

## Key Problem Set #1

24 a)  $\bar{x} = \frac{3+5+4+6+10+5+6+4+2+8}{10} = 5.8$

b) 2 3 4 5 5 6 6 8 9 10      $m = 5.5$   
↑  
m

c) mode from part b we can see is 5 and 6

30 a)  $\bar{x} = \frac{10+9+13+8+9+10+10+6+5+11}{10} = 9.1$

b) 5 6 8 9 9 10 10 10 11 13      $m = 9.5$   
↑  
m

c)  $\bar{x}$  and  $m$  are similar enough that it doesn't really matter what you use to quantify the central tendencies

34 a)  $\bar{x} = \frac{2+4+0+3+1}{5} = 2$

b)  $s^2 = \frac{(2-2)^2 + (4-2)^2 + (0-2)^2 + (3-2)^2 + (1-2)^2}{5-1} = 2.5$       $s = \sqrt{2.5} \approx 1.58$

c)  $CV = \frac{1.58}{2} \times 100 = 79\%$

35  $\bar{x} = 36$       $s = 3$

a) 68% will fall between 33 and 39  
 95% of observations will fall between 30 and 42  
 almost all will fall between 27 and 45

b) we could only say that at least 0% of the obs fall between 33 and 39  
 at least  $1 - (1/2)^2 = 3/4$  of the obs will fall between 30 and 42  
 at least  $1 - (1/3)^2 = 8/9$  of the obs will fall between 27 and 45

44 b) by putting into STATA (using the summarize command)

$$\bar{x} = 26.23 \quad s = 26.22$$

$$c) \quad 26.23 \pm 26.22 = (.01 \text{ to } 52.45)$$

44 out of 50 lie in this interval 88%

$$26.23 \pm 2*(26.22) = (-26.21 \text{ to } 78.67)$$

47 out of 50 lie in this interval 94%

$$26.23 \pm 3*(26.22) = (-52.43 \text{ to } 104.89)$$

49 out of 50 lie in this interval 98%

- this matches with Tchebysheff's theorem where the % needed to be at least 0% 75% and 89% respectively

- this does not match the empirical rule where we would expect 68%, 95%, almost all respectively

53 a)  $\bar{x} = \frac{3+9+6+5+5+4+7+6+8+2+6+7+3}{13} \approx 5.46$

$$s^2 = \frac{(3-\bar{x})^2 + (9-\bar{x})^2 + (6-\bar{x})^2 + 2(5-\bar{x})^2 + (4-\bar{x})^2 + (7-\bar{x})^2 + 2(6-\bar{x})^2 + (8-\bar{x})^2 + (2-\bar{x})^2 + (3-\bar{x})^2}{12}$$

$$\approx 4.269 \quad \text{so } s \approx \sqrt{4.269} \approx 2.066$$

b)  $z_{\text{smallest}} = \frac{2-5.46}{2.066} = -1.67$  - we are within 2 s.d. of the mean so this is reasonable

$z_{\text{largest}} = \frac{9-5.46}{2.066} = 1.713$  - still within 2 s.d. of the mean so reasonable

58  $\bar{x} = 410 \quad s = 14$

$z_{\text{score}} = \frac{430-410}{14} = 1.43$  - this is not unusually high because it is still within 2 s.d. of the mean

$$77 \quad a) \quad \bar{X}_{13} = \frac{10 + 9 + 13 + 8 + 4 + 10 + 10 + 6 + 5 + 11}{10} = 9.1$$

$$s_{13}^2 = \frac{.9^2 + .1^2 + 3.9^2 + 1.1^2 + .1^2 + .9^2 + .9^2 + 3.1^2 + 4.1^2 + 1.9^2}{9} = 5.433 \quad s = 2.33$$

$$\bar{X}_{30} = \frac{28 + 21 + 28 + 26 + 17 + 12 + 15 + 9 + 20 + 18}{10} = 19.4$$

$$s_{30}^2 = \frac{8.6^2 + 1.6^2 + 8.6^2 + 6.6^2 + 2.4^2 + 7.4^2 + 4.4^2 + 10.4^2 + .6^2 + .6^2}{9} = 42.711 \quad s = 6.535$$

$$b) \quad CV_{13} = \left( \frac{2.33}{9.1} \right) \times 100 = 25.6\%$$

$$CV_{30} = \left( \frac{6.535}{19.4} \right) \times 100 = 33.7\%$$

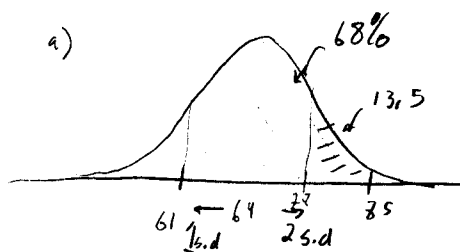
c) the 30 gallon bags are more variable

$$84. \quad \bar{x} = 69 \quad s = 8$$

a) notice that 53 and 85 are 2 standard deviations (16) away from the mean so by Chebyshev we know at least 75% of the CD prices will be between 53 and 85

b) if at least  $\frac{3}{4}$  are in the interval then at most  $\frac{1}{4}$  are outside the interval - that could be split less than 53 and more than 85 but we can't say how - so the only thing we can say is that at most  $\frac{1}{4}$  of the games will be more than \$85

$$85 \quad \bar{x} = 69 \quad s = 8 \quad - \text{we can now use general rule of thumb}$$

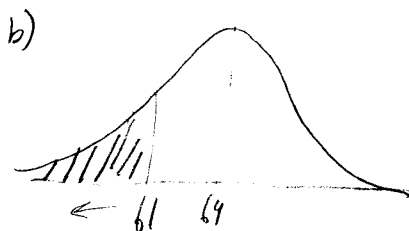


adding a deviation adds  $95\% - 68\% = 27\%$

that has to be split between left and

right so each addition is  $\frac{27}{2} = 13.5\%$

- so  $68\% + 13.5\% = 81.5\%$  will be between 61 and 85



$100\% - 68\% = 32\%$  must be split between both sides

$$\frac{32}{2} = 16\%$$

so 16% will be less than \$61