

NAME _____

Honor _____

Exam 2

Answer each question. **Show your work**, even if you do most of the answer on a calculator you need to set up the problem and show me what you were using to get that answer. **Circle your final answer**. The honor code is in affect.

1. The time you have to wait in order to get out of the parking lot after the July 4th fireworks show is exponentially distributed with a mean of 30 minutes.

- a. What is the probability that your wait will be less than 20 minutes long?

$$\mu = 30$$

$$30 = \frac{1}{\lambda} \quad \lambda = \frac{1}{30}$$

$$\begin{aligned} P(X < 20) &= 1 - P(X > 20) \\ &= 1 - e^{-\frac{1}{30}(20)} \\ &= .487 \end{aligned}$$

space

- b. What is the wait time that insures that with a probability of .8 you will still be waiting in the parking lot.

$$x?$$

$$P(X > x_0) = .8$$

$$e^{-\frac{1}{30}x} = .8$$

$$-\frac{1}{30}x = \ln .8$$

$$x = -30(\ln .8)$$

$$x_0 = 6.69 \text{ minutes}$$

2. A recent survey of parents showed that 55% of them gave their children a weekly allowance

sample a. If you were to go and sample 100 parents and ask them whether they give an allowance, would it be proper to estimate this binomial distribution with a normal distribution? Why?

$x = \#$ of parents who give allowance
 give allowance don't
 $p = .55$ $q = .45$
 Binomial

yes because $100(.55) = 55$
 and $100(.45) = 45$
 both are greater than 5

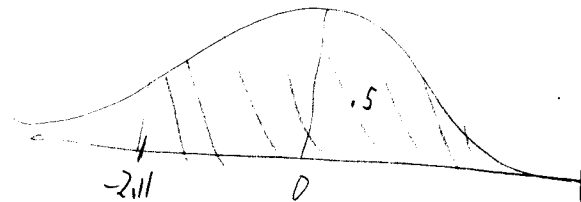
b. In our sample, what is the expected number of parents that give allowances and the standard deviation?

$$E(x) = 100(.55) = 55$$

$$\sigma = \sqrt{100(.55)(.45)} = 4.97$$

c. Using the normal approximation what is the probability that at least 45 of the parents surveyed would give their children a weekly allowance?

$$\begin{aligned}
 P(x \geq 45) &= P(x > 44.5) \\
 &= P\left(z > \frac{44.5 - 55}{4.97}\right) \\
 &= P(z > -2.11) \\
 &= .5 + .4826 \\
 &= .9826
 \end{aligned}$$



3. At the local Weight Watchers it was found that out of the last group that completed the program the average weight loss was 8 pounds with a standard deviation of 3.3 pounds.

$$\bar{x} = 8$$
$$s = 3.3$$

a. Why can we not use the z-distribution to determine a confidence interval?
the sample size is too small so we cannot guarantee that the sample distribution is normal

b. In what way(s) is/are the t-distribution different^{/similar} than the z-distribution?
the t-distribution has thicker tails, so it is a little flatter depending on the size of n, if n is large enough then they are basically the same

c. What is our estimate for the average weight loss of the whole population of those who participate in Weight Watchers? Is this the actual average weight loss of the population? Why or why not?

$$\mu = \bar{x} = 8 \text{ pounds}$$

- it is not necessarily the actual population mean because \bar{x} is a random variable which has a distribution from which it is drawn.

d. Construct a lower one-sided ^{95%} confidence interval above which the average weight loss will lie.

$$\alpha = .05 \quad \text{d.f.} = 9 \quad t_{.05} = 1.833$$

$$8 - 1.833 \left(\frac{3.3}{\sqrt{10}} \right) = 6.087$$

e. Interpret your one-sided interval found in part d.

with 95% confidence the actual average weight loss of those participating in weight watchers will be greater than 6.087 pounds.

4. In a survey of weekly allowances it was found that average weekly allowance for an African American child was \$11.30 with a sample variance of \$8.15. A white child's average weekly allowance was \$9.20 with a sample variance of \$8.00. If these statistics were based on a random sampling of 100 parents from each ethnicity answer the following questions.

$$\begin{aligned} \bar{x}_1 &= 11.30 & \bar{x}_2 &= 9.20 \\ s_1^2 &= 8.15 & s_2^2 &= 8 \\ n_1 &= 100 & n_2 &= 100 \end{aligned}$$

- a. From this sample what is the estimated difference between African American and white children's average weekly allowance?

$$\mu_1 - \mu_2 = \hat{\mu}_1 - \hat{\mu}_2 = (\bar{x}_1 - \bar{x}_2) = 11.30 - 9.20 = \$2.1$$

- b. Find a 95% confidence interval for the difference between average allowances.

$$\alpha = .05 \quad z_{\alpha/2} = 1.96$$

$$2.1 \pm 1.96 \sqrt{\frac{8.15}{100} + \frac{8}{100}} = 2.1 \pm 0.788 = \{1.312, 2.888\}$$

- c. If someone were to ask you whether the difference between allowances was zero, what would you say using your confidence interval?

because zero is not in my confidence interval I can be 95% confident that the difference between these means is actually not 0

- d. If you were only able to survey 10 African American parents and 15 white parents what value on the t-table would you use to construct a 95% two-confidence interval around the differences in the sample mean?

$$\begin{aligned} n_1 &= 10 & \text{d.f.} &= 10 + 15 - 2 = 23 & t_{.025} &= 2.069 \\ n_2 &= 15 & \alpha &= .05 \end{aligned}$$

- e. What would be the 95% confidence interval with our small sample from part d if the means and standard deviations did not change with the smaller sample?

$$s^2 = \frac{(10-1)8.15 + (15-1)8}{(10-1) + (15-1)} = 8.059$$

$$s = \sqrt{8.059} = 2.84$$

$$2.1 \pm (2.069)(2.84) \sqrt{\frac{1}{10} + \frac{1}{15}}$$

$$2.399$$

$$2.1 \pm 2.399$$

$$\{-.299, 4.499\}$$

n_1 and n_2 are large enough so I use the z distribution