chapter 6 key

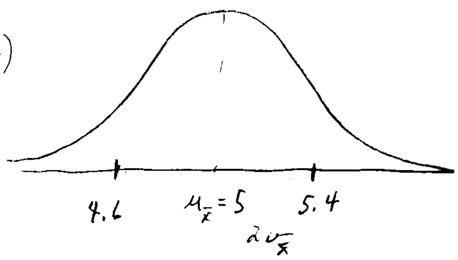
1 a) $E(\bar{x}) = 10$ $\sigma_{\bar{x}} = \frac{\sqrt{9}}{\sqrt{25}} = 3/5$ b) $E(\bar{x}) = 5$ $\sigma_{\bar{x}} = \frac{\sqrt{4}}{\sqrt{100}} = .2$

c) $E(\bar{x}) = 120$ $\sigma_{\bar{x}} = \frac{\sqrt{1}}{\sqrt{6}} = 1/\sqrt{6}$

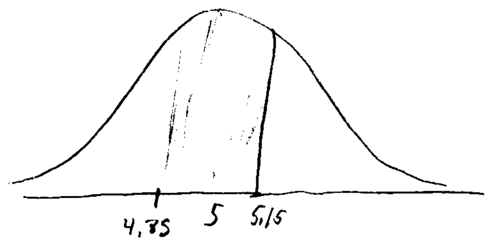
2 a) if x is normal then we know $\bar{x}$ will always be distributed normally

b) we could assume a and b sample distributions are normal b/c $n \geq 25$ but we could not say anything abut the sample distribution in c

3 a)



b)



c) $P[4.85 < x < 5.15] = P\left(\frac{4.85-5}{.2} < z < \frac{5.15-5}{.2}\right) = .75 < z < .75 = 2(.2734) = .5468$

c) $P[4.6 < \bar{x} < 5.4) = P\left[\frac{4.6-5}{.2} < z < \frac{5.4-5}{.2}\right] = -2 < z < 2 = 2(.4772) = .9544$
empirical rule

13 a) we don't know if original sample is normal but $\mu = 26$ $\sigma = 12$

we do know that the sampling distribution of $\bar{x}$ is approximatly normally distributed with $\mu_{\bar{x}} = 26$ $\sigma_{\bar{x}} = \frac{12}{\sqrt{35}} = 2.028$

so $P(25 < x < 27) = P\left(\frac{-1}{2.028} < x < \frac{1}{2.028}\right) = 2P(x < .49) = 2(.1879) = .3758$

b) No, there are many possible values for x, the actual % tax savings, as given by the prob distribution for x.

14 The small average increases are significant b/c of the Larg # of students grades used to calculate the average grade - n is large $\sigma_{\bar{x}}$ is small

16 a) because the original strengh measurements are normally distributed we know the sample mean $\bar{x}$ will be also

$n=10 \qquad \sigma = 2 \qquad \sigma_{\bar{x}} = \frac{2}{\sqrt{10}}$

b) $\mu = 21$ so $\mu_{\bar{x}} = 21 \qquad P(\bar{x} < 20) = P\left(z < \frac{20-21}{.632}\right) = .5 - P(z < 1.58)$ .4429

$= .0571$

.499

.001

-3.08

c) $P(\bar{x} < 20) = .001 \Rightarrow P\left(z < \frac{20-\mu_0}{.632}\right) = .001$

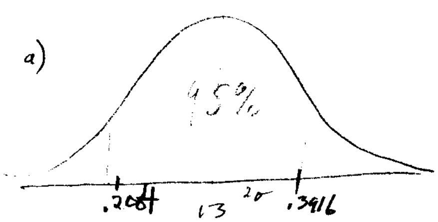
$3.08 = \frac{20-\mu_0}{.632} \Rightarrow \mu_0 = 21.948$

18 a) $\mu_{\hat{p}} = .3 \qquad \sigma_{\hat{p}} = \sqrt{\frac{(.3)(.7)}{100}} = .0458 \qquad$ b) $\mu_{\hat{p}} = .1 \qquad \sigma_{\hat{p}} = \sqrt{\frac{(.1)(.9)}{400}} = .015$

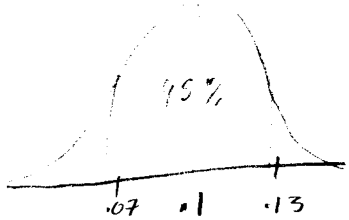
c) $\mu_{\hat{p}} = .6 \qquad \sigma_{\hat{p}} = \sqrt{\frac{.6 \cdot .4}{250}} = .0310$

n large so approximatly normal

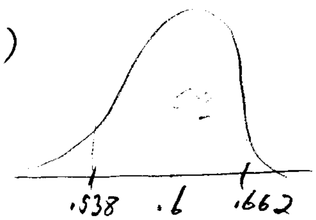
19 a)



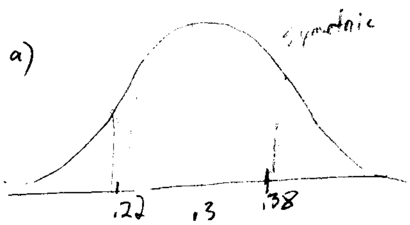
b)



c)



20 a)



b) $P(.22 < \hat{p} < .38) = P\left(\frac{-.08}{.0458} < z < \frac{.08}{.0458}\right)$

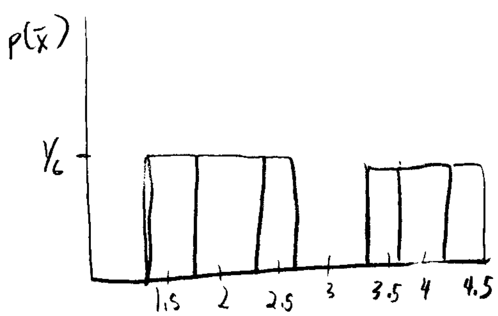
$= 2P(z < 1.75) = 2(.4599) = .9198$

35 a) $\binom{4}{2} = \frac{4!}{2!2!} = 6 \qquad$ b–c)

obs	$\bar{x}$
6,1	3.5
6,3	4.5
6,2	4
1,3	2
1,2	1.5
3,2	2.5

d)

$\bar{x}$	$P(\bar{x})$
3.5	1/6
4.5	1/6
4	1/6
2	1/6
1.5	1/6
2.5	1/6

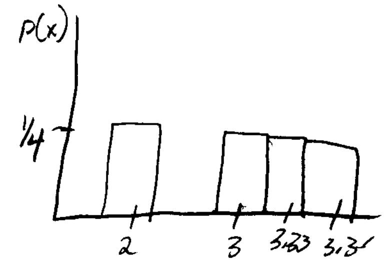


e) $\mu = \frac{6+1+3+2}{4} = 3 \qquad$ no sample will produce exactly the pop mean when $n=2$

36 $\binom{4}{3} = \frac{4!}{3!(1!)} = 4$

obs	$\bar{X}$
6, 1, 3	10/3
6, 1, 2	3
6, 3, 2	11/3
1, 3, 2	2

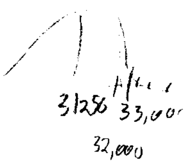
$\bar{X}$	$P(x)$
10/3	1/4
3	1/4
11/3	1/4
2	1/4



42 a) we can assume the sample mean is approximitally normally distributed through the central limit theorem (CLT)

$\mu = 31{,}256 \quad \sigma = 1550 \quad n = 25$

a) $\mu_{\bar{x}} = 31{,}256 \qquad \sigma_{\bar{x}} = \frac{1550}{\sqrt{25}} = 310$



c) $P(\bar{x} > 32{,}000) = .5 - P\left(z < \frac{32{,}000 - 31{,}256}{310}\right) = .5 - P(z < 2.4) = .0104$ (.4918)

$P(\bar{x} > 33000) = .5 - P\left(z < \frac{33000 - 31256}{310}\right) = .5 - P(z < 5.63) \approx 0$

d) within 2 standard deviations (emperical rule of thumb)

$2(310) = 620$

$31{,}256 \pm 620$

26. a) $n = 250$ $p = .5$ b/c 112,000 is the median so half must be over it

so $\hat{p} \sim N(.5, .03162)$ $\sigma_{\hat{p}} = \sqrt{\frac{(.5)(.5)}{250}} = .03162$
s.e.

b) $P(\hat{p} > .66) = P\left(\hat{p} > \frac{.66 - .5}{.03162}\right) = P(\hat{p} > 5.06) \approx 0$

c) this would be very unlikely - it is possible that
or 1) $p = .5$ is wrong
2) our sample is not random

43 a) $n = 1000$ $p = .86$ $\hat{p} = .86$ $\sigma_{\hat{p}} = \sqrt{\frac{(.86)(.14)}{1000}} = .01097$ $\hat{p} \sim N(.86, .01097)$
s.e.

b) $P(.84 < \hat{p} < .88) = P\left(\frac{-.02}{.01097} < z < \frac{.02}{.01097}\right) = P(-1.82 < z < 1.82)$
$= 2(.4656) = \underline{.9312}$

c) $P(\hat{p} > .9) = P\left(z > \frac{.9 - .86}{.01097}\right) = P(z > 3.65) \approx 0$
not likely