On Line Appendix 1

Implementing Long-Run Restriction

In order to identify different types of shocks, this paper employs long-run restrictions on structural vector autoregressions (SVARs) as introduced by Blanchard and Quah (1989). Reduced form VAR representations, such as those introduced by Sims (1980), do not allow for instantaneous relationship among the dependent variables. As a result, there is a correlation structure among the error term that is left undetermined. SVAR[1] models have been introduced that use economic restrictions on the reduced form VAR to decouple the correlation structure in the reduced form error. Correct restrictions allow one to identify the previously unidentified structural errors or economic shocks. Impulse response functions can be constructed to look at the reaction of dependent variables to structural shocks.

Unlike the reduced form VAR, which is easily estimated by maximum likelihood or equation by equation OLS, the SVAR allows for contemporaneous responses among the variables and can be represented by

$$A_0 y_t = A_1 y_{t-1} + A_2 y_{t-2} + ... + A_p y_{t-p} + \varepsilon_t \, . \qquad (1)$$

Where (y_t) is a (n_y x 1) vector of explanatory variables and (p) is the number of lags. Each A is a (n x n) matrix and ε_t is an (n_ε x 1) vector of unobservable structural shocks which can have an economic interpretation with $E(\varepsilon_t \varepsilon_t ') = \sum_\varepsilon$. From this representation, it is not clear how to estimate all of the parameters in the SVAR. Pre-multiplying by A_0, however, allows the structural VAR to be written as an easily estimated reduced form VAR

$$y_t = \Phi_1 y_{t-1} + \Phi_2 y_{t-2} + ... + \Phi_p y_{t-p} + e_t \, , \qquad (2)$$

where e_t = $A_0^{-1}\varepsilon_t$ and has an independent multivariate normal distribution with a covariance matrix $E(e_t e_t ') = \sum_e$.

The SVAR (1) has $(p+1)\cdot n^2$ parameters along with $\left(\frac{n\cdot(n+1)}{2}\right)$ variances and covariances to estimate. The reduced form VAR (2) only provided estimates for

[1] More in depth information on SVARs can be found in Amisano and Gianni (1997) and Hamilton (1994)

$(p \cdot n^2)$ parameters and $\left(\frac{n \cdot (n+1)}{2}\right)$ variances and covariances. In order to identify the structural shocks, (n^2) restrictions must be placed on the system. The first restriction, common to all SVARs, is to assume that structural shocks are independent of each other. Thus, Σ_ε is a diagonal matrix, which provides $\left(\frac{n(n-1)}{2}\right)$ restrictions. In addition, the main diagonal of A_0 is normalized to be one, providing (n) more restrictions. This leaves just $\left(\frac{n(n-1)}{2}\right)$ more restrictions that need to be imposed on the SVAR.

There have been many different types of restriction suggested in order to complete the identification of the structural shocks. A summery of different types of restriction can be found in Chirstiano, Eichenbaum, and Evans (1999), but this paper will concentrate on the long-run restrictions introduced by Blanchard and Quah (1989).

To use long-run restrictions, rewrite the SVAR in its moving average representation

$$y_t = C(L) \cdot \varepsilon_t . \qquad (3)$$

Where (L) is the lag operator and C(L) is a matrix polynomial that can be of infinite order. In order for the identification to work one must assume that 1) C(L) is invertible i.e. det[C(L)]=0 has all of its roots outside the unit circle, 2) the number of shocks evaluated is equal to the number of parameters used in the estimation, $n_y=n_\varepsilon=n$, 3) A_0 is invertible, and 4) the infinite representation VAR(∞) can be approximated with a VAR(p) that sufficiently captures the serial correlation in the data.

Because the restrictions placed on the variable have to do with the long run, (3) needs to be written as its infinite representation

$$y_t = C(1) \cdot \varepsilon_t . \qquad (4)$$

Where C(1) is and (n x n) matrix of the sum of the infinite order VMA coefficients from the Wold decomposition of the VAR, i.e. $\sum_{i=0}^{\infty} C_i$. This representation shows the effect a structural shock will have on the independent variables (y_t) in the long run. At this point one must turn to economic theory to make the final restrictions. In the two variable case one additional restriction is needed. Using output and prices, basic theory of aggregate supply and aggregate demand can be used. The long-run aggregate supply curve is vertical so a structural demand shock has no long-run effect on output. This means that if output is ordered first in the VAR then the upper right entry of the C(1) matrix is restricted to be zero.

The restrictions now allow for structural interpretations based on the reduced form estimation of the VAR. To illustrate how this is done, the reduced form VAR (2) is rewritten as

$$\Phi(1)\, y_t = e_t \,, \tag{5}$$

where Φ_0 is the identity matrix. Stationarity allow one to write (5) as:

$$y_t = \left(\Phi(1)\right)^{-1} e_t \,. \tag{6}$$

Equating the structure VMA (4) to (6), a relationship between the structural and reduced form error terms is established

$$C(1) \cdot \varepsilon_t = \left(\Phi(1)\right)^{-1} e_t \tag{7}$$

Taking the variance of this expression and recalling that the variance of the structural error term is the identity matrix we get:

$$C(1) \cdot C(1)' = \Phi(1)^{-1} \Sigma_e \Phi(1)^{-1}\,' \,. \tag{8}$$

Based on the restriction taken from economic theory C(1) can now be estimated. The restrictions in our two variable AS/AD framework entail that C(1) is lower triangular[2]. A Choleski factorization of $\Phi(1)^{-1} \Sigma_e \Phi(1)^{-1}\,'$, which is available from the reduced form VAR, provides a lower triangular C(1). The structural error terms, or shocks, can now be identified from (7):

$$\varepsilon_t = C(1)^{-1} \Phi(1)^{-1} e_t \,. \tag{9}$$

In summary, the following procedure is used. The reduced form VAR is estimated and its residuals are recorded. The variance covariance matrix is then estimated, Σ_e. The VAR is then inverted to obtain $\Phi(1)^{-1}$. C(1) is then estimated using a Choleski factorization on $\Phi(1)^{-1} \Sigma_e \Phi(1)^{-1}\,'$. C(1) is then used to calculate the different types of shock according to (9). In addition, the estimation of C(1) make calculations of impulse response functions possible according to (4).

[2] See Shapiro and Watson (1998) for estimation using long-run restriction without a lower triangular relationship

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