

PROBLEM SET I

1. Given the following model:

$$y_i = \beta x_i + \varepsilon_i$$

where all terms are scalars and the Gauss Markov assumptions are satisfied.

a. Write down the formula for the OLS estimator and show that it is unbiased and determine its variance.

b. Consider an alternative estimator:

$$\tilde{\beta} = \frac{\bar{Y}}{\bar{X}}$$

- 1). Show that this estimator is a linear estimator.
- 2). Show that this estimator is unbiased.
- 3). Derive its variance and show that it is inefficient relative to the OLS estimator.

2. For the model of equation (1), define the least squares residuals as:

$$\hat{\varepsilon}_i = y_i - \hat{\beta} x_i$$

a. Derive the mean and variance of this residual. Does it follow a normal distribution?

b. Consider the residual for observation j ($i \neq j$). Does it have the same variance? Is it correlated with the residual for observation i ?

3. Given the following model:

$$Y = X\beta + \varepsilon$$

Where Y is $N \times 1$, X is $N \times K$, β is $K \times 1$, and ε is $N \times 1$. We further assume that the Gauss Markov assumptions are satisfied except that $\varepsilon_i = z_i \mu_i$, where the μ 's follow the Gauss Markov assumption.

a. Derive the $N \times N$ covariance matrix for the disturbances.

b. Derive the $K \times K$ covariance matrix for the OLS of β .

4. The problem uses the national STATA dataset.

a. Generate the quarterly percentage change in the NASDAQ, employment, and CPI.

b. Run a regression with change in the NASDAQ as the dependent variable and the other two new variables, the ten year bond rate and a constant as the independent variables.

c. Interpret the R^2 and the adjusted R^2 in terms of what you think you know about stock market returns.

d. Perform a t test of the null hypothesis each of the explanatory variables is zero.

e. Test the null hypothesis the change in employment and change in CPI have the same effect.