Economics 272
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FINAL EXAM

1. Given the following model:

$$y_t = x_t \beta + \varepsilon_t$$

where all terms are scalars and the ϵ's are iid normal with mean zero and variance 1 and the x's are non-stochastic.

a. Set up the likelihood function for this model and derive the maximum likelihood estimator for β.

b. Derive the OLS estimator for β and its variance.

c. Define the alternative estimator for β (the z's are non-stochastic):

$$\tilde{\beta} = \frac{\sum z_t y_t}{\sum z_t x_t}$$

1). Show that this alternative estimator is unbiased and derive its variance.
2). Show that the OLS estimator is efficient relative to the alternative estimator. Note that the Cauchy-Schwartz Inequality implies that $(\sum z_t^2)(\sum x_t^2) \geq (\sum z_t x_t)^2$.

d. Modify the model so that the slope is a random variable: $\beta_t = \beta + \mu_t$. Where the μ's are iid with mean zero and variance 1.

1). Substitute this expression for the random slope into the regression equation and derive the error covariance matrix for the new composite error term.
2). Discuss the implications of the new specification for the OLS estimator of β.
3). Discuss a method of obtaining an asymptotically correct standard error for the OLS estimator of β.

2. Given the following model:

$$Y_{i1} = \beta X_{i1} + \varepsilon_{i1}$$
$$Y_{i2} = \alpha Y_{i1} + \varepsilon_{i2}$$

where all terms are scalars and standard simultaneous equation assumptions hold.

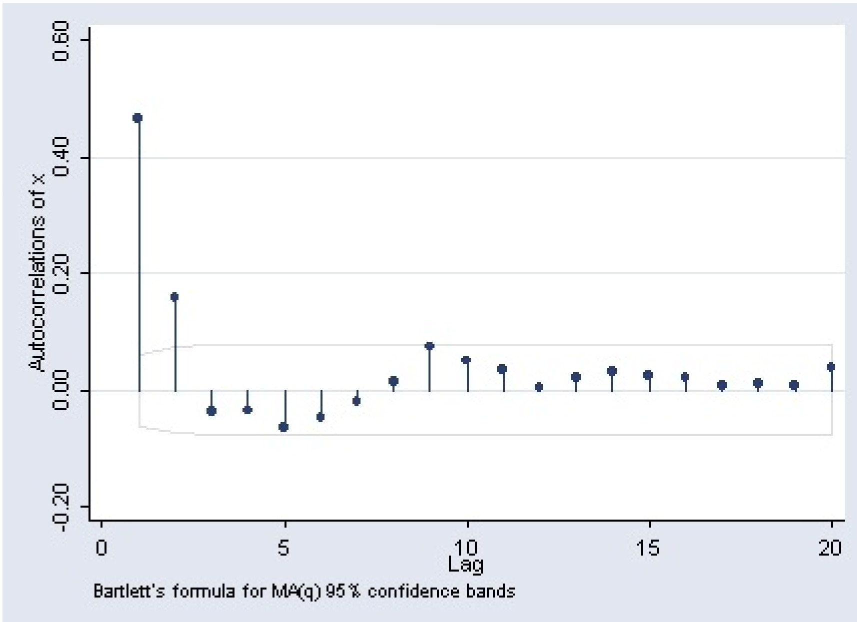
a. Check the order conditions for identification of each equation.

b. Present an instrumental variables estimator (X_{i1} as the instrument for Y_{i1}) for the second equation.

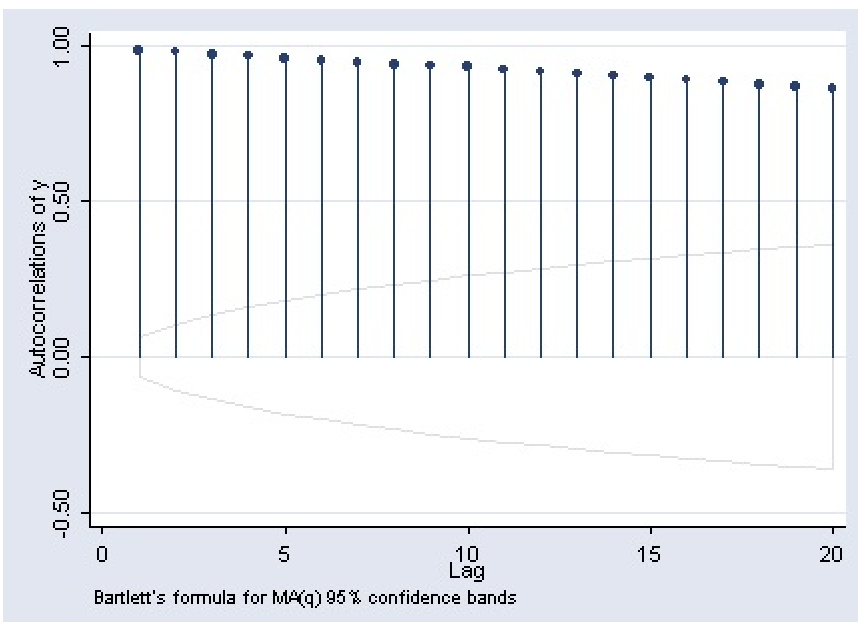
c. Present the two-stage least squares estimator for the second equation.

3. For each part of the following question, briefly explain the STATA output.

a.



b.



c.

```
. dfuller y,lags(1)

Augmented Dickey-Fuller test for unit root         Number of obs   =       998

                               ---------- Interpolated Dickey-Fuller ---------
                  Test         1% Critical       5% Critical      10% Critical
               Statistic           Value             Value             Value
------------------------------------------------------------------------------
 Z(t)             -2.465            -3.430            -2.860            -2.570
------------------------------------------------------------------------------
```

d. In this part, be sure to interpret the regression coefficients as part of your answer.

```
    curuse - current use of contraception
    educf - years of education
    ageyr - age
    city  - dummy variable for resides in city

probit curuse  ageyr educf  city

Iteration 0:   log likelihood = -2630.4878
Iteration 1:   log likelihood = -2571.0843
Iteration 2:   log likelihood = -2571.0277

Probit estimates                                  Number of obs   =       4190
                                                  LR chi2(3)      =     118.92
                                                  Prob > chi2     =     0.0000
Log likelihood = -2571.0277                       Pseudo R2       =     0.0226

------------------------------------------------------------------------------
      curuse |      Coef.   Std. Err.      z     P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       ageyr |   .0239306   .0023815    10.05   0.000      .019263    .0285983
       educf |    .021133   .0066047     3.20   0.001      .008188    .0340779
        city |   .1432316    .045654     3.14   0.002     .0537514    .2327117
       _cons |  -1.318087    .091802   -14.36   0.000    -1.498016   -1.138158
------------------------------------------------------------------------------
```

e.

```
. replace city=0
(1413 real changes made)

. predict use_not_city
(option p assumed; Pr(curuse))

. su use_not_city

    Variable |       Obs        Mean    Std. Dev.       Min        Max
-------------+-----------------------------------------------------
use_not_city |      4196    .3036213    .0732348   .1687473   .5564535

. replace city=1
(4201 real changes made)

. predict use_city
(option p assumed; Pr(curuse))

. su use_city

    Variable |       Obs        Mean    Std. Dev.       Min        Max
-------------+-----------------------------------------------------
    use_city |      4196    .3542473    .0773915   .2072799   .6122603
```

4. Given the following model:

$$y_{ti}=x_{ti}\beta+\mu_i+\varepsilon_{ti}$$

a. What are the standard assumptions that we make for this panel data model?

b. Discuss the implications of estimating β by OLS.

c. Modify the model as follows:

$$y_{ti}=x_{ti}\beta+\alpha y_{t-2,i}+\mu_i+\varepsilon_{ti}$$

1). Discuss the implications of estimating this model by OLS.
2). Present the first differences estimator. Will the OLS estimator for β in the first differenced model be consistent? Explain.