is consumption of both X and Y do not change This is possible, in general, only when one good = 0 leaving the entire income to be spent on the other good (e.g. X = 0, Y = I/P_y the maximum Y amount).

c. From above TaR = $X^*\ P_x{}^*\tau$ = 6000

For EV, we need U_1 = .5*ln(1200) + .5*ln(3000) = 7.548.

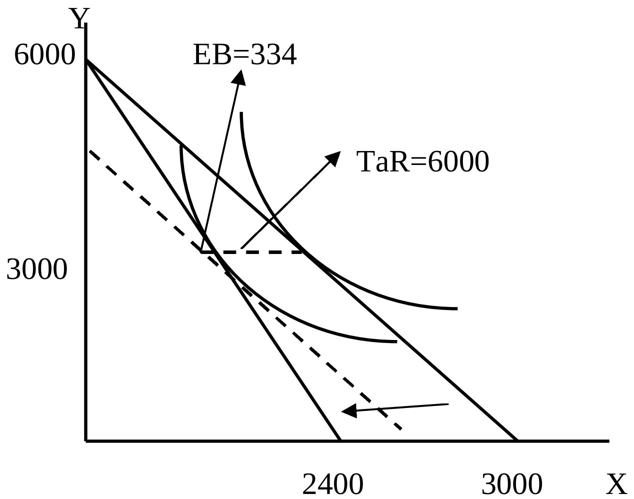
$$
\begin{aligned}
7.548 &= .5\ln\frac{.5\times I_1}{20} + .5\ln\frac{.5\times I_1}{10} \\
&= .5(\ln .5 - \ln 20 + \ln I_1) + .5(\ln .5 - \ln 10 + \ln I_1) \\
&= \ln .5 - .5\ln 20 - .5\ln 10 + \ln I_1 \\
&= -3.34231 + \ln I_1 \\
I_1 &= e^{7.548+3.34231} = 53665.63
\end{aligned}
$$

Therefore

$$\text{EV} = 60000 - 53665.63 = 6334.37$$

We now have EB = EV-TAR = 6334.37-6000=334.37.

d.



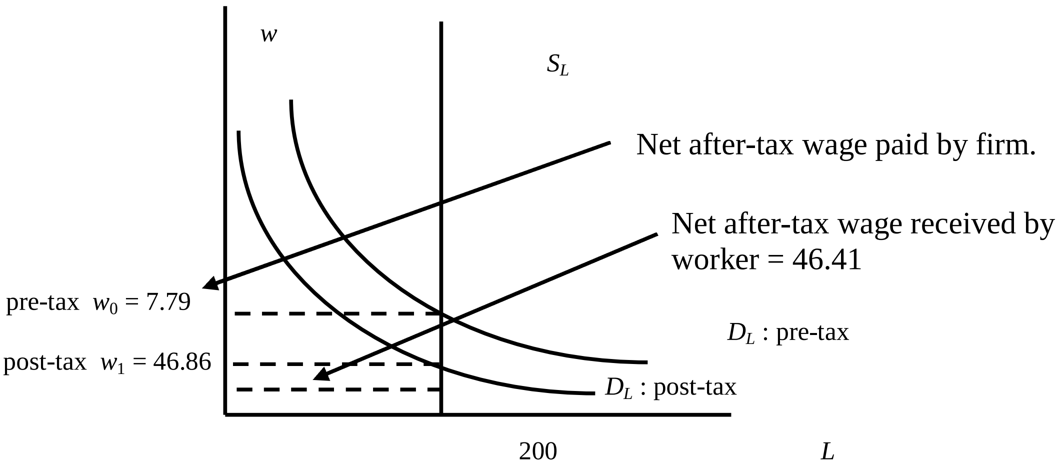
3. (Recall labor *supply* is fixed at $L^* = 150$, capital is fixed at $K_0 = 1$, output is described by a Cobb-Douglas function: $Q(L) = 20L^{1/3}$, the market price of the good is $P = 33$, and $\tau = .08$.

a. Labor demand is willingness-to-pay for labor, which is the value of marginal product of labor:

$$P \times MP_L = P\frac{\partial Q(L)}{\partial L} = P \times \frac{1}{3}\frac{20}{L^{2/3}} = \frac{33 \times 20}{3}\frac{1}{L^{2/3}} = \frac{220}{L^{2/3}}$$

Pre-tax labor demand as willingness-to-pay, and supply, are

$$\text{pre-tax WTP} = P \times MP_L = \frac{220}{L^{2/3}}$$



b. The firm optimizes profit by choosing labor inputs L:

$$\max_L \pi(L) = \max_L P \times Q(L) - C(L) = \max_L 33 \times 20L^{1/3} - wL = \max_L 660L^{1/3} - wL$$

The profit maximization condition is w = *value of marginal product* = $P \times MP_L$:

$$w = P \times MP_L \quad \text{or} \quad w = \frac{220}{L^{2/3}}$$

Since labor supply fixed at $L = 150$ the pre-tax equilibrium wage w_0 satisfies

$$w_0 = \frac{220}{150^{2/3}} = 7.79283$$

c. The firm is willing to pay w_0 = \$7.79/hour for $L = 150$ hours. After the tax $\tau = .08$ the willingness-to-pay drops to $w_1 = w_0/(1+\tau)$ since then the firm pays $w_1(1+\tau) = w_0$. Since wage drops by the entire portion of the payroll tax paid by the firm, economic incidence falls entirely on the worker.

This implies a new labor demand, or "post-tax willingness to pay" by the firm:

$$\text{WTP}: \frac{220}{(1+\tau)L^{2/3}} = \frac{220}{(1+.08)L^{2/3}} = \frac{203.7037}{L^{2/3}}$$

d. The post-tax equilibrium wage:

$$w_1 = \frac{203.7037}{150^{2/3}} = 7.215583$$

The firm pays net 7.215583(1+.08) = 7.79283, and the worker receives net 7.215583 (1-.08) = 6.638336.

Since the firm's net wage is identical to the pre-tax wage, incidence falls entirely o the worker.

4.

a. The price is the present discounted value under perfect competition (notice *r* and *T* are plugged in):

$$P_t^{post-tax} = PDV_t^{post-tax} = A_0 - u_0 + \frac{A_1 - u_1}{1+.02} + \frac{A_2 - u_2}{(1+.02)^2} + \cdots + \frac{A_{20} - u_{20}}{(1+.02)^{20}}$$

$$= \left\{A_0 + \frac{A_1}{1+.02} + \cdots + \frac{A_{20}}{(1+.02)^{20}}\right\} - \left\{u_0 + \frac{u_1}{1+.02} + \cdots + \frac{u_{20}}{(1+.02)^{20}}\right\}$$

This shows clearly the post-tax price is the pre-tax price less the discounted value of the 20 year stream of taxes:

$$P_t^{post-tax} = P_t^{pre-tax} - \left\{u_0 + \frac{u_1}{1+.02} + \cdots + \frac{u_{20}}{(1+.02)^{20}}\right\}$$

b. The price formula is

$$P_t^{pre-tax} = A + \frac{A}{1+t} + \cdots + \frac{A}{(1+t)^T} = A \times \left(\frac{1-\rho^{T+1}}{1-\rho}\right) \text{ where } \rho = \frac{1}{1+r}$$

hence

$$P_t^{pre-tax} = 2500 \times \left(\frac{1-.980392^{21}}{1-.980392}\right) = 43{,}378.58 \text{ where } \rho = \frac{1}{1+.02} = 0.980392$$

The stream of taxes has a PDV

$$\text{PDV(tax)} = u \times \left(\frac{1-\rho^{T+1}}{1-\rho}\right) = 250 \times \left(\frac{1-0.980392^{21}}{1-0.980392}\right) = 4{,}337.858$$

(*somewhat trivially, taxes are 10% of the return A, so the PDV of tax is 10% of the pre-tax price*):

c. The only individual in this economy who pays the tax is the owner at the time the tax is levied. If she never sells the land then she legally pays the tax. If she sells, the market price has already dropped by the present discounted value of tax, hence she pays by receiving a lower market price.