7.5 - 7.7

16 a) 2.015 b) 2.306 c) 1.33 d) 1.96

19 a) $n = 12$ so $d.f. = 11$ $\alpha = .02$ $t_{.01} = 2.718$ $\mu = \bar{x} = 125.12$

$$125.12 \pm 2.718 \frac{12.3}{\sqrt{12}} = \{125.12 \pm 9.661\}$$

b) $\alpha = .01$ $t_{.005} = 3.106$ $125.12 \pm 3.106 \frac{12.3}{\sqrt{12}} = \{114.092, 134.771\}$

c) intervals constructed will enclose μ 98% (99%) of the time in repeated sampling.

21 $$\bar{x} = \frac{8710 + 6370 + 9620 + 8200 + 10350}{5} = 8660$$

$$s^2 = \frac{(8710 - 8660)^2 + (6370 - 8660)^2 + (9620 - 8660)^2 + (8200 - 8660)^2 + (10350 - 8660)^2}{4} = 2055760$$

$\alpha = .05$ $t_{.025}$ at 4 d.f. $= 2.776$ so $8660 \pm 2.776 \sqrt{\frac{2055760}{5}}$

$= 8660 \pm 1780$

$= \{6880, 10440\}$

b) because 8500 is included in our range we cannot say that for certain that average profit will exceed 8500

23 entering the set into STATA and using the summary command I find

$\bar{x} = 15.6$ $s = 1.48728$

a) $\alpha = .02$ $t_{.01}$ at 10 d.f. $= 2.764$

$$15.6 \pm 2.764 \frac{1.48728}{\sqrt{11}} = \{12.836, 16.84\}$$

.239

b) no, projections are never completely accurate, rarely are they exactly equal

25 $\bar{X}_1 - \bar{X}_2 = 2.9 - 5.1$ $\alpha = .1$ $z_{.05} = 1.645$ large sample sd

$$-2.2 \pm 1.645\sqrt{\frac{.83}{64} + \frac{1.67}{64}} = \{-2.525, -1.875\}$$

- intervals constructed in this manner will inclose $(\mu_1 - \mu_2)$ 90% of the time

28 $n1 = 4$ $\bar{X}_1 = 7$ $s_1 = 3.916$ $s_1^2 = 15.335$ from STATA

$n2 = 5$ $\bar{X}_2 = 8.6$ $s_2 = 3.209$ $s_2^2 = 10.297$

$$s_p^2 = \frac{(4-1)15.335 + (5-1)10.297}{(4-1)+(5-1)} = 12.46$$

29 $\alpha = .05$ $d.f. = 4+5-2 = 7$ $t_{.025} = 2.365$

$$(7 - 8.6) \pm 2.365\sqrt{12.46}\left(\frac{1}{4} + \frac{1}{5}\right)^{1/2} = -1.6 \pm 5.6 = \{-7.2, 4\}$$

31 $\alpha = .1$ n1 and n2 large $z_{.05} = 1.645$

$$(2.4 - 3.1) \pm 1.645\sqrt{\frac{1.44}{100} + \frac{2.64}{100}} = -.7 \pm .332 \quad \{-1.032, -.368\}$$

intervals constructed in this manner will enclose $(\mu_1 - \mu_2)$ 90% of the time.

34 $n1 = 57$ $\bar{X}_1 = 78{,}100$ $s_1 = 6300$ n1 and n2 are large enough

$n2 = 66$ $\bar{X}_2 = 82{,}700$ $s_2 = 7100$ $\alpha = .02$ $z_{.01} = 2.326$

$$(78{,}100 - 82{,}700) \pm 2.326\sqrt{\frac{6300^2}{57} + \frac{7100^2}{66}} = \{7410.62, -1789.38\}$$

-4600 ± 2810.62

35 $n1 = 5$ $\bar{x}_1 = 3.44$ $s_1 = .4159$ from STATA $\alpha = .1$ small sample so

$n2 = 5$ $\bar{x}_2 = 3.68$ $s_2 = .3114$ $df = 5+5-2 = 8$ $t^{8}_{.05} = 1.86$

$$s^2 = \frac{(5-1).4159^2 + (5-1).3114^2}{5+5-2} = .13497$$

$$(3.44 - 3.68) \pm 1.86 (\sqrt{.135}) \left(\frac{1}{5} + \frac{1}{5}\right)^{1/2} = \{-.672, .192\}$$

$-.24 \pm .432$

78 $n1 = 11$ $\bar{x}_1 = 60.4$ $s_1^2 = 31.4$ $\alpha = .1$ small sample so

$n2 = 11$ $\bar{x}_2 = 65.3$ $s_2^2 = 44.82$ $df = 11+11-2 = 20$ $t^{20}_{.05} = 1.725$

$$s^2 = \frac{(11-1)\,31.4 + (11-1)\,44.82}{11+11-2} = 38.11$$

$$(60.4 - 65.3) \pm 1.725 (\sqrt{38.11}) \left(\frac{1}{11} + \frac{1}{11}\right)^{1/2} = \{-9.44, -.36\}$$

-4.9 ± 4.54

note: on choosing n – if you know a Range [illegible] equal to 4σ so $4\sigma \approx R$ $\sigma \approx R/4$

7.7 – 7.9

37 $n = 300$ $x = 263$ $\hat{p} = .877$ $\sigma_{\hat{p}} = \sqrt{\frac{.877(.123)}{300}} = .01896$

$\alpha = .1$ $z_{\alpha/2} = 1.645$

$.877 \pm 1.645(.01896) = .877 \pm .031$ $\{.846, .908\}$

44 a) $\hat{p} = .57$ $n = 1002$ $\sigma_{\hat{p}} = \sqrt{\frac{.57(.43)}{1002}} = .016$

$1.96(.016) = .031$

b) $\hat{p} = .09$ $n = 1002$ $\sigma_{\hat{p}} = \sqrt{\frac{.09(.91)}{1002}} = .009$

$1.64(.009) = .0147$

c) $\hat{p} = .37$ $n = 1002$ $\sigma_{\hat{p}} = \sqrt{\frac{(.37)(.63)}{1002}} = .015$

$\alpha = .05$ $z_{\alpha/2} = 1.96$

$.37 \pm \underbrace{1.96(.015)}_{.03} = \{.34, .4\}$

48 $n_1 = 315$ $n_2 = 207$ $x_1 = 108$ $x_2 = 102$ $\hat{p}_1 = \frac{108}{315} = .344$ $\hat{p}_2 = \frac{102}{207} = .493$

a) $\alpha = .05$ $z_{\alpha/2} = 1.96$

$(.344 - .443) \pm 1.96\sqrt{\frac{.344(.493)}{315} + \frac{(.443)(.507)}{207}} = -.149 \pm .086$

with 95% confidence the population difference in proportions will lie between −.235 and −.063

b) samples must be independent, sample size is large

50 a) $\hat{p}_1 = .1$ $n_1 = 5000$ $\hat{p}_2 = .08$ $n_2 = 5000$ $\alpha = .05$ $z_{\alpha/2} = 1.96$

$(.1 - .08) \pm 1.96\sqrt{\frac{(.1)(.9)}{5000} + \frac{(.08)(.92)}{5000}} = .02 \pm .011$ $\{.009, .031\}$

b) the sample sizes are independent

53 $\sigma = 12.7$ $D = 1.6$ $\alpha = .05$

$$1.96\left(\frac{12.7}{\sqrt{n}}\right) \le 1.6 \qquad \sqrt{n} \ge \frac{1.96(12.7)}{1.6} = 15.56 \qquad n \ge 242.04 \text{ or } 243$$

57 $\sigma = R/4$ $\sigma = 100{,}000/4 = 25{,}000$ $D = 5000$ assume $\alpha = .05$ (you do any whatever you want)

$z_{\alpha/2} = 1.96$

$$1.96\left(\frac{25000}{\sqrt{n}}\right) \le 5000$$

$$\sqrt{n} \ge \frac{1.96(25000)}{5000} \qquad n \ge 96.04 \text{ or } n \ge 97 \text{ obs}$$

58 $D = 1000$ $\alpha = .05$ difference between means from 7.34 $\sigma_1 \approx 6300$, $\sigma_2 \approx 7100$

$z_{\alpha/2} = 1.96$

$$1.96\sqrt{\frac{6300^2 + 7100^2}{n}} \le 1000$$

$$\sqrt{n} \ge \frac{1.96\sqrt{6300^2+7100^2}}{1000} \Rightarrow n \ge 346.128 \quad n \ge 347$$

60 $D = .04$ $\alpha = .05$ $z_{\alpha/2} = 1.96$ we will use $p_1 = p_2 = .5$ difference in proportion

$$1.96\sqrt{\frac{(.5)(.5)+(.5)(.5)}{n}} \le .04 \qquad \sqrt{n} \ge \frac{1.96\sqrt{.5}}{.04}$$

$n \ge 1200.5$

$n \ge 1201$ in each market

63 $D = .03$ $\alpha = .05$ $z_{\alpha/2} = 1.96$ use $p = .5$ to get max n estimating proportion

$$1.96\sqrt{\frac{(.5)(.5)}{n}} \le .03 \qquad \sqrt{n} \ge \frac{1.96\sqrt{.25}}{.03} \qquad n \ge 1067.11 \qquad n \ge 1068$$

85 a) $n = 400$ $x = 25$ $\hat{p} = 25/400 = .0625$

$$1.96\sqrt{\frac{(.0625)(.9375)}{400}} = .0237$$

b) $D = .02$ $\hat{p} = .0625$ use $\alpha = .05$ $z_{\alpha/2} = 1.96$

$$1.96\sqrt{\frac{(.0625)(.9375)}{n}} \le .02$$

$$\sqrt{n} \ge \frac{1.96\sqrt{.0586}}{.02} \qquad \sqrt{n} \ge 23.72 \qquad n \ge 562.73$$

$n \ge 563$ obs

87 $D = 500$ $\quad \alpha = .05$ $\quad z_{\alpha/2} = 1.96$ $\quad R = 13000 - 4800 = 8200$

$s = \frac{8200}{4} = 2050$

$$1.96 \frac{2050}{\sqrt{n}} \le 500$$

$$\sqrt{n} \ge \frac{1.96(2050)}{500} \qquad \sqrt{n} \ge 8.036 \qquad n \ge 64.577$$

$$n \ge 65$$