

Tips for Choosing which Test to Use to Determine Convergence or Divergence of a Series

1. Check to see if the series is a p-series or a geometric series. Then determine the convergence or divergence based on facts about these (p-series converges if $p > 1$, geometric series converges if $|r| < 1$ and they diverge otherwise)
2. Check to see what happens to the terms a_n as $n \rightarrow \infty$. If this limit is not zero or does not exist then the series diverges. This is the Test for Divergence.
3. If the series seems similar to a p-series or a geometric series, try and use the Comparison Test. If the inequality between them does not seem to be the right one to come to a conclusion (or you can't figure out which inequality sign to use between the two), use the Limit Comparison Test instead with the b_n coming from the appropriate p-series or geometric series.
4. For series of the form $\frac{\text{polynomial}}{\text{polynomial}}$ or a rational function involving roots of polynomials, try and use the Comparison Test using the reduced form of n to a power (written as $\frac{1}{n^p}$ so that it looks like a p-series, possibly with coefficients in the numerator and denominator) that comes from using the top degree term of the numerator divided by the top degree of the denominator term. Again if it isn't clear which inequality sign goes between them, use the Limit Comparison Test instead. For the Limit Comparison, you don't have to worry about the coefficients, just use the p-series that you get when reducing the powers of n .
5. If the series is of the form $\sum(-1)^n b_n$ or $\sum(-1)^{n+1} b_n$ try and use the Alternating Series Test. Remember this involves showing that $b_{n+1} < b_n$ and $b_n \rightarrow 0$ as $n \rightarrow \infty$. If you cannot show both of these then you must use another test.
6. If you can easily integrate expression for a_n (that is the function $f(x)$ where $f(n) = a_n$) you may want to use the Integral Test. Remember that to use this test, the series must have positive terms and the terms must be decreasing. (You have to show me that the terms decrease.)
7. If the series involves factorials or constants raised to the n power, use the Ratio Test. Remember that the Ratio Test can be used for series that have all positive terms or positive and negative terms. Remember also that the Ratio Test shows that a series is *absolutely convergent*. Then say that because it is absolutely convergent, the series is also convergent.
8. If a series involves sin or cos terms, take the absolute value and use the Comparison Test to show that the absolute value of the original series converges, which is the same thing as saying that the original series is absolutely convergent, hence convergent.

Other Things to Keep in Mind

- For some series, there is more than one test that can be used.
- When you need the fact that a_n or b_n decrease, this has to be true for n greater than some finite number. It's okay if the terms don't decrease initially as long as they do eventually.
- You can ALWAYS use the Ratio Test, but other tests may be easier for certain series. Also, the Ratio Test is sometimes inconclusive.
- If your first test doesn't work, don't give up! Try another.
- When you state that a series converges or diverges, you must support that with work. This involves telling me what test you are using and using it correctly. In other words, if you must show that you can use the test (ex. for Integral Test show the terms are positive and decrease) and then after using the test come to the correct conclusion.