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Econ 400 Summer 2007
Exam 1

Show all of your work, partial credit awarded if I can tell what you are doing. Answer all the questions. The honor code is in effect. This exam counts for a total of 17 points, points are awarded as stated on each problem.

1. (4 pts) Feeling the crunch of high gas prices you head out across Chapel Hill to find the cheapest gas. You visit 6 local gas stations and find the price of a gallon of gas to be \$2.90, \$2.95, \$3.01, \$2.89, \$2.95, \$2.98.

- a. (1/3) What is the median gas price?

$$2.89 \quad 2.9 \quad 2.95 \quad 2.95 \quad 2.98 \quad 3.01 = \underline{2.95}$$

- b. (1/3) What is the mode?

$$2.95$$

- c. (1/3) What is the mean? (round to nearest penny)

$$= 2.95$$

- d. (1) What is the variance and standard deviation

$$\frac{(2.89 - 2.95)^2 + (2.9 - 2.95)^2 + (2.95 - 2.95)^2 + (0)^2 + (2.98 - 2.95)^2 + (3.01 - 2.95)^2}{5}$$

$$= \frac{.0036 + .0025 + 0 + 0 + .0009 + .0036}{5}$$

$$s^2 = .00212$$

$$s = .046$$

- e. (1) If you were holding out until you found gas for \$2.80, discuss how likely it is that you would find this price.

$$\frac{2.8 - 2.95}{.046} = -3.26$$

so this is over 3 standard deviations from the mean

Tsb says $(1 - \frac{1}{(3.26)^2}) = .906$ of the obs should fall within this interval so chances are low.

- f. (1) If you went and did a similar survey of Durham gas stations and found that the average price of gas was \$2.85 per gallon with a standard deviation of ????, which town has more variable gas prices? → show why for full credit

$$CH \cdot \left(\frac{.046}{2.95}\right) \cdot 100\% = 1.56\%$$

$$DR \cdot \left(\frac{.05}{2.85}\right) \cdot 100\% = 1.75\%$$

Durham is more variable

$I = I$ sleeps
 $C = C$ sleeps

$P(I) = .81$

$P(C|I) = .6$

$P(I|C) = .9$

2. (5 pts) Say you head over to your sister's house to spend the weekend and she has 2 (I and C) kids who share a room and are awful sleepers. The probability that (I) sleeps through the night is 0.81. You know that if (I) sleeps through the night then the probability that (C) sleeps through the night is 0.6. You also know that if (C) sleeps through then the probability that (I) sleeps through the night is 0.9.

a. (1) Are (C) and (I) sleeping through the night mutually exclusive events?

Why or why not?

$P(C \cap I) = P(I) P(C|I) = (.81)(.6) = .486 \neq 0$

not mutually exclusive

b. (1) Are (C) and (I) sleeping through the night independent events? Why or why not?

$P(I|\bar{C}) = .4 \neq P(I) = .81$ not independent events

c. (1) What is the probability that (I) sleeps through the night and (C) does not sleep through the night?

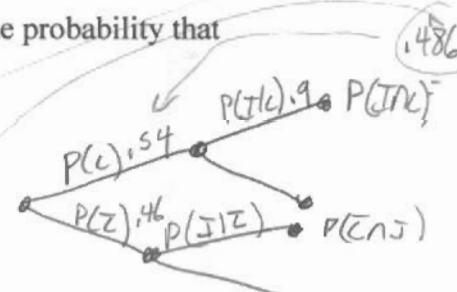
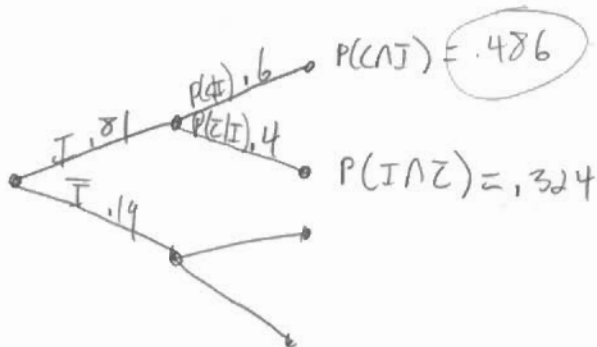
$P(I \cap \bar{C}) = P(I) P(\bar{C}|I) = .81 \cdot .4 = .324$

d. (1) What is the probability that (C) sleeps through the night

$P(C) = \frac{P(C \cap I)}{P(I|C)} = \frac{.486}{.9} = .54$

e. (1) Given (C) does not sleep through the night what is the probability that (I) sleeps through the night?

$P(I|\bar{C}) = \frac{P(I \cap \bar{C})}{P(\bar{C})} = \frac{.324}{.46} = .704$



	I	\bar{I}	
C	.486	.054	.54
\bar{C}	.324	.136	.46
	.81	.19	

20 min

3. (3 pt) North Carolina Lottery time - if there are (n) balls in the hopper and a number is picked that consists of (r) of those balls
- a. (1/4) How many combinations are possible if the chosen ball is put back into the hopper after it has been picked and the order it comes out matters?

$$n^r$$

- b. (1/4) How many combinations are possible if the ball is put back into the hopper after it has been picked and the order it comes out in does not matter?

$$\frac{(n+r-1)!}{r!(n-1)!}$$

- c. (1/4) How many combinations are possible if the ball is not put back into the hopper and the order it comes out matters?

$$P_r^n = \frac{n!}{(n-r)!}$$

- d. (1/4) How many combinations are possible if the ball is not put back into the hopper and the order it comes out does not matter?

$$C_r^n = \frac{n!}{r!(n-r)!}$$

- e. (1) Suppose there are 20 balls and 4 of the numbers are chosen as in part (c), what is the probability that you will have the winning combination?

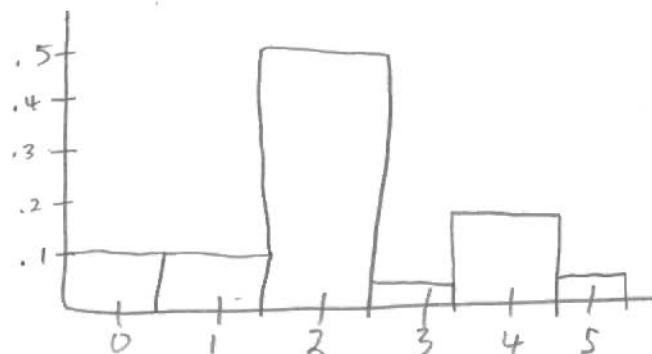
$$n=20 \quad r=4 \quad P_4^{20} = \frac{20!}{(20-4)!} = 116280$$

$$P(\text{win}) = \frac{1}{116280} \approx .0000086$$

- f. (1) If the ticket costs \$10 and the prize is \$1,000,000 what is your expected return on playing the lottery?

$$(1,000,000) \left(\frac{1}{116280} \right) + (-10) \left(1 - \frac{1}{116280} \right) = -1.3999$$

4. (1 pt) Graph the probability distribution of a discrete random variable that can take on the value of 0, 1, 2, 3, 4, and 5 with the respective probabilities of 0.1, 0.1, 0.5, 0.05, 0.2, and 0.05.



5. (3.5 pts) A recent survey found that 23% of Americans believe that American lives are worth more than the lives of those in another country (Pew Research Center, 2007)
- a. (1/2) If (x) is the number of people that agree that American lives are more valuable than those of another country, what distribution does this random variable follow?

Binomial

- b. (1) If you were to ask 10 people what is the probability that 8 of them would say American lives are worth more?

$$n=10 \quad x = \# \text{ that agree} \quad p=.23$$

$$P(x=8) = C_{8}^{10} (.23)^8 (.77)^{10-8} = \frac{10!}{8!(10-8)!} \\ = 45 (.23)^8 (.77)^2 = .0002089$$

- c. (1) What is the expected number and standard deviation of people that would say American lives are worth more?

$$E(x) = 10(.23) = 2.3$$

$$\sigma = \sqrt{10(.23)(.77)} = 1.33$$

- d. (1) Within what range would you expect to find that 0.75 of the variables would fall? (you don't know the shape of the distribution)

Tchebysheff says that $(1 - \frac{1}{2k}) = .75$ of the values will fall within 2 standard deviations of the mean so I expect to see .75 with k at least

$$2.3 \pm 2(1.33) = \{-.36 \quad 4.96\}$$

6. (1.5 pts) On average the number of people who fail Econ 400 per semester is 2.5
- a. (1/2) If (x) is the number of people that fail Econ 400 this semester, what distribution does this random variable follow?

Poisson

- b. (1) What is the probability that no more than 1 person will fail Econ 400 this year?

$x = \# \text{ who fail}$

$$\mu = 2.5$$

$$P(x \leq 1) = P(x=0) + P(x=1)$$

$$P(x \leq 1) = \frac{2.5^0 e^{-2.5}}{0!} + \frac{2.5^1 e^{-2.5}}{1!}$$

$$.0821 + .2052 = .287$$