Economics 771
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# Midterm Exam

1. Given the following model:

$$Y_{i1} = \beta_1 X_{i1} + \beta_2 X_{i2} + \beta_3 X_{i3} + \varepsilon_{i1}$$
$$Y_{i2} = \alpha_1 Y_{i1} + \alpha_2 X_{i4} + \alpha_3 X_{i5} + \varepsilon_{i2}$$

a. Check the order condition for identification in the second equation.

b. Under what conditions can you consistently estimate the second equation by OLS? Explain.

c. Given the following STATA output:

```
. regress  y1 x1 x2 x3

      Source |       SS       df       MS              Number of obs =    1000
-------------+------------------------------           F(  3,   996) =  447.97
       Model |  1315.35878     3  438.452928           Prob > F      =  0.0000
    Residual |  974.843117   996   .97875815           R-squared     =  0.5743
-------------+------------------------------           Adj R-squared =  0.5731
       Total |   2290.2019   999   2.2924944           Root MSE      =  .98932

------------------------------------------------------------------------------
          y1 |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
          x1 |   .4240564   .0312679    13.56   0.000     .3626979    .4854149
          x2 |   .8065833   .0315655    25.55   0.000     .7446409    .8685258
          x3 |  -.7561827   .0318268   -23.76   0.000     -.818638   -.6937274
       _cons |   .4966116    .031313    15.86   0.000     .4351646    .5580587
------------------------------------------------------------------------------

. predict error,residual


. regress  y2 y1 x4 x5 error

      Source |       SS       df       MS              Number of obs =    1000
-------------+------------------------------           F(  4,   995) = 1697.22
       Model |  2537.19811     4  634.299529           Prob > F      =  0.0000
    Residual |  371.860097   995  .373728741           R-squared     =  0.8722
-------------+------------------------------           Adj R-squared =  0.8717
       Total |  2909.05821   999  2.91197018           Root MSE      =  .61133

------------------------------------------------------------------------------
          y2 |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
          y1 |   .5004827   .0168964    29.62   0.000     .4673259    .5336394
          x4 |   .2020537   .0247587     8.16   0.000     .1534684     .250639
          x5 |   -.811964   .0237034   -34.26   0.000    -.8584784   -.7654497
       error |   .7745524   .0258443    29.97   0.000     .7238369    .8252679
       _cons |   .2405561   .0210031    11.45   0.000     .1993406    .2817716
------------------------------------------------------------------------------

. ivregress 2sls  y2 (y1= x1 x2 x3) x4 x5

Instrumental variables (2SLS) regression               Number of obs =    1000
                                                       Wald chi2(3)  =  995.30
                                                       Prob > chi2   =  0.0000
                                                       R-squared     =  0.6706
                                                       Root MSE      =  .97884

------------------------------------------------------------------------------
          y2 |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
          y1 |   .4992444   .0270972    18.42   0.000     .4461349    .5523539
```

```
         x4 |   .2106808   .0396531     5.31   0.000     .1329622    .2883995
         x5 |  -.8368746   .0380091   -22.02   0.000    -.9113711   -.762378
      _cons |   .2413048   .0336393     7.17   0.000      .175373    .3072365
------------------------------------------------------------------------------
Instrumented:  y1
Instruments:   x4 x5 x1 x2 x3

. estat overid

  Tests of overidentifying restrictions:

  Sargan (score) chi2(2) =  .132506  (p = 0.9359)
```

1).What is being tested in the first two regressions? What is the result of the test?

2).Explain in the detail the method of estimation used in the third regression.

3).What is being tested by the Sargon test? What are the results and what are the implications of the results?

2. Given the following model:

$$Y_i = \beta X_i + \alpha Z_i + \varepsilon_i$$

where the $\varepsilon_i \sim N(0,\sigma^2)$ and both X and Z are standardized (mean zero variance 1) with correlation ρ and both are uncorrelated with ε.

a. Write down the formula for the OLS estimator for β and α and derive the covariance matrix. What value for the correlation between X and Z results in the smallest variance for β?

b. Suppose we run on OLS regression with Y a function of X alone. Under what conditions we will get an unbiased estimator for β? Prove your assertion.

3. Given the following model:

$$Y_t = \beta X_t + \varepsilon_t$$

where all terms are scalars and

$$\varepsilon_t = \rho_1 \varepsilon_{t-1} + \rho_2 \varepsilon_{t-2} + \mu_t$$

with $\mu_t \sim N(0,\sigma_\mu{}^2)$.

a. Show that the OLS estimator for β is unbiased.

b. Develop a feasible GLS estimation method for β (ignore the first and second observations)

c. Suppose that $\mu_t \sim N(0,\sigma_{t\mu}{}^2)$ where the form of the heteroskedasticity is not known. Show how asymptotically correct statistical inferences about β can be made using the OLS estimator.