Mathematical modeling is invaluable towards understanding fundamental phenomena in physics, chemistry, biology, medicine, and engineering. This course will give students a broad introduction to modeling scientific applications using ordinary differential equations, difference equations, and stochastic processes.

More specifically, by taking this course the student will be able to:

- Write down a system of equations to model a specific application.
- Understand the assumptions made when constructing a mathematical model and interpret the solutions in terms of the application.
- Numerically solve differential equations and difference equations in Matlab.
- Construct and interpret bifurcation diagrams.
- Construct phase plane diagrams and use them to understand the behavior of two-dimensional systems of ODE’s.
- Construct, analyze, and interpret probabilistic models.

Prerequisites

Students taking this course should have the equivalent of Calculus I and II, as well as a basic introduction to linear algebra and multivariable calculus. However, students that are weak in linear algebra are welcome to take the course, given the motivation to do some outside work. Mathematics, chemistry, physics, biology, engineering, and social science majors with an interest in mathematical modeling are welcome to enroll in the course. Scientific applications will be drawn from the interests of the students enrolled.

Reading


Supplemental material on discrete and stochastic modeling and applications will be provided during the semester.
Mathematical Modeling
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Software

Students are expected to bring a laptop to every class to participate in interactive simulations of the mathematical models presented. We will use Matlab for all computer work, and this software is freely available to students at UNC.

Instructions for obtaining Matlab
Go to the following website and scroll down to the Matlab link:
https://software.unc.edu/available.php
Login using your Onyen. Scroll down to “Placing an Order,” and click on the student’s link. The student online order form will appear, and you will receive an email when your copy of Matlab is ready. Instructions for installation are on the UNC Matlab page. Note that you will need the Personal License Password and/or the license.dat file provided online to get through the installation process.

Create a Math564 folder on your computer and download dfield and pplane from the following website:
http://math.rice.edu/~dfield/index.html

You will find this software very useful for homework assignments and projects.

Course Requirements

Students in this course will be coming from a wide range of backgrounds and interests. The course work and evaluation is designed to allow for flexibility in training and scientific interests. The course grade will be based on a take home midterm, a take home final, homework assignments, and a class project.

<table>
<thead>
<tr>
<th>Assignment</th>
<th>Weight</th>
<th>Points</th>
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</thead>
<tbody>
<tr>
<td>Homework assignments (10 total)</td>
<td>20%</td>
<td>200 points</td>
</tr>
<tr>
<td>Take home midterm</td>
<td>25%</td>
<td>250 points</td>
</tr>
<tr>
<td>Take home final</td>
<td>25%</td>
<td>250 points</td>
</tr>
<tr>
<td>Written project</td>
<td>20%</td>
<td>200 points</td>
</tr>
<tr>
<td>Project presentation</td>
<td>10%</td>
<td>100 points</td>
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Grades will be assigned using the following point distribution:

<table>
<thead>
<tr>
<th>Points</th>
<th>Grade</th>
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<tbody>
<tr>
<td>900 – 1000</td>
<td>A</td>
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<tr>
<td>800 – 899</td>
<td>B</td>
</tr>
<tr>
<td>700 – 799</td>
<td>C</td>
</tr>
<tr>
<td>600 – 699</td>
<td>D</td>
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<tr>
<td>less than 599</td>
<td>F</td>
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Take home tests
Students will be given 1 week to complete the take home exams. The midterm will be assigned on Wednesday, October 10th and will be due in class on Wednesday, October 17th. The take home final will be assigned on Wednesday, December 5th and will be due
during the final exam period for the class (Friday, December 14th, 8 – 11 am). 10% will be taken off the total score of the take home midterm for each day it is late. The take home final will not be accepted after 5 pm, December 14th, and 10% will be deducted for each hour the exam is late (after 11 am).

Students will be allowed to use any outside sources (text book, journal articles) as well as any computer software. All references must be cited using correct APA format: http://owl.english.purdue.edu/owl/resource/560/01/

Students will not be allowed to work together or consult tutors, instructors, teaching assistants, fellow students, or any other person for assistance. Students will be expected to follow the UNC Honor Code, and those who violate the Honor Code will be reported to Honor Court.

**Homework Assignments**

There will be 10 homework assignments during the semester. Students are allowed to work together on the assignments, but rote copying is forbidden. 10% of the total score will be deducted for each day any assignment is late.

**Class projects**

Students will be allowed to work individually or in pairs on a class project that involves a mathematical model of any area of application. Students should meet with Dr. Miller during the semester to discuss project ideas and to have the proposed project approved. A written report of the project will be due the last day of class (December 5th). Each individual or pair will be required to give a 15 minute presentation on their project during class at the end of the semester.

**Policies and Late Assignments**

Homework assignments and take home assignments turned in late will have points deducted each day (see above). Exceptions will only be made in cases of extreme circumstances (prolonged illness or death in the family) or travel related to university activities. Please contact me as soon as possible if such a situation arises. Do not wait until the end of the semester!!
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Course Schedule

**August 22 – 24**
Reading: Chapter 1, Strogatz
- Linear algebra background quiz
- Introduction to mathematical modeling and nonlinear differential equations
- Spring-mass-damper example introduced

**August 27 – 31**
Reading: Chapter 2, Strogatz
- One dimensional, first-order system review
- Fixed points, stability, phase portraits
- Linear stability analysis
- Intro to solving ODE’s on the computer
- Applications: population growth, circuits, economics

**September 5 – 14**
Reading: Chapter 3, Strogatz
- Saddle-Node, transcritical, and pitchfork bifurcations
- Dimensional analysis and scaling
- Phase plane analysis
- Applications: Insect outbreaks, flapping flight, lasers, consumer-producer problem

**September 17 – 19**
Reading: Chapter 4, Strogatz
- Uniform and nonuniform oscillators
- Application: fireflies, superconducting Josephson Junctions

**September 21 – 28**
Reading: Chapters 5 and 6, Strogatz and handouts
- Linear systems of differential equations and linear algebra review
- Working with pplane
- Nonlinear systems
- Linearization about fixed points and classification of nonlinear systems
- Applications: Predator prey model, pendulum, Fitzhugh Nagumo (neurobiology), ecological models

**October 1 – 5**
Reading: Chapter 7, Strogatz
- Limit Cycles
- Ruling out closed orbits
- Poincare-Bendixson Theorem
- Applications: Circadian rhythms, van der Pol oscillators, cardiac dynamics, Duffing equation (nonlinear structures)
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October 8 – 15
Reading: Chapter 8, Strogatz
• More on bifurcations
• Hopf bifurcations
• Applications: kidneys, oscillating chemical reactions, coupled oscillators in physiology

October 22 – 29
Reading: Chapter 9, Strogatz and handouts
• Discrete dynamic systems
• Fixed points and cobwebbing
• Logistic map, period doubling and chaos
• Applications: Population growth, gas exchange in the lung, economic bifurcations and chaos

October 31 – November 9
Reading: Handouts
• Stochastic discrete time models
• Markov chains and random walks
• Monte Carlo methods
• Applications: genetics, polymer physics, fixational eye movements, reliability engineering

November 12 – 19
Reading: Handouts
• Stochastic differential equations overview
• The Langevin equation
• Exponential decay with and without noise
• Applications: predator-prey, math finance, and others to be decided.

November 26 – December 5
Project presentations