Robust Estimation and Inference for Extremal Dependence in Time Series

Appendix D: Omitted Figures and Tables

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<table>
<thead>
<tr>
<th>TABLE 1 - EVAR (n = 500)</th>
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</thead>
<tbody>
<tr>
<td>Two-Tailed Median ( \hat{q}<em>{m}(h) \pm K</em>{m}(h) )</td>
</tr>
<tr>
<td>( h )</td>
</tr>
<tr>
<td>1</td>
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<tr>
<td>2</td>
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</tbody>
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<table>
<thead>
<tr>
<th>TABLE 2 - SAV (n = 500)</th>
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</thead>
<tbody>
<tr>
<td>Two-Tailed Median ( \hat{q}<em>{m}(h) \pm K</em>{m}(h) )</td>
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<table>
<thead>
<tr>
<th>TABLE 3 - E-VAR (n = 1000)</th>
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</thead>
<tbody>
<tr>
<td>Two-Tailed Median ( \hat{q}<em>{m}(h) \pm K</em>{m}(h) )</td>
</tr>
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<td>( h )</td>
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<td>2</td>
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<tr>
<td>3</td>
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<td>4</td>
</tr>
</tbody>
</table>

| Two-Tailed Median \( \hat{r}_{m}(h) \pm K_{m}(h) \) |
| \( h \) | \( \hat{r}_{m} \pm K_{m} \) | p-val | rej % | \( \hat{r}_{m} \pm K_{m} \) | p-val | rej % | \( \hat{r}_{m} \pm K_{m} \) | p-val | rej % |
| 1 | -.021±.039 | .406 | .01 | .191±.055 | .001 | 1.0 | .270±.058 | .000 | 1.0 |
| 2 | -.002±.039 | .530 | .01 | .176±.053 | .000 | 1.0 | .213±.055 | .000 | 1.0 |
| 3 | .000±.039 | .640 | .02 | .128±.051 | .000 | .96 | .149±.052 | .000 | 1.0 |
| 4 | .001±.039 | .691 | .02 | .080±.050 | .001 | .94 | .105±.050 | .000 | 1.0 |
TABLE 4 - SAV (n = 1000)

<table>
<thead>
<tr>
<th>h</th>
<th>$q_m \pm K_m$</th>
<th>p-val</th>
<th>%rej</th>
<th>$q_m \pm K_m$</th>
<th>p-val</th>
<th>%rej</th>
</tr>
</thead>
<tbody>
<tr>
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<td>.445</td>
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<td>.123±.078</td>
<td>.016</td>
<td>.975</td>
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<td>.552</td>
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<td>.083±.069</td>
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<td>.617</td>
<td>.000</td>
<td>.054±.057</td>
<td>.092</td>
<td>.322</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>h</th>
<th>$r_m \pm K_m$</th>
<th>p-val</th>
<th>%rej</th>
<th>$r_m \pm K_m$</th>
<th>p-val</th>
<th>%rej</th>
</tr>
</thead>
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<td>.000</td>
<td>.105±.06</td>
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<td>.978</td>
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<tr>
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<td>.756</td>
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<td>.064±.05</td>
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<td>.843</td>
</tr>
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Figure 1: Rolling Fractile Two-Tailed $\hat{r}_{\text{Fractile}}(h)$

- **E-VAR: No Spillover**
  - Two-Tailed Tail Dependence Coef. $r(h)$ and Robust Confidence Bands
  - $\text{med} r(1) = -0.014 \pm 0.050$

- **SAV: No Spillover**
  - Two-Tailed Tail Dependence Coef. $r(h)$ and Robust Confidence Bands
  - $\text{med} r(1) = -0.001 \pm 0.063$

- **E-VAR: Strong Spillover**
  - Two-Tailed Tail Dependence Coef. $r(h)$ and Robust Confidence Bands
  - $\text{med} r(1) = 0.273 \pm 0.073$

- **SAV: Spillover**
  - Two-Tailed Tail Dependence Coef. $r(h)$ and Robust Confidence Bands
  - $\text{med} r(1) = 0.115 \pm 0.071$
Figure 2: Rolling Fractile Two-Tailed $\hat{q}_{\alpha^*}(h)$

$n = 500, h = 1...4$

E-VAR: No Spillover
Two-Tailed Exceedance Correlation $q(h)$
and Robust Confidence Bands

$med\ q(1) = -0.006 \pm 0.064$

E-VAR: Strong Spillover
Two-Tailed Exceedance Correlation $q(h)$
and Robust Confidence Bands

$med\ q(1) = 0.336 \pm 0.199$

SAV: No Spillover
Two-Tailed Exceedance Correlation $q(h)$
and Robust Confidence Bands

$med\ q(1) = -0.002 \pm 0.071$

SAV: Spillover
Two-Tailed Exceedance Correlation $q(h)$
and Robust Confidence Bands

$med\ q(1) = 0.126 \pm 0.101$
Figure 3: Two-Tailed Median $\hat{q}_{m,n}(h) \pm \hat{K}_{m,n}(h)$

$n = 500$, $h = 1 \ldots 20$
Figure 4: Rolling Window Two-Tailed Median $\hat{\varphi}_{mn}(1) \pm \hat{K}_{mn}(1): n = 500$

The above figures are plots of the median $\hat{\varphi}_{mn}(1)$ over rolling fractile windows $M_n$. The windows are $M_n = \{5, \ldots, 25\}$ to $M_n = \{5, \ldots, 450\}$, and $M_n = \{345, \ldots, 350\}$ to $M_n = \{5, \ldots, 350\}$. 


Figure 5: Rolling Fractile Two-Tailed Serial $i_{m,1}(1)$

NASDAQ
Two-Tailed First-Order Tail Dependence Coefficient
Median = .10 ± .08

SP500
Two-Tailed First-Order Tail Dependence Coefficient
Aver = .13 ± .09

LSE
Two-Tailed First-Order Tail Dependence Coefficient
Median = .18 ± .05

NIKKEI
Two-Tailed First-Order Tail Dependence Coefficient
Median = .02 ± .04
Figure 6: Equity-to-Equity Rolling Fractile Plots

Two-Tailed $r_{m,n}(1) \pm K_{m,n}$

NASDAQ(t-1) --> SP500(t)
First Order Two-Tailed Extremal $r(1,m)$ Spillover
Median = .17 ± .08

SP500(t-1) --> NASDAQ(t)
First Order Two-Tailed Extremal Volatility $r(1,m)$ Spillover
Median = .093 ± .08

NASDAQ(t-1) --> LSE (t)
First Order Two-Tailed Extremal $r(1,m)$ Spillover
Median = .12 ± .05

SP500(t-1) --> LSE(t)
First Order Two-Tailed Extremal Volatility $r(1,m)$ Spillover
Median = .14 ± .05

NASDAQ(t-1) --> Nikkei(t)
First Order Two-Tailed Extremal $r(1,m)$ Spillover
Median = .05 ± .04

SP500(t-1) --> Nikkei(t)
First Order Two-Tailed Extremal Volatility $r(1,m)$ Spillover
Median = .08 ± .054
Figure 6 Cont.: Equity-to-Equity Rolling Fractile Plots
Two-Tailed $r_{mn}(1) \pm \sigma_{mn}$

- LSE (t-1) $\rightarrow$ NASDAQ(t)
  First Order Two-Tailed Extremal $r_{1,m}$ Spillover
  Median = .09 ± .04

- Nikkei(t-1) $\rightarrow$ NASDAQ(t)
  First Order Two-Tailed Extremal $r_{1,m}$ Spillover
  Median = .01 ± .04

- LSE (t-1) $\rightarrow$ SP500(t)
  First Order Two-Tailed Extremal $r_{1,m}$ Spillover
  Median = .10 ± .04

- Nikkei(t-1) $\rightarrow$ SP500(t)
  First Order Two-Tailed Extremal $r_{1,m}$ Spillover
  Median = .01 ± .04

- LSE (t-1) $\rightarrow$ Nikkei(t)
  First Order Two-Tailed Extremal $r_{1,m}$ Spillover
  Median = .10 ± .04

- Nikkei(t-1) $\rightarrow$ NLSE(t)
  First Order Two-Tailed Extremal $r_{1,m}$ Spillover
  Median = .01 ± .04
Figure 7: NASDAQ Cross-Tailed Median $\hat{r}_{m_{h}}(h) \pm \tilde{K}_{m_{h}}(h)$

Note, $\hat{r}_{m_{h}}(h, 1, 2)$ measures left-to-right tail dependence, and $\hat{r}_{m_{h}}(h, 2, 1)$ measures right-to-left tail dependence.
Figure 7 Cont.: SP500 Cross-Tailed Median $\hat{r}_{mn}(h) \pm \hat{K}_{mn}(h)$

- **SP500 -> NASDAQ**
  - Median Cross-Tailed Extremal
  - Volatility $r(h,1,2)$ Spillover

- **SP500 -> LSE**
  - Median Cross-Tailed Extremal
  - Volatility $r(h,1,2)$ Spillover

- **SP500 -> NIKKEI**
  - Median Cross-Tailed Extremal
  - Volatility $r(h,1,2)$ Spillover
Figure 7 Cont.: LSE Cross-Tailed Median $\hat{r}_{mn}(h) \pm \hat{K}_{mn}(h)$

- LSE --> NASDAQ
  Median Cross-Tailed Extremal Volatility (r(h,1,2) Spillover)

- LSE --> SP500
  Median Cross-Tailed Extremal Volatility (r(h,1,2) Spillover)

- LSE --> NIKKEI
  Median Cross-Tailed Extremal Volatility (r(h,1,2) Spillover)
Figure 7 Cont.: NIKKEI Cross-Tailed Median $\hat{r}_{\text{Mn}}(h) \pm \hat{K}_{\text{Mn}}(h)$

NIKKEI $\rightarrow$ NASDAQ
Median Cross-Tailed Extremal Volatility $r(h,1,2)$ Spillover

NIKKEI $\rightarrow$ LSE
Median Cross-Tailed Extremal Volatility $r(h,1,2)$ Spillover

NIKKEI $\rightarrow$ NASDAQ
Median Cross-Tailed Extremal Volatility $r(h,2,1)$ Spillover

NIKKEI $\rightarrow$ LSE
Median Cross-Tailed Extremal Volatility $r(h,2,1)$ Spillover