

# Homework 2

Math 232 section 006

Due: Thursday, September 5th

1. In your own words, answer the following questions:

(a) What is a sequence?

(b) Explain what it means to say

$$\lim_{n \rightarrow \infty} a_n = \infty.$$

(c) What is a series?

(d) Explain what it means to say

$$\sum_{n=1}^{\infty} a_n = 5.$$

2. Determine if the following series' converge or diverge. If they converge what is the limit?

(a)

$$a_n = \frac{n!}{(n+2)!}$$

(b)

$$\left\{ \frac{2n}{n^2 - 4} \right\}_{n=3}^{\infty}$$

(c)

$$a_n = \sin(1/n)$$

(d)

$$\left\{ \frac{n}{2 + \sqrt{n}} \right\}$$

3. Determine if the following sequences are monotonic:

(a)

$$\left\{ \frac{n}{1+n^2} \right\}_{n=0}^{\infty}$$

(b)

$$\left\{ \frac{2}{3+n} \right\}$$

(c)

$$a_n = n + \frac{1}{n} \quad n \geq 1$$

4. Determine if the following series converge or diverge:

(a)

$$\sum \frac{1}{4n}$$

(b)

$$\sum_{n=0}^{\infty} \frac{2}{3^n}$$

(c)

$$\sum_{n=0}^{\infty} [(1 - 2^n) 3^n]$$

(d)

$$\sum_{n=0}^{\infty} (2^{2n} 7^{2-n} 3^{4+n})$$

5. Express the rational number  $2.121212\dots$  as the fraction of two integers.

6. For which values of  $x$  does

$$\sum_{n=0}^{\infty} \frac{2^n}{x^n}$$

converge? For those values of  $x$  determine the sum.

**Bonus:** Suppose that the series

$$\sum_{n=1}^{\infty} a_n$$

converges and the partial sum  $s_n$  is given by the formula:

$$s_n = \frac{n-1}{n+1}.$$

Find  $a_n$  and the value of

$$\sum_{n=1}^{\infty} a_n.$$