

# Why Do Term Structures in Different Currencies Comove? \*

Chotibhak Jotikasthira

The University of North Carolina at Chapel Hill  
pab\_jotikasthira@unc.edu

Anh Le

The University of North Carolina at Chapel Hill  
anh\_le@unc.edu

Christian Lundblad

The University of North Carolina at Chapel Hill  
christian\_lundblad@unc.edu

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## Abstract

Yield curve fluctuations across different currencies are highly correlated. This paper investigates this phenomenon by exploring the channels through which macroeconomic shocks are transmitted across borders. Macroeconomic shocks affect current and expected future short-term rates as central banks react to changing economic environments. However, policy reactions are not the only channel through which the macroeconomy may affect bond yields. Investors may also respond to these shocks by altering their required compensation for risk. Macroeconomic shocks thus influence bond yields both through a “policy” channel as well as through a “risk compensation” channel. In a no-arbitrage vector autoregressive framework, we employ deviations from the expectations hypothesis to identify the two transmission channels, with particular attention to the degree to which each channel contributes to the co-variation among term structures across the U.S., the U.K., and Germany. We find that a world inflation factor explains over 70% of the co-variance of medium- to long-maturity yields between the U.S. and the U.K., as well as between the U.S. and Germany. Further, we find that the world inflation effect operates almost exclusively through the risk compensation channel for long-term bonds.

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# 1 Introduction

Yield curve fluctuations across different currencies are highly correlated (see Figure 1). We address this phenomenon by exploring how macroeconomic shocks are transmitted across borders. In a rational expectation equilibrium, shocks can affect the term structure in two ways. One is through the joint dynamics of state variables, including those upon which monetary policy actions are based. The other is through the market prices of risk (i.e. term premia) that investors assign to these state variables. We refer to these two channels as the “policy” and the “risk compensation” channels, respectively.

The first channel arises as central banks target the short-term rate, as a policy tool, in order to manage domestic growth and inflationary pressures. Thus, co-movement at the short end of the yield curve is likely to reflect co-movement in the economic fundamentals to which the monetary authorities are responding. Indeed, Kose, Otrok, and Whiteman (2003) find that a common world factor is an important source of volatility for macro aggregates in most countries, providing evidence for a global business cycle.<sup>1</sup> If central banks indeed set interest rates as a function of global economic conditions, then the world business cycle would suggest that short-term interest rates are also highly correlated across countries.

However, a reasonable question is whether policy exposures to common global factors, coupled with the expectations hypothesis, can explain other equally important features of international yield curves across the maturity spectrum. While the policy channel would imply that long-maturity yields will also be correlated, it is unlikely that this channel alone can explain the significant degree of variation and cross-market co-variation observed for long maturity bond yields. In a single-country setting, several papers (see Ang and Piazzesi (2003), Kuttner (2001), Evans and Marshall (2007), and Dai and Philippon (2005) for example) have documented that macroeconomic shocks, including monetary and fiscal policy

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<sup>1</sup>However, the evidence suggests that the cross-country correlations are significantly higher for interest rates than for macroeconomic variables.

surprises, do significantly impact bond yields. However, these shocks work their way to the long end of the term structure largely through market prices of risk, which allow long-maturity yields to load more or less on economic and monetary factors than predicted by the expectations hypothesis (see Dai and Singleton (2002), Le, Dai, and Singleton (2010), and Duffee (2002)).<sup>2</sup> These findings imply that macroeconomic factors may indeed be important in explaining the observed co-movement among yield curves in the international setting, but that these factors may impact long-maturity yields through the alternative risk compensation channel. Employing standard Campbell and Shiller (1991) regressions, Sutton (2000) provides early evidence that bond yields in different major currencies indeed display excess co-movement relative to that implied by the expectations hypothesis; we formally explore his conjecture that these findings are the result of positively correlated term premia.<sup>3</sup>

To provide some context, there are numerous examples where long-maturity bond yields tend to co-move across borders even when the short-term policy rates diverge, in contrast to the pattern implied by the standard policy (expectations hypothesis) channel. For instance, short-term policy rates diverged in the U.S. and U.K. in 1997, reflecting different inflationary pressures across the two countries, whereas ten-year bond yields in both currencies moved in concert (see Figure 2). Similarly, during the broad economic recession in 1990-1991, the Federal Reserve and the Bank of England responded by sharply cutting policy rates. During the same period, however, the German economy grew rapidly and the Bundesbank raised its policy rate by over 250 basis points from late 1989 to end of 1991. Despite these quite different policy directions, ten-year bond yields fell sharply across all three markets.

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<sup>2</sup>In related work, Gallmeyer, Hollifield, Palomino, and Zin (2009) and Wachter (2006), in a single country setting, argue that long-maturity bond yield volatility is difficult to produce using a usual rational expectations model with stationary state variables. Both papers thus propose models with habit formation to generate large market prices of risk. Also, for equity markets, Bansal and Lundblad (2002) and Colacito and Croce (2010) argue that time-varying expected returns explain the sharp contrast in correlation between international equity returns (high) versus aggregate market dividend growth rates (low).

<sup>3</sup>A few other papers examine the expectations hypothesis and the predictive power of term spreads in countries other than the U.S. However, they do not study linkages between term structures of different countries. Examples include Hardouvelis (1994) and Jorion and Mishkin (1991).

Generating these sorts of (admittedly anecdotal) patterns requires a more general model that facilitates fluctuations in market prices of risk.

In this paper, we quantify the contributions of both the policy and risk compensation channels by estimating a discrete-time affine term structure model with the observed underlying factors following a vector autoregressive (VAR) process. The model is similar in spirit to Ang and Piazzesi (2003), but allows for un-spanned macroeconomic variables as in Joslin, Priebsch, and Singleton (2010). We estimate the model (jointly) by maximum likelihood using macroeconomic and term structure data from the U.S., the U.K., and Germany (from 1987:04 to 2008:03, at monthly frequency).<sup>4</sup>

We find that measured macroeconomic and yield-curve factors are stationary and therefore have a limited impact on long-maturity yields under the expectations hypothesis. Further, we find that yield volatilities exceed those predicted by the expectations hypothesis and the physical dynamics of macroeconomic variables and monetary factors. The model attributes the remainder to risk premia fluctuations; similar to the single country research mentioned above, this is particularly important for long-maturity bond yields. With regards to cross-border *co-movement*, we find that the policy channel and the risk compensation channel account for 10-58% and 42-90%, respectively, of the cross-country co-variances of five-year bond yields. As we move to the ten-year maturity, these numbers shift considerably in favor of the risk compensation channel.<sup>5</sup> Digging a bit deeper, we find that while both macroeconomic and yield-curve factors (level, slope, and curvature) are almost equally important in explaining the variances of long-term yields, it is the global macroeconomic variables that largely explain cross-country co-variation. The most important state variable

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<sup>4</sup>As discussed by Dong (2006), the operating procedures of the Federal Reserve prior to 1982 focused on meeting non-borrowed aggregate reserve targets instead of managing short interest rates in the post-Volcker era. For now, we exclude data during the recent sub-prime crisis (a better understanding of the link between policy, risk premia, and the term structure of interest rates during the crisis is an important avenue for future research).

<sup>5</sup>This is true despite the fact that by construction, the co-movement of short-term rates is 100% fundamentally driven.

is the world inflation rate as it alone explains over half of the cross-country co-variances in bond yields. Interestingly, the global inflation factor operates through the policy channel at the short end, as central banks respond to global inflationary pressures, but almost exclusively through the risk compensation channel at the long end, as global investors alter their required compensation for risk.

This paper contributes to the finance and economics literature in three important ways. First, it is the first to examine the economics behind the international co-variation of the term structure of interest rates, providing a first step in understanding the co-movement of other risky assets and, possibly, economic activity. Shiller (1989) studies the co-movement of the U.S. and U.K. stock prices in a simple present-value context and shows that the correlation between dividends in the two countries is too small to account for the high degree of co-variation in stock price. The macroeconomic literature also singles out the interest-rate channel as one of the most important linkages between different countries' stock markets. For example, Ehrmann and Fratzscher (2006) find that the transmission of U.S. monetary policy to other countries' equity markets occurs largely through the reaction of U.S. short-term interest rates, foreign short-term interest rates, and exchange rates.<sup>6</sup> Examining the impact of U.S. monetary policy on other countries' economic activity, Canova (2005) and Kim (2001) arrive at the same conclusion: interest rate reactions are the most important transmission channel.

Second, from a modeling standpoint, this paper is the first to apply an affine term structure model to the exploration of the cross-country transmission of macroeconomic shocks. Building on recent advances in the term structure modeling literature, pioneered by Joslin, Singleton, and Zhu (2010) and Joslin, Priebsch, and Singleton (2010), our model design allows us to jointly estimate multiple term structures denominated in different currencies in a particularly robust and efficient manner. Possibly due to the curse of dimensionality,

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<sup>6</sup>See also Wongswan (2006).

most models studying the transmission of macroeconomic shocks and the term structures are employed in a single-currency setting. A few manage to extend the models to a two-countries setting, aiming at explaining the exchange rate dynamics and the failure of the uncovered interest rate parity.<sup>7</sup> However, due to the size of the parameter space, these papers typically model and estimate separate pairs of countries. If one considers the U.S.-U.K. and U.S.-Germany pairs as we do in this paper, the traditional models will almost surely imply different estimated dynamics for the US term structure. Since joint maximum likelihood estimation can be easily implemented within our framework, this issue is completely avoided.

Third, the results in this paper may help to further develop our understanding of the channels through which crises and contagion spread. Forbes and Rigobon (2002) show that stock markets co-move highly in all periods but, due to the large volatility of shocks, they may appear to co-move more during crises. That is, international co-movement and contagion rely on the similar linkages between economies. A number of papers have documented the specific channels by which a shock in one country spreads to other countries. The proposed channels can be divided into two broad groups: (i) fundamental economic factors, such as trade (e.g. Forbes (2004)) and (ii) investor linkages, such as portfolio re-balancing (e.g. Boyer, Kumagai, and Yuan (2006) and Jotikasthira, Lundblad, and Ramadorai (2010)). The relative importance of these groups remains hotly debated due to the difficulty in identifying the independent effects of different channels. This paper identifies risk compensation (implicitly the role of global investors) as a critical channel for understanding the determinants of long maturity bond yields across markets. During crises, the effects arising from the behavior of global investors may be particularly pronounced.

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<sup>7</sup>For example, Dong (2006) uses a similar model to show that inflation and the output gap explain the majority of variation in the USD/EUR exchange rate. Backus, Foresi, and Telmer (2001) also use affine term structure models to study exchange rates, but with only two common latent factors and relatively restricted market price of risk functions. Inci and Lu (2004) use quadratic models, with only three common latent factors.

The remainder of the paper is organized as follows. Section 2 presents the model. In Section 3, we describe the data and the empirical implementation of the model. Section 4 presents the results. Section 5 concludes.

## 2 The Model

We build a model of multiple term structures of interest rates across countries, each indexed by  $i$ . Underlying each economy is a vector of macro variables  $M_t^i$  and yield curve pricing factors  $P_t^i$ . The length of these vectors can be unequal across currencies. In our implementation, we use the U.S. economy as proxy for the global economy and denote by the superscript  $W$  the U.S. macro variables ( $M_t^W$ ) and pricing factors ( $P_t^W$ ). Following Joslin, Singleton, and Zhu (2010) and Joslin, Pribsch, and Singleton (2010), among others, under the  $\mathbb{Q}$ -measures, only the pricing factors  $P$  can contemporaneously affect bond yields. Under the  $\mathbb{P}$ -measures, both  $M$  and  $P$  can contemporaneously influence and predict bond yields, through their joint dynamics. This way, we avoid over-fitting bond yields with a large number of pricing factors while allowing for both spanned and un-spanned macro risks.<sup>8</sup>

### 2.1 Short Rates and Bond Yields

We assume that the central bank of country  $i$  sets the short-term (policy) rate as a linear function of the world macro variables and all state variables of that country (similar in spirit to the backward-looking Taylor rule).<sup>9</sup> These state variables can be collapsed and rotated

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<sup>8</sup>In reality, there may be several other omitted latent and observed variables. However, Duffee (2006) shows that the estimation is unbiased and consistent so long as the observed (and estimated) factors (i) do not affect the market price of risk of the omitted factors, and (ii) do not contain information about the future values of the omitted factors.

<sup>9</sup>Ang, Dong, and Piazzesi (2005) examine the term structure implications of backward-looking vs. forward-looking Taylor rules. Gallmeyer, Hollifield, and Zin (2005) study the implications of the Taylor rule vs. the McCallum rule. The latter allows information in the yield curve to be used in setting the policy interest rate.

into  $N^i$  pricing factors  $X^i$  on which the short rate  $y_1^i$  is based:

$$y_{1,t}^i = r_\infty^i + \iota' X_t^i \quad (1)$$

where  $\iota$  is a vector of ones. To allow only  $X^i$  to affect country  $i$ 's bond yields, we follow Joslin, Priebsch, and Singleton (2010) in assuming the following risk-neutral dynamics for  $X^i$ :

$$X_{t+1}^i = \lambda^i X_t^i + \Sigma_{X^i} \epsilon_{i,t+1}^{\mathbb{Q}} \quad (2)$$

where  $\lambda^i$  is a diagonal matrix. Note that every VAR(1) specification can be transformed to (2) through standard shifts and rotations of the latent factors. For simplicity, we assume that elements of  $\lambda^i$  are real and distinct.<sup>10</sup> From (1) and (2), the yield of bond of country  $i$  with maturity  $n$  is given by<sup>11</sup>

$$y_{n,t}^i = A_{n,X^i}^i(\lambda^i, r_\infty^i, \Sigma_{X^i}) + B_{n,X^i}^i(\lambda^i)' X_t^i. \quad (3)$$

It is clear from (3) that the yields on all bonds of country  $i$  will only depend on  $X^i$  and a few parameters associated with its risk-neutral dynamic. For notational simplicity, we will drop the superscript  $i$  in showing how  $A_n^i$  and  $B_n^i$  are computed:

$$B_{n,X} = \frac{1}{n} \text{diag}((I - \lambda)^{-1}(I - (\lambda)^n)), \quad (4)$$

$$A_{n,X} = r_\infty - \frac{1}{2n} \text{tr} \left( \Sigma_X \Sigma_X' \sum_{k=0}^{n-1} k^2 B_{k,X} B_{k,X}' \right) \quad (5)$$

$$= r_\infty - \frac{1}{2n} \text{tr} (\Sigma_X \Sigma_X' C_{n,X}) \quad (6)$$

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<sup>10</sup>Joslin, Singleton, and Zhu (2010) consider the case of complex  $\lambda^i$ . Real-valued  $\lambda^i$ 's are found to be empirically adequate.

<sup>11</sup>See proofs in Duffie and Kan (1996) and Dai and Singleton (2000) for continuous-time versions and Ang and Piazzesi (2003) for discrete-time versions.

where

$$C_{n,X} = \sum_{k=0}^{n-1} k^2 B_{k,X} B'_{k,X}. \quad (7)$$

It is important to note that  $B_{n,X}$  and  $C_{n,X}$  are dependent on  $\lambda$  only. This feature significantly reduces the dimensionality of the problem, which is particularly helpful in simultaneously dealing with multiple term structures. One shortcoming of this affine structure is that it leads to constant factor loadings  $B_{n,X}$ . This implies that our model cannot correctly capture the time-varying nature of the relative importance of the factors. In addition, the Gaussian structure of the state variables implies that the conditional covariances of bond yields will also be constant.

## 2.2 From Latent to Observed Pricing Factors

An affine model with latent factors can be difficult to estimate since the factor values have to be inferred from bond prices in each and every iteration of the estimation process. Therefore, maximum likelihood estimation requires that the parameters of both the physical and risk neutral dynamics be jointly estimated. Joslin, Singleton, and Zhu (2010) show that by assuming that a few portfolios of bonds are priced without error and using their yields as observed pricing factors, one can bypass this challenge. The reason is that the likelihood function can now be split into two smaller parts corresponding to the physical and risk-neutral dynamics, and each of these parts can (more or less) be maximized separately.<sup>12</sup>

We follow this approach and assume that the fixed weight ( $W$ ) portfolios of yields corresponding to the first  $N$  principal components of yields,  $P_t$ , are priced without error. Note again that all calculations in this subsection are specific to each currency  $i$  and so we drop

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<sup>12</sup>The only parameter that appears in both is  $\Sigma_X$ . However, the covariances of residuals from the estimated physical dynamics provide a good starting values for the estimation.

the superscript  $i$  for simplicity. We first show that  $P_t$  is nothing more than a rotation of  $X_t$ .

$$P_t = W y_{\tilde{n},t} = W A_{\tilde{n},X} + W B'_{\tilde{n},X} X_t = U + V X_t \quad (8)$$

where  $\tilde{n}$  is a vector of maturities that constitute the perfectly priced portfolios, and let

$$U = W A_{\tilde{n},X} \quad \text{and} \quad V = W B'_{\tilde{n},X}. \quad (9)$$

From this, we can re-write the term structure model now with  $P_t$ , as opposed to  $X_t$  as the pricing factors:

$$y_{n,t} = A_{n,X} + B'_{n,X} X_t = A_{n,X} + B'_{n,X} V^{-1} (P_t - U) = A_{n,P} + B'_{n,P} P_t \quad (10)$$

where

$$A_{n,P} = A_{n,X} - B'_{n,X} V^{-1} U \quad \text{and} \quad B_{n,P} = (V^{-1})' B_{n,X}. \quad (11)$$

## 2.3 Physical Dynamics and Restrictions

Now that we can price all bonds as a function of *observed* pricing factors,  $P$ , we will present the physical dynamics of  $P$  as opposed to those of  $X$  (which simply reflects a rotation). To disentangle the part of  $P_t^W$  that is correlated with  $M_t^W$  from its pure yield curve component, we first project  $P_t^W$  on  $M_t^W$ :

$$P_t^W = b^W M_t^W + P_t^{W,e} \quad (12)$$

to obtain  $P_t^{W,e}$  as the residual yield curve factors. Likewise, projecting  $P_t^i$  on  $M_t^i$  and  $M_t^W$ :

$$P_t^i = a^i M_t^i + b^i M_t^W + P_t^{i,e} \quad (13)$$

we obtain  $P_t^{i,e}$  as the residual yield curve factors for the  $i^{th}$  economy.

We model the joint physical dynamics of the U.S. and the  $i^{th}$  economy by a Gaussian VAR:

$$Y_{t+1}^{i,e} = \kappa_0^{i,e} + \kappa_1^{i,e} Y_t^{i,e} + \Sigma_{Y^{i,e}} \epsilon_{i,t+1}^{\mathbb{P}} \quad (14)$$

where  $Y_t^{i,e} = (M_t^{W'}, P_t^{W,e'}, M_t^{i'}, P_t^{i,e'})'$ .

Similar to Ang and Piazzesi (2003) and Evans and Marshall (2007), we impose a block structure on the dynamics of  $Y_t^{i,e}$ . In particular, the residual yield curve factors of each economy are not allowed to feed back into the macro variables of any economy, U.S. (world) or otherwise. The macro variables and the residual yield curve factors of a non-U.S. economy do not feed into any other economy, except its own. In addition, except for the contemporaneous correlation between macro variables of the  $i^{th}$  economy and the U.S. economy, all other cross-economy contemporaneous correlations are shut down. With this structure, interactions between any two non-U.S. economies occur through their respective correlations with the U.S. To be concrete, our restrictions imply that  $\kappa_1^{i,e}$  and  $\Sigma_{Y^{i,e}}$  take the following forms:

$$\kappa_1^{i,e} = \begin{bmatrix} xx & 0 & 0 & 0 \\ xx & xx & 0 & 0 \\ xx & 0 & xx & 0 \\ xx & xx & xx & xx \end{bmatrix}, \text{ and } \Sigma_{Y^{i,e}} = \begin{bmatrix} xx & 0 & 0 & 0 \\ 0 & xx & 0 & 0 \\ xx & 0 & xx & 0 \\ 0 & 0 & 0 & xx \end{bmatrix} \quad (15)$$

where  $xx$  denotes a non-zero block of a matrix.

The assumed dynamics of  $Y_t^{i,e}$  imply a similar VAR(1) structure for  $Y_t^i = (M_t^{W'}, P_t^{W'}, M_t^{i'}, P_t^{i'})'$

because equations (12) and (13) together imply that  $Y_t^i = \Pi^i Y_t^{i,e}$  where:

$$\Pi^i = \begin{bmatrix} I & 0 & 0 & 0 \\ b^W & I & 0 & 0 \\ 0 & 0 & I & 0 \\ b^i & 0 & a^i & I \end{bmatrix}. \quad (16)$$

It follows that:

$$Y_{t+1}^i = \kappa_0^i + \kappa_1^i Y_t^i + \Sigma_{Y^i} \epsilon_{i,t+1}^{\mathbb{P}} \quad (17)$$

where

$$\kappa_1^i = \Pi^i \kappa^{i,e} \Pi^{i-1} \text{ and } \Sigma_{Y^i} = \Pi^i \Sigma_{Y^{i,e}} \text{ and } \kappa_0^i = \Pi^i \kappa_0^{i,e}. \quad (18)$$

Given this structure and our bond pricing solution, bond yields are related across currencies by the exposures of (country-specific) pricing factors to the common macro variables, both directly and indirectly through the influence of the world macro variables on the local macro variables.

Our model differs from those of Ang and Piazzesi (2003) and Dong (2006) along several important dimensions. First, although we allow the macro variables to affect the pricing factors, they remain un-spanned by bond yields. This is consistent with the evidence provided by Joslin, Priebsch, and Singleton (2010) and others. Under a rigid application of the models in Ang and Piazzesi (2003) or Dong (2006), the macro variables can be expressed as linear functions of bond yields. Second, we adopt the risk-neutral setup of Joslin, Priebsch, and Singleton (2010), and therefore can restrict the number of pricing factors to a small number as required by the data. The first three principal components of bond yields in the U.S. explain about 99.99% of their observed variation. Therefore, a term structure model that implies five

or six factors for bond yields will likely overfit the data. Third, since our yield curve factors are observable, the relationship between macro variables and bond yields will be more easily identified from the data. On this last point, although the contemporaneous relationship should, in theory, be apparent from the estimate of  $\Sigma_{Y^i}$ , identifying this relationship from the data can be challenging given the measurement error, the exact timing of the macro variables, and the high degree of persistence of both the macro variables and bond yields.

## 2.4 Implied Monetary Policy Rule

Unlike Ang and Piazzesi (2003) and others, we do not start with a monetary policy rule (e.g. Taylor rule). However, our model implies a certain monetary rule given that the short rate loads on the pricing factors (as in (1)) and the pricing factors load on the macro variables (as in (12) and (13)). If we define a general form of monetary policy rule as

$$y_{1,t} = \delta M_t + y_{1,t}^* \tag{19}$$

where the coefficient  $\delta$  represents the monetary policy responses to macroeconomic shocks, then our model implies that for the world (or the U.S.),  $\delta^{W'} = B_{1,P}^{W'} b^W$  and for a non-U.S. country  $i$ ,  $\delta^{i'} = B_{1,P}^i [b^i a^i]$  where both  $M^W$  and  $M^i$  affect country  $i$ 's monetary policy. If  $M$  includes only the inflation rate and the output gap, then our model is equivalent to a Taylor rule.<sup>13</sup>

## 2.5 Exchange Rates

Given our focus on cross-country interest rates, one natural question is what our model implies for the dynamics of exchange rates. If we were to assume that bond markets are complete and fully integrated, then the difference in *SDFs* of two countries (used in our

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<sup>13</sup>See also Joslin, Priebsch, and Singleton (2010) for a general discussion on this point.

bond pricing) determines the change in their exchange rate (see Backus, Foresi, and Telmer (2001) and Inci and Lu (2004) among others).<sup>14</sup> Let  $S_t^i$  denote the time- $t$  exchange rate, expressed as the number of dollars per one unit of non U.S. currency  $i$ , and  $s_t^i = \log(S_t^i)$ . Since we have fully specified the  $\mathbb{P}$  and  $\mathbb{Q}$  dynamics of the pricing factors as well as the short rate for each economy, the stochastic discount factor for the  $i^{\text{th}}$  economy can be written as:

$$SDF_{t+1}^i = e^{-y_{1,t}^i} \frac{f_t^{\mathbb{Q}}(P_{t+1}^i)}{f_t^{\mathbb{P}}(P_{t+1}^i)} \quad (20)$$

where  $f_t^{\mathbb{Q}}(\cdot)$  and  $f_t^{\mathbb{P}}(\cdot)$  denote the density under the risk-neutral and physical measures, respectively. Then, no arbitrage implies that

$$\begin{aligned} \Delta s_{t+1}^i &\equiv s_{t+1}^i - s_t^i = \log(SDF_{t+1}^i / SDF_{t+1}^W) \\ &= (y_{1,t}^W - y_{1,t}^i) + \log \left( \frac{f_t^{\mathbb{Q}}(P_{t+1}^i)}{f_t^{\mathbb{P}}(P_{t+1}^i)} \frac{f_t^{\mathbb{P}}(P_{t+1}^W)}{f_t^{\mathbb{Q}}(P_{t+1}^W)} \right) \end{aligned} \quad (21)$$

Notice that the expected change in exchange rate does not follow the uncovered interest rate parity (UIRP). For an investor with dollar as the numeraire currency, the second term in expression (21) is the excess return on currency  $i$ , the foreign exchange risk premium, which is generally state-dependent. Dong (2006) shows that his model, which in spirit is similar to ours, produces the path of exchange rates that matches the data very well. Since fitting the exchange rate path is not of direct interest in this paper and doing so will worsen the fitting of bond yields (plus additional assumptions are needed), we do not use equation (21) in our estimation. In unreported results, we show that despite high correlations among bond yields in different currencies (and thus high correlations of  $SDF$ s), the implied exchange rates from our term structure estimation are still significantly more volatile than the actual rates. This is consistent with the finding of Brandt, Cochrane, and Santa-Clara (2006).

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<sup>14</sup>In fact, the assumption that the modeled  $SDF$ s are minimum-variance  $SDF$ s are sufficient.

## 3 Empirical Implementation

### 3.1 Data

We study the U.S., the U.K., and German term structures, using LIBOR rates from the British Bankers Association and swap rates from Reuters. LIBOR-swap rates are more readily available than government bond yields for a broader set of countries (which we plan to study in future research). Moreover, LIBOR rates are of much better quality than the government rates at short maturities, due to the lack of liquidity in the U.K. and German bill markets. In addition, several researchers have argued that government bonds are not an ideal proxy for the unobservable risk-free rates due to favorable taxation treatment, repo specials, scarcity premia, and benchmark status. Swaps may arguably provide a good alternative given that the credit risk priced into swap contracts is very small.<sup>15</sup> We extract the zero-coupon yields from the swap rates, using the un-smoothed Fama-Bliss procedure (or bootstrapping assuming constant forward rates over absent maturities).<sup>16</sup> We use the following maturities: 3 months, 6 months, 1 year, 2 years, 5 years, 7 years, and 10 years. In general, the swap-implied zero-coupon rates look quite similar to (upward-sloping etc.) and are highly correlated with government bond yields. To save space, general summary statistics are not reported, but certain important moments can be found in Table 5.

Following Ang and Piazzesi (2003) among others, we use two macroeconomic variables in the vector  $M$  for each country – the inflation rate and Industrial Production (IP) growth (denoted by  $\pi$  and  $g$  respectively), calculated as the overlapping 12-month (year-on-year) logged changes in the seasonally-adjusted CPI and IP indices, respectively. The data are at the monthly frequency for the periods from 1987:04 to 2008:03. The starting date is

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<sup>15</sup>See Blanco, Brennan, and Marsh (2005), Duffie and Huang (1996), and J. Hull and White (2004) for further elaboration.

<sup>16</sup>See Fama and Bliss (1987) for details.

restricted by the starting date of LIBOR.<sup>17</sup> For now, we exclude the most recent data (though the crisis period remains an interested laboratory for future research). Table 1 reports the cross-country correlations of these variables. As discussed, the long-term yields are more highly correlated than both the short-term rates and the macro variables. This is not due to a simple explanation that the long-term yields are less volatile than short-term rates. First, the standard deviations of long-term yields are only slightly lower than those of the short-term rates. As pointed out by Gallmeyer, Hollifield, Palomino, and Zin (2009) and Wachter (2006), this pattern is difficult to produce in a usual rational expectation model with stationary state variables. Second, the covariances, not just the correlations, are higher at the long end (once the predictability has been accounted for) (see Figure 1). This means that long-term yields are likely to have greater loadings on common state variables than do short-term rates. Together, the data and the theoretical literature hints at the significance of volatile and correlated market prices of risk in explaining the cross-country covariation of yields. This paper examine this channel in detail under a no-arbitrage VAR framework.

Another interesting observation from Table 1 is that the correlations of short-term rates are only slightly higher than those of inflation rates, suggesting that inflation may drive the correlations at the short end of the term structure. Whether or not inflation is as important in explaining the co-movement across the long maturity bond yields remains a question. For the U.S., Ang, Bekaert, and Wei (2008) finds that the term structure of real interest rates is flat and that the inflation risk premium largely explains the variation in long-term yields.<sup>18</sup> These findings thus suggest that inflation is likely to also be an important driver of

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<sup>17</sup>Our starting date comes after most developed central banks have managed to better control inflation, so we do not face the problem of high and non-stationary inflation rates. Clarida, Gali, and Gertler (1998) provide some guidance on when controlling inflation became a major focus. For the Bundesbank, they identify March 1979, the time Germany entered the European Monetary System, at the starting point. They identify October 1979 for the Fed, when Volcker clearly signalled his intention to rein in inflation. Clarida et al. experiment with the post-1982 sample period, as the operating procedures of the Fed prior to 1982 focused on meeting non-borrowed aggregate reserve targets instead of managing short interest rates.

<sup>18</sup>See also Wright (2009) for international evidence.

cross-country covariances of bond yields. Finally, both the macro variables and short-term rates exhibit higher correlations for the U.S. and U.K. pair than for the U.S. and Germany pair, implying possibly that the U.S. is more economically integrated with the U.K. than to Germany. As described in the previous section, we model the relationships among countries as exposures to common world factors (taking the U.S. as the world). In general, the patterns in the data seem to indicate that macro variables, in general, are critical in the determination of bond yield co-movements.

### 3.2 Construction of Pricing Factors

We construct the vector of pricing factors  $P$  as the first  $N$  principal components of yields for each currency. We choose  $N^W$ ,  $N^{UK}$ , and  $N^{DE}$  to be 3, 2, and 2, respectively. Table 2 reports the variations explained by these principal components and their loadings on the zero-coupon yields. In all currencies,  $P^1$  explains around 95% of yield variations, and  $P^1$  and  $P^2$  together explain over 99% of yield variations. Thus, without imposing no-arbitrage, it appears that using these factors will allow us to fit the term structures quite well. The loadings of  $P$  on yields suggest that in all currencies,  $P^1$  is the “level” factor since it loads almost equally on all yields.  $P^2$  appears to be the “slope” factor since it loads negatively on the short-term rates and positively (increasingly so) on the long-maturity rates.  $P^{W,3}$  appears to be the “curvature” factor as it loads positively on the middle part of the yield curve but negatively at the short and long ends. Our results are thus consistent with what others have documented about the behavior of the term structure, particularly that over 90% of the time, the term structure moves in a parallel fashion.<sup>19</sup> Note that these pricing factors are *observed*, resulting in an easier estimation environment relative to the latent factors found in most term structure models.

In order to understand how important the macro variables are in explaining these pricing

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<sup>19</sup>See Litterman and Scheinkman (1991) for example.

factors, we regress each pricing factor on the macro variables and document the R-squared in the last four columns of Table 2. In all currencies, over half of the variation in  $P^1$  can be accounted for by the world and local macro variables. This proportion declines for  $P^2$ , particularly for the U.S. Still, given that  $P^1$  explains over 95% of variation bond yields, the high R-squared's from regressing  $P^1$  on the macro variables suggest that macro variables are important in explaining the movements of term structures. We will perform variance decompositions as part of our analysis to confirm this conjecture. Another interesting finding from Table 2 is that the world macro and pricing factors can explain a large portion of the local term structure variability, mostly through  $P^{i,1}$ .

Together, dropping the superscript  $i$  as we stack together the state variables of all three countries, the vector  $Y$  in (17) is  $13 \times 1$ , including 2 world macro variables, 3 world pricing factors, 2 local macro variables for each non-U.S. country, and 2 pricing factors for each non-U.S. country. Given the structure of our model, the challenge in estimation lies in searching for  $\lambda$  (this will be clear in the next subsection), and given that  $\lambda$  is diagonal, it only grows linearly with the number of pricing factors. These 13 state variables, including only 7 pricing factors, will not yield a specification too large to be estimated.

### 3.3 Estimation

Starting with the observed pricing factors ( $P$ ), we obtain the residual yield curve factors ( $P^e$ ) by the projections described in (12) and (13). We estimate the model parameters jointly by maximum likelihood (ML). We write down explicitly the log-likelihood function of the

observed data for all three countries– state variables  $Y_t^e$ , and bond yields  $y_{n,t}$  – as follows:

$$\begin{aligned}\mathcal{L} &= \sum_{t=1}^{T-1} \ln(f(Y_{t+1}^e, ye_{t+1}|Y_t^e)) \\ &= \sum_{t=1}^{T-1} [\ln(f(Y_{t+1}^e|Y_t^e)) + \ln(f(ye_{t+1}|Y_{t+1}^e))] \end{aligned} \quad (22)$$

where  $f(y|x)$  is the Gaussian density function and  $ye$  is a vector of all pricing errors.  $Y_t^e$  is the union of all the  $Y_t^{i,e}$  vectors described earlier. In our estimation, we take the yields for maturities 6 months, 5 years, and 10 years as being priced with IID errors. These pricing errors are assumed to be the same for all maturities but different across countries.<sup>20</sup> This choice of IID errors and constant variance structure is standard. The rationale is that it forces the estimation to fit the data as much as possible through the structure of the model as opposed to pricing errors. The first term in (22) corresponds to the physical dynamics (time series fit) and the second term to the risk-neutral pricing (cross-sectional fit). It is easy to see that unlike models with latent factors, the two terms in (22) are *almost* independent (since  $Y_{t+1}^e$  is observed). This feature will be helpful in our estimation, given the total number of parameters we need to estimate is quite large.

To save notations, let's denote the Gaussian density function by:

$$f^G(X, \Sigma) = -\frac{1}{2} \ln((2\pi)^N |\Sigma \Sigma'|) - \frac{1}{2} \text{tr} (X'(\Sigma \Sigma')^{-1} X)$$

where  $N$  is the size of  $\Sigma$ . We can then write (22) as a function of parameters to be estimated below:

$$\mathcal{L} = \sum_{t=1}^{T-1} [f^G(Y_{t+1}^e - \kappa_0^e - \kappa_1^e Y_t^e, \Sigma_{Y^e}) + f^G(y_{t+1} - (A_P + B'_P \Pi Y_{t+1}^e), \Sigma_{ye})] \quad (23)$$

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<sup>20</sup>Recall that  $P$  comes from portfolios of 6 yields for each currency. Since the maximum size of  $P$  is 3 for  $P^W$ , we can only have 3 yields at most that can be priced with IID errors.

where  $\Sigma_{ye}$  is a diagonal matrix whose diagonal elements correspond to the standard deviations of the pricing errors;  $y_{t+1}$  is the vector of yields priced with errors;  $\Pi$  is obtained by appropriately combining together  $\Pi^i$ , described in the model section.

We implement our estimation procedure, relying on the fact that holding  $\lambda$  (the risk-neutral drift fixed), the ML estimates of all other parameters can be derived (quasi-)analytically. We omit the detailed proof<sup>21</sup> but briefly sketch out our procedure here. Given the functional forms of  $A_P$  and  $B_P$ , the second term only depends on  $\lambda$ ,  $r_\infty$ ,  $\sigma_{ye}$ , and  $\Sigma_{Y^e}$ . Among these, only  $\Sigma_{Y^e}$  shows up in the first term. Therefore, if  $\Sigma_{Y^e}$  is given, then the two parts of (23) can be separately estimated. The physical dynamics parameters in the first part can be estimated analytically by typical least squares procedures. In doing this, we force the model-implied means to match the sample means of the pricing factors. Among the risk-neutral parameters that appear in the second part, the global estimates of  $r_\infty$  and  $\Sigma_{ye}$ , can be solved for analytically since the first order conditions with respect to these parameters have affine forms. These leave us with only  $\Sigma_{Y^e}$ , which is shared by both the physical dynamics part and the risk-neutral dynamics part of the likelihood function. The first order condition of  $\Sigma_{Y^e}$  is non-linear. We solve this non-linear equation by iterations, starting with the values implied by the VAR(1) of  $Y_{t+1}^e$ . We find in our implementation that convergence is very fast, typically obtained after four or five iterations.

Since the estimates of all other parameters follow (quasi-)analytically given the values of  $\lambda$ , maximizing  $\mathcal{L}$  only requires numerically searching over  $\lambda$  which has only seven unique elements in our case. Our maximization is fast and the convergence is robust. For this reason, we are able to compute the small-sample standard errors of our estimates by Monte Carlo simulations.

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<sup>21</sup>The detailed proof is available from the authors upon request.

## 4 Empirical Results

### 4.1 Model Estimates and Fit

Table 3 reports the ML estimates of model parameters. Overall, the model fits the data quite well as indicated by the yield measurement errors of 3-13 basis points, in line with others in the literature. The risk-neutral persistence parameters  $\lambda$  are around one for all unobserved pricing factors  $X$ , suggesting that for pricing purposes shocks to these factors are treated as if they are permanent. This means that even long-term yields respond strongly to shocks. These persistence parameters, transformed to obtain the counterparts for  $P^i$  are higher than their physical values, implying that the market price of risk is important in explaining observed variation in long-maturity bond yields.

The next set of parameters ( $a$  and  $b$  in equations (12) and (13)) show that the pricing factors load strongly on the inflation rate, particularly the *local* inflation rate. Recall from Table 2 that the observed pricing factors for each currency  $P^i$  (which are linear transformations of  $X^i$ ) can be interpreted as the level, slope, and curvature factors, and that macro variables explain over half of the variation in the level factors. The loadings documented here thus demonstrate that, among the macro variables, inflation rates are most important in explaining bond yield variation. Indeed, Table 4 shows that the model-implied monetary policy rule looks similar to the Taylor rule (taking the one-month rate as the policy rate) – the coefficients on local inflation are not significantly different from 1.5 for all countries. However, the coefficients on industrial production growth (as a proxy for output gap) are close to zero as opposed to 0.5 as Taylor suggests. For the U.K. and Germany, the estimated responses to the world inflation are positive but relatively small, possibly as a result of high correlations between the world and local inflation rates, rendering the identification of each effect imprecise. These small responses may potentially lead to small model-implied correlations of yields, given the importance of inflation.

We report estimates of the feedback matrix ( $\kappa_1$ ) and the contemporaneous relationships ( $\Sigma_Y$ ) by country. Note that these are  $Y$ , not  $Y^e$ , which already reflects the contemporaneous relationships between  $P$  and  $M$  (through  $\Pi$ ). The estimates of  $\Sigma_Y$  show a strong influence of the world and local inflation rates on the level factor  $P^1$  in all countries. Interestingly for the U.K., even if the loading of  $P^{UK,1}$  on the local IP growth  $g^{UK}$  is not big, the high volatility of  $g^{UK}$  makes this factor quite important in explaining the variance of  $P^{UK,1}$  and as a result the variance of U.K. yields. Other than these relationships, the contemporaneous links among the state variables are economically insignificant. The estimates of  $\kappa_1$  give the same message. Every state variable is persistent, though less so than in the risk-neutral world, feeding mostly on itself. The cross feedbacks are weak with only few exceptions. First, in all countries, IP growth responds negatively to lagged inflation (though the effects are not necessarily statistically significant). Second, IP growth in the U.K. feeds on the lagged world IP growth. This effect does not exist for Germany. Third, the world curvature factor  $P^{W,3}$  strongly predicts every country's level factor  $P^1$ . This is consistent with existing evidence that factors that do not matter much in explaining contemporaneous variations in yields may be important predictors of future interest rates and excess bond returns (see Cochrane and Piazzesi (2005) for example).

We examine the general goodness of fit of the model in Table 5. To assess the pricing fit (or fit of the risk-neutral parameters), we compare the sample moments of fitted yields with those of the data. The table shows that, conditional on observing all state variables, the model provides a close match to the means, standard deviations, first-order autocorrelations, and cross-country correlations of the bond yields for all countries and maturities. We conduct the tests of difference in moments using GMM standard errors with twelve lags (Newey and West (1987)'s weights). The differences are statistically significant for a few moments, but even for these moments, the economic differences are very small. Most importantly, the model appears to match the cross-currency correlations of bond yields well. The fitted

values captures between 94-98% of cross-country yield covariance.

We also assess the fit of the overall model, including both the physical and risk-neutral dynamics, by comparing the model-implied unconditional moments at the ML estimates with the sample moments from the data. The model matches the averages of bond yields by construction, but overestimates the volatilities, particularly at the short end of the U.S. term structure and the long end of German term structure. The model slightly underestimates the persistence of yields. Finally, the model significantly underestimates the cross-currency correlations of yield changes and produces unconditional correlation structures that are too flat. Thus, our covariance analysis based on the estimated model parameters may only partially capture the observed co-movement of yields.<sup>22</sup>

In Table 6, we examine how well the model explains the behavior of risk premiums observed in the data using Campbell and Shiller (1991)'s, henceforth C-S, regressions for a single term structure and Sutton (2000)'s modification for a pair of term structures. These coefficients speak to the correlations between risk premia and the short rate and among risk premia in different currencies. Consistent with the evidence in C-S and Dai and Singleton (2000) among others, the coefficients estimated from the data are generally negative (increasingly so at longer maturities).<sup>23</sup> The magnitudes are smaller than those reported in earlier studies, possibly due to risk premia becoming smaller and less volatile in recent years. For the U.S., our model implied C-S coefficients are flat, too negative at the short end and less negative than the empirical estimates at the long end. For the U.K., our model implied C-S coefficients are too negative throughout but show the same downward sloping structure as do the empirical estimates. For Germany, our model implied C-S coefficients are flat

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<sup>22</sup>Clearly, with these low implied correlations, our model also produces the exchange rates that are more volatile than in the actual data (unreported).

<sup>23</sup>C-S and Dai and Singleton (2000) show that the expectations hypothesis would predict that the C-S regression coefficients be unity for all maturities. Negative coefficients imply positive correlations between bond excess returns and term spreads, which Dai and Singleton (2000) argue are driven by negative correlations between the short rates and risk premia.

and positive for all maturities, matching the short end well but missing significantly at long maturities. These results indicate that in most cases, our model may underestimate either the negative correlations between risk premia and short rates or risk premium volatilities or both. In fact, given that we attempt to match the data with the least number of parameters (as we are dealing with several term structures), our model’s performance in each currency is surprisingly good compared to many models considered by Dai and Singleton (2000).

We now turn to the model’s performance in matching the co-movement of excess bond returns or realized risk premia. The last two rows of Table 6 show that the data exhibit positive cross-currency correlations of risk premia as shown in Sutton (2000). Our model does capture this feature of the data, which is important for our study given that our goal is to measure the importance of risk premia in explaining cross-currency co-movements of bond yields. Still, in both currency pairs, our model produces risk premia correlations that are (i) too low and (ii) too flat in structure, compared to the empirical estimates from the data. This pattern resembles the mismatch in the overall cross-country correlations of yield changes in Table 5, suggesting that underestimating the correlations of term premia contributes to underestimating the overall co-movements. This means that the extent to which we may overestimate the importance of risk premia as a result of underestimating the overall yield co-movements but overestimating the risk premium co-movements is limited.

## 4.2 Impulse Responses

To understand the joint dynamics of bond yields, we first examine the joint dynamics of the state variables. In Figure 3, we plot the responses of inflation rates, IP growth rates, and the level factors in each currency to a one standard deviation shock in the world macro variables. A one standard deviation shock to the world (U.S.) inflation rate causes the U.S. inflation rate to initially increase by approximately 0.30%. This response dies down over

2-3 years, suggesting that under the expectations hypothesis, long-term yields should not respond much to this shock. Inflation rates in the U.K. and Germany hardly respond to the world inflation shock (less than 0.10%). This is surprising and may be a result of the model attributing much of the observed correlations in inflation to the high degree of persistence in these variables (i.e. spurious correlations). The world inflation rate has a negative impact on U.S. IP growth (about 0.3% or a third of growth standard deviation) but no discernible impact on the U.K. and German IP growths. The response of U.S. IP growth to inflation grows slowly, peaks in about 1 year, and decays to zero over the next few years. Despite weak responses of the U.K. and German inflation and growth to the world inflation shock, the level factors of all three countries respond strongly (0.5-1%) and in the same direction to the world inflation shock. This suggests that the world inflation is important in explaining the cross-country co-movements of bond yields. On the other hand, the world IP growth only affects the U.S. IP growth itself and the U.S. level factor and therefore appears to contribute little to yield co-movement.

In Figure 4, we turn our attention to the responses of bond yields to various economic shocks. Since the slope and curvature factors explain only a small portion of movement in yields, we focus only on the inflation rate, IP growths rates, and the level factors. We also decompose the responses into those that operate through the policy channel and those that go through the risk compensation channel. The policy component is calculated based on the expectations hypothesis: the long-term yield is the expected average of the short-term rates from now to maturity (i.e. the market price of risk is zero). Thus, the policy component is a result of the monetary policy loadings to various state variables and how these state variables evolve through time. The risk compensation component is the difference between the total responses and the policy component. Note that the two components can be of opposite signs, in which case the sign of the total responses will depend on which component dominates.

Panel A-1 shows that a one-standard deviation increase in the world inflation rate ( $\pi^W$ ) increases the U.S. and German 6-month rates equally by about 40 basis points. Since these responses are relatively large and in the same direction (positive), the contributions of  $\pi^W$  to the co-movement of short-term rates must be significant and positive. The impact of the world inflation on the U.K. 6-month rate is significantly smaller (about 20 basis points). Importantly, at this maturity, the world inflation affects yields almost exclusively through the policy channel.

The responses of the U.S. and German 10-year yields to  $\pi^W$  are in the same direction and slightly smaller than those of the short-term yields. For the U.K., the 10-year yield response is about the same as that of the short-term yield. Due to the stationarity of  $\pi^W$ , the policy components of the responses are indistinguishable from zero. Almost all of the responses come from the risk compensation channel. Germany is an exception. The policy and risk compensation components are initially about half and half of the total response of 10-year yield, but the proportion becomes much smaller for the policy component after 1-2 years. Thus, at long maturities, the world inflation rate is an important factor in explaining variation in bond yields. In contrast to what we observe at short maturities, it works mostly through the risk compensation channel for long-maturity bond yields.

Panel A-2 of Figure 4 plots the responses of the U.K. and German yields to their own local inflation rate ( $\pi^{UK}$  and  $\pi^{DE}$ ). For the U.K., the responses to  $\pi^{UK}$  are twice as big as those to  $\pi^W$  at 6-month maturity but are of about the same size at the 10-year maturity. For Germany, the responses to  $\pi^{DE}$  are about 5-10 basis points smaller than those to  $\pi^W$ . Interestingly, unlike the world inflation, the local inflation rate seem to impact 10-year yields equally through the policy and risk compensation channels.

Panels B-1 and B-2 of Figure 4 illustrate the responses of yields to shocks in the world IP growth and the local IP growth rates, respectively. Three interesting patterns emerge. First, the yield responses to  $g^W$  are significantly smaller than those to  $\pi^W$ . The difference

is very stark at the long end, indicating that the world inflation is a key driving factor of risk premiums.<sup>24</sup> Second, for the U.K. and Germany, the impacts of  $g^W$  on yields manifests slowly, becoming larger at longer horizons. This implies that, for these countries,  $g^W$  may lead other factors that matter in pricing by several months. Third, the local IP growth rate ( $g^{UK}$  and  $g^{DE}$ ) have no discernable impact on German yields but large impact on the U.K. yields. The importance of local variables for U.K. yields (both  $\pi^{UK}$  and  $g^{UK}$ ) may explain why cross-country yield correlations are lower for the U.S.-U.K. pair than for the U.S.-Germany pair.

Panels C-1 and C-2 of Figure 4 illustrate the responses of yields to shocks in the world (or U.S.) level factor and the local level factors, respectively. The responses to these factors appear to be large (over 60 basis points) but local in that the U.S. yields only respond to  $P^{US,1}$ , the U.K. yields to  $P^{UK,1}$ , and German yields to  $P^{DE,1}$ . Thus, it appears that the level factors generally cannot explain the cross-country co-movement of bond yields (except through the macro components of these factors). Taking all these impulse-responses together, the world inflation rate seems to be the dominant factor that affects, in the same direction, yields for all maturities and countries. We thus expect the world inflation to account for the largest fraction of the observed yield covariances. Finally, the world inflation rate operates primarily through the policy channel at the short end but through the risk compensation at the long end. This is consistent with the general findings of Ang, Bekaert, and Wei (2008) for the U.S. and Wright (2009) for several developed countries.

### 4.3 Variance and Covariance Decomposition

Following Ang and Piazzesi (2003) and others, we construct variance and covariance decompositions to obtain the contribution of each state variable to the variances and covariances of

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<sup>24</sup>The U.K. 6-month rate is the only rate that responds almost equally to the world IP growth and the world inflation. This suggests that for other pairs, the co-movements are driven mostly by inflation but for the U.S.-U.K. 6-month rates, the world inflation and IP growth are both important.

forecast errors of bond yields. These decompositions measure the proportion of the forecast variance or covariance (typically positive, but can be negative for the covariance) attributable to each factor, and are closely related to the impulse responses at different horizons. In addition, we decompose each coefficient matrix in the  $MA(\infty)$  representation of a yield into the matrix resulting from the physical dynamics alone and that resulting from the time-varying market price of risk. The relative contributions of the policy channel (again assuming the expectations hypothesis holds) and the risk compensation channel for each shock can then be obtained.

Table 7 reports the sum of proportions of yield forecast variance attributable to all macro variables (left panel) and those to all state variables through the risk compensation channel (right panel). The contributions of macro variables to yield variances are of similar magnitudes to those reported by Ang and Piazzesi (2003). The importance of macro variables becomes greater at longer forecast horizons. The contributions to yield variances of all variables ( $M$  and  $P$ ) through the risk compensation channel show a clear pattern that is similar for all countries. These contributions are very small (less than 20%) at all forecast horizons, but significantly larger at short horizons than at long horizons. As we move to five-year yields, the risk compensation channel becomes more important, with contributions to variances ranging from 57-81%. The most striking finding is that as we move to ten-year yields, the contributions through the risk compensation channel are extremely large, in the range of 85-94%. This can be explained by the pattern of impulse-responses discussed in the previous section, consistent with the fact that the state variables are stationary and therefore converge to the means in the long run. This evidence indicates that the longer-term yields move with the state variables more than predicted by the expectations hypothesis. In line with this finding, Kuttner (2001) finds that although unanticipated monetary policy shocks affect bond yields of all maturities, these shocks contain little information about future monetary policy actions. The importance of time-varying market prices of risk has been

discussed in detail by Campbell and Shiller (1991), Dai and Singleton (2002), and Duffee (2002).

Next, we turn to the covariance decompositions reported in Table 8. The results for the U.S.-U.K. pair (in Panel A) show that, consistent with the variance decompositions presented above, the total contribution of all factors that go through the market prices of risk are small for short-term rates (even negative in some cases) and very large for long-term yields. The risk-compensation channel contributes 42-50% to the co-movements of the U.S. and U.K. five-year yields, depending on the forecast horizon. These contributions increase to roughly 80% at the ten-year maturity. The results for the U.S.-Germany pair in Panel B are similar. The contributions of the risk compensation channel to the covariances between the U.S. and German yields appear to be even larger. Collectively, the evidence suggests that international macro-models need to facilitate a role for the effects fluctuations in the macro model imply not just for monetary policy but also for time-varying risk compensation (implicitly the role of international investors).

To provide some guidance on this point, the state variable that unambiguously contributes the most to the co-movements (and hence risk-premia variation) at all maturities is the world inflation rate. At the short end, the world IP growth and the world (or U.S.) level factors are secondary. The contributions of the world inflation rate and level factors decay over long horizons, whereas the contribution of the world IP growth rate increases over time. This can be explained by the patterns of yield responses to these factors in Figure 4. At the long end, the world inflation clearly dominates, accounting for 80-90% and 90-100% of yield covariances at five-year and ten-year maturities, respectively. Further, the importance does not diminish over longer horizons (unlike at the short end).<sup>25</sup> Taken together, we conclude that the evidence suggests that the world inflation rate is the single most important variable

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<sup>25</sup>Interestingly, the contributions of the world IP growth rate through the policy and risk compensation channels appear to cancel out, although each individually is quite significant.

for explaining the cross-country co-movements of bond yields (at least for the portion of co-movements captured by our model). The world inflation works mostly through the policy channel at the short end but almost exclusively through the risk compensation channel at the long end.

## 5 Conclusions

This paper provides a possible explanation for the observed degree of co-movement in yield curves across different currencies and maturities. This issue is important as a first step in better understanding the linkages among international asset markets. We jointly model the term structures of interest rates in several countries using an affine model in which the state variables follow a vector autoregressive (VAR) process. We use only a few pricing factors based on linear combinations of yields, but also allow macro variables (including the inflation rate and industrial production growth) to influence the pricing factors in the physical dynamics. The model is estimated to match the data for the U.S., the U.K., and Germany. The policy channel refers to the propagation of shocks through the physical dynamics of short-term rates, as predicted by the expectations hypothesis. The risk compensation channel refers to the responses of bond yields to shocks that are in excess of those transmitted through the policy channel. We find that the policy and the risk compensation channels account for 10-58% and 42-90%, respectively, of the cross-country covariances of 5-year bond yields. As we move to longer maturities, these numbers shift considerably in favor of the risk compensation channel. The most important state variable is the world inflation rate, which explains over half of the co-variances among bond yields at almost all maturities and forecast horizons. An acknowledgement of the risk compensation channel, often absent or secondary in the literature, is critical for understanding the behavior of the yield curve, in general, and long maturity (benchmark) bonds, in particular.

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	Level	1-Month Change	1-Year Change	Conditional (Macros Only)	Conditional (Macros and Yields)
<b>Panel A: U.S. - U.K.</b>					
Inflation Rate	0.742	0.174	0.386	-	-
IP Growth	0.635	0.024	0.488	-	-
6-Month Yield	0.767	0.233	0.528	0.414	0.039
1-Year Yield	0.783	0.268	0.536	0.470	0.106
2-Year Yield	0.843	0.387	0.615	0.563	0.210
5-Year Yield	0.908	0.456	0.691	0.562	0.272
7-Year Yield	0.919	0.499	0.705	0.527	0.304
10-Year Yield	0.924	0.495	0.694	0.496	0.328
<b>Panel B: U.S. - Germany</b>					
Inflation Rate	0.319	0.270	0.075	-	-
IP Growth	0.095	0.203	0.314	-	-
6-Month Yield	0.407	0.399	0.230	0.509	0.265
1-Year Yield	0.454	0.432	0.305	0.548	0.269
2-Year Yield	0.560	0.497	0.441	0.644	0.328
5-Year Yield	0.745	0.554	0.607	0.720	0.398
7-Year Yield	0.797	0.573	0.660	0.721	0.423
10-Year Yield	0.832	0.594	0.691	0.711	0.450

**Table 1: Cross-Country Correlations.** This table reports the cross-country correlations of (1-year moving) inflation rate, (1-year moving) IP growth rate, and the LIBOR-swap zero-coupon yields for maturities 6 months, 1 year, 2 years, 5 years, 7 years, and 10 years. The sample period is from 1987:04 to 2008:03. The frequency is monthly. The correlations are calculated based on: (i) the level, (ii) the monthly change, (iii) the annual change (overlap), (iv) the forecast errors based on a linear model with inflation and output gap up to 12 lags, and (v) the forecast errors based on a linear model with inflation, output gap, and bond yields up to 12 lags. Panel A reports the correlations between the U.S. and the U.K. variables. Panel B reports the correlations between the U.S. and German variables.

Principal Component	Variance Explained	Loading on Yield							10-Year	% Explained $M^W$	% Explained $M^W, P^{US}$	% Explained $M^W, M^i$	% Explained $M^W, P^{US}, M^i$
		6-Month	1-Year	2-Year	5-Year	7-Year							
U.S.													
1st	94.35	0.45	0.44	0.44	0.39	0.37	0.35	0.35	50.80	-	-	-	-
2nd	5.45	-0.51	-0.40	-0.13	0.30	0.43	0.53	0.53	4.32	-	-	-	-
3rd	0.17	-0.61	0.14	0.65	0.21	-0.10	-0.36	-0.36	9.51	-	-	-	-
U.K.													
1st	96.49	0.48	0.45	0.42	0.38	0.37	0.36	0.36	48.18	87.30	71.57	93.86	
2nd	3.28	-0.52	-0.39	-0.09	0.29	0.43	0.55	0.55	5.80	42.79	29.26	58.01	
Germany													
1st	94.87	0.48	0.46	0.45	0.38	0.34	0.31	0.31	31.49	66.12	59.94	88.17	
2nd	4.80	-0.46	-0.37	-0.14	0.30	0.47	0.57	0.57	18.57	40.62	35.98	58.19	

**Table 2: Principal Components of Yields.** This table reports (i) the variance of yields explained by the first few principal components in each currency, (ii) the loadings of these principal components on yields, and (iii) the relationships between these principal components with the world and local macro variables. The macro variables ( $M$ ) include inflation ( $\pi$ ) and IP growth ( $g$ ). For currencies other than the U.S., the relationships with the U.S. principal components ( $P$ ) are also reported. The superscripts  $W$ ,  $US$ , and  $i$  denote the world, the U.S., and each of the local currencies - the U.K. and Germany, respectively.

Parameters Specific to Pricing Factors							
	$X^{US,1}$	$X^{US,2}$	$X^{US,3}$	$X^{UK,1}$	$X^{UK,2}$	$X^{DE,1}$	$X^{DE,2}$
$\lambda$	0.998 (0.000)	0.946 (0.004)	0.902 (0.010)	1.001 (0.001)	0.950 (0.003)	0.996 (0.001)	0.973 (0.002)
	$P^{W,1}$	$P^{W,2}$	$P^{W,3}$	$P^{UK,1}$	$P^{UK,2}$	$P^{DE,1}$	$P^{DE,2}$
$b(\pi^W)$	2.985 (0.855)	0.014 (0.337)	-0.045 (0.033)	0.948 (0.924)	0.360 (0.491)	1.710 (1.075)	0.264 (0.434)
$b(g^W)$	0.531 (0.314)	-0.080 (0.113)	0.011 (0.014)	-0.008 (0.301)	-0.007 (0.156)	0.076 (0.390)	0.131 (0.153)
$a(\pi^i)$	-	-	-	2.173 (0.643)	-0.350 (0.321)	1.976 (1.198)	-0.374 (0.494)
$a(g^i)$	-	-	-	0.559 (0.256)	0.150 (0.106)	-0.063 (0.177)	-0.077 (0.066)
Parameters Specific to Each Currency							
	US	UK	DE				
$r_\infty$	0.017 (0.003)	-0.009 (0.036)	0.011 (0.008)				
Measurement Error in Basis Points	3.4 (0.1)	12.9 (0.4)	12.0 (0.5)				

**Table 3: Maximum Likelihood Estimates.** This table reports maximum likelihood estimates of the model parameters (notations as described in the model section). The priced factors ( $P$ ) are the principal components of yields. The macro variables ( $M$ ) which partially explain the dynamics of the priced factors include inflation ( $\pi$ ) and IP growth ( $g$ ). The superscripts  $W$ ,  $US$ ,  $UK$ , and  $DE$  denote the world, the U.S., the U.K., and Germany, respectively. The superscript  $i$  denotes the currency corresponding to each row or column to which it applies. Small-sample standard errors are calculated by Monte Carlo simulation at the maximum likelihood parameter estimates, and are in parentheses.

Dynamics of World and U.S. Factors

	$K_1$					
	$K_0$	$\pi^W$	$g^W$	$P^{W,1}$	$P^{W,2}$	$P^{W,3}$
$\pi^W$	0.001 (0.001)	0.960 (0.026)	0.010 (0.008)	0	0	0
$g^W$	0.005 (0.002)	-0.126 (0.059)	0.949 (0.027)	0	0	0
$P^{W,1}$	-0.001 (0.009)	0.175 (0.220)	0.055 (0.054)	0.933 (0.033)	-0.019 (0.097)	0.367 (0.489)
$P^{W,2}$	0.001 (0.003)	-0.030 (0.043)	-0.023 (0.013)	0.006 (0.008)	0.969 (0.026)	-0.305 (0.149)
$P^{W,3}$	0.000 (0.001)	-0.008 (0.014)	-0.001 (0.004)	-0.001 (0.002)	0.000 (0.008)	0.822 (0.039)

	$\Sigma$				
	$\pi^W$	$g^W$	$P^{W,1}$	$P^{W,2}$	$P^{W,3}$
$\pi^W$	0.0030 (0.0001)	0	0	0	0
$g^W$	0.0002 (0.0005)	0.0083 (0.0004)	0	0	0
$P^{W,1}$	0.0092 (0.0025)	0.0044 (0.0025)	0.0116 (0.0009)	0	0
$P^{W,2}$	0.0000 (0.0010)	-0.0007 (0.0009)	0.0002 (0.0004)	0.0025 (0.0005)	0
$P^{W,3}$	-0.0001 (0.0001)	0.0001 (0.0001)	0.0002 (0.0001)	0.0004 (0.0001)	0.0010 (0.0001)

Table 3, cont'd: Maximum Likelihood Estimates.

Dynamics of U.K. Factors

	$K_1$									
	$K_0$	$\pi^W$	$g^W$	$P^{W,1}$	$P^{W,2}$	$P^{W,3}$	$\pi^{UK}$	$g^{UK}$	$P^{UK,1}$	$P^{UK,2}$
$\pi^{UK}$	0.000 (0.001)	-0.011 (0.028)	-0.003 (0.012)	0	0	0	0.989 (0.018)	0.039 (0.011)	0	0
$g^{UK}$	-0.002 (0.004)	0.108 (0.106)	0.144 (0.044)	0	0	0	-0.083 (0.053)	0.731 (0.046)	0	0
$P^{UK,1}$	-0.002 (0.014)	0.071 (0.268)	-0.025 (0.081)	0.049 (0.047)	0.058 (0.148)	0.398 (0.647)	0.248 (0.151)	0.067 (0.081)	0.864 (0.048)	0.168 (0.122)
$P^{UK,2}$	0.003 (0.004)	0.074 (0.079)	-0.002 (0.024)	0.029 (0.014)	-0.007 (0.047)	-0.110 (0.186)	-0.051 (0.049)	0.008 (0.021)	-0.020 (0.015)	0.917 (0.034)

	$\Sigma$									
	$\pi^W$	$g^W$	$P^{W,1}$	$P^{W,2}$	$P^{W,3}$	$\pi^{UK}$	$g^{UK}$	$P^{UK,1}$	$P^{UK,2}$	
$\pi^{UK}$	0.0004 (0.0002)	0.0001 (0.0002)	0	0	0	0.0030 (0.0001)	0	0	0	
$g^{UK}$	0.0003 (0.0007)	0.0007 (0.0007)	0	0	0	0.0017 (0.0008)	0.0120 (0.0006)	0	0	
$P^{UK,1}$	0.0040 (0.0029)	0.0006 (0.0025)	0	0	0	0.0076 (0.0020)	0.0067 (0.0029)	0.0123 (0.0011)	0	
$P^{UK,2}$	0.0010 (0.0014)	0.0000 (0.0012)	0	0	0	-0.0008 (0.0010)	0.0018 (0.0012)	-0.0007 (0.0006)	0.0035 (0.0005)	

Table 3, cont'd: Maximum Likelihood Estimates.

Dynamics of German Factors

	$K_1$									
	$K_0$	$\pi^W$	$g^W$	$P^{W,1}$	$P^{W,2}$	$P^{W,3}$	$\pi^{DE}$	$g^{DE}$	$P^{DE,1}$	$P^{DE,2}$
$\pi^{DE}$	0.000 (0.001)	0.027 (0.027)	-0.012 (0.010)	0	0	0	0.952 (0.024)	-0.001 (0.007)	0	0
$g^{DE}$	0.001 (0.006)	0.192 (0.176)	0.025 (0.070)	0	0	0	-0.205 (0.170)	0.842 (0.042)	0	0
$P^{DE,1}$	-0.004 (0.012)	0.115 (0.217)	-0.037 (0.066)	0.025 (0.040)	-0.055 (0.113)	0.307 (0.548)	0.204 (0.233)	0.010 (0.031)	0.913 (0.044)	0.146 (0.100)
$P^{DE,2}$	0.002 (0.003)	-0.006 (0.073)	-0.003 (0.017)	-0.001 (0.012)	-0.040 (0.038)	0.025 (0.173)	-0.042 (0.074)	-0.014 (0.011)	0.011 (0.014)	0.946 (0.040)

	$\Sigma$									
	$\pi^W$	$g^W$	$P^{W,1}$	$P^{W,2}$	$P^{W,3}$	$\pi^{DE}$	$g^{DE}$	$P^{DE,1}$	$P^{DE,2}$	
$\pi^{DE}$	0.0009 (0.0002)	0.0002 (0.0002)	0	0	0	0.0028 (0.0001)	0	0	0	
$g^{DE}$	0.0023 (0.0012)	0.0035 (0.0013)	0	0	0	-0.0020 (0.0011)	0.0184 (0.0009)	0	0	
$P^{DE,1}$	0.0068 (0.0030)	0.0009 (0.0033)	0	0	0	0.0057 (0.0033)	-0.0012 (0.0031)	0.0095 (0.0013)	0	
$P^{DE,2}$	0.0003 (0.0012)	0.0007 (0.0012)	0	0	0	-0.0009 (0.0013)	-0.0014 (0.0011)	0.0001 (0.0008)	0.0027 (0.0005)	

Table 3, cont'd: Maximum Likelihood Estimates.

	U.S.	U.K.	Germany
World Inflation	1.364 (0.438)	0.215 (0.318)	0.692 (0.367)
World IP Growth	0.261 (0.156)	0.000 (0.105)	-0.041 (0.135)
Local Inflation	-	1.320 (0.229)	1.208 (0.373)
Local IP Growth	-	0.170 (0.108)	0.015 (0.067)
R-Squared	0.448	0.688	0.583

**Table 4: Implied Monetary Policy Rules.** This table reports the implied monetary policy rules at the maximum likelihood parameter estimates. The loadings are calculated taking the 1-month interest rates as the policy rates. U.S. macro variables are used as the world factors. Small-sample standard errors, calculated by Monte Carlo simulation at the maximum likelihood parameter estimates, are in parentheses.

	U.S.			U.K.			Germany		
	6-Month	5-Year	10-Year	6-Month	5-Year	10-Year	6-Month	5-Year	10-Year
Mean									
Data	0.051	0.063	0.067	0.072	0.073	0.073	0.047	0.055	0.060
Fitted	0.051	0.063	0.067	0.071	0.073	0.073	0.047	0.054	0.060
Different? ( <i>p</i> -value)	(0.001)	(0.431)	(0.079)	(0.057)	(0.000)	(0.642)	(0.011)	(0.000)	(0.000)
Model Implied	0.051	0.063	0.067	0.071	0.073	0.073	0.047	0.054	0.060
Standard Deviation									
Data	0.021	0.018	0.017	0.030	0.023	0.023	0.022	0.017	0.015
Fitted	0.021	0.018	0.017	0.029	0.023	0.023	0.022	0.017	0.015
Different? ( <i>p</i> -value)	(0.005)	(0.037)	(0.389)	(0.126)	(0.049)	(0.370)	(0.954)	(0.511)	(0.202)
Model Implied	0.028	0.021	0.020	0.028	0.025	0.026	0.029	0.028	0.028
AR(1) Coefficient									
Data	0.992	0.987	0.986	0.991	0.992	0.994	0.995	0.992	0.993
Fitted	0.992	0.986	0.986	0.990	0.994	0.993	0.994	0.993	0.992
Different? ( <i>p</i> -value)	(0.004)	(0.036)	(0.333)	(0.109)	(0.104)	(0.519)	(0.993)	(0.905)	(0.094)
Model Implied	0.967	0.958	0.961	0.962	0.962	0.964	0.974	0.984	0.986
Yield Change Correlation									
Data	-	-	-	0.233	0.456	0.495	0.399	0.554	0.594
Fitted	-	-	-	0.253	0.465	0.525	0.415	0.560	0.600
Different? ( <i>p</i> -value)	-	-	-	(0.679)	(0.096)	(0.003)	(0.094)	(0.243)	(0.571)
Model Implied	-	-	-	0.117	0.164	0.177	0.277	0.325	0.330
Portion of Covariance									
Explained by Fitted Values	-	-	-	0.974	0.960	0.947	0.973	0.965	0.943

**Table 5: Goodness of Fit.** This table compares moments of actual yields, model-fitted yields, and unconditional moments implied by the model at the maximum likelihood parameter estimates. The test of difference in value of the sample moment is based on GMM with one degree of freedom. Corresponding *p*-values are in parentheses. The last row reports the proportion of yield covariance explained by the model-fitted values for each pair of country and each maturity.

	U.S.			U.K.			Germany		
	1-Year	5-Year	10-Year	1-Year	5-Year	10-Year	1-Year	5-Year	10-Year
C-S Regression Coefficient									
Data	0.642	-0.962	-1.171	0.283	0.072	-0.695	0.529	-0.584	-1.024
Model Implied	-0.531	-0.613	-0.576	-0.179	-0.353	-1.001	0.434	0.524	0.725
Excess Bond Return Correlation									
Data	-	-	-	0.426	0.637	0.685	0.389	0.598	0.680
Model Implied	-	-	-	0.227	0.236	0.223	0.308	0.378	0.373

**Table 6: Deviations from Expectations Hypothesis.** This table compares (i) coefficients of Campbell and Shiller (1991) regression and (ii) correlations of excess bond returns across currencies, estimated from the data, with those implied by the model at the maximum likelihood parameter estimates.

Horizon	All Macros			All through Market Price of Risk		
	6-Month	5-Year	10-Year	6-Month	5-Year	10-Year
U.S.						
3 Months	0.49	0.43	0.42	0.15	0.81	0.94
1 Year	0.59	0.51	0.49	0.14	0.80	0.94
3 Years	0.68	0.58	0.56	0.12	0.76	0.92
5 Years	0.70	0.59	0.56	0.10	0.73	0.92
U.K.						
3 Months	0.48	0.49	0.48	0.23	0.77	0.93
1 Year	0.64	0.61	0.57	0.13	0.74	0.92
3 Years	0.76	0.67	0.61	0.07	0.68	0.90
5 Years	0.79	0.67	0.61	0.05	0.65	0.89
Germany						
3 Months	0.53	0.49	0.46	0.10	0.57	0.85
1 Year	0.68	0.59	0.54	0.06	0.59	0.87
3 Years	0.79	0.66	0.59	0.07	0.68	0.90
5 Years	0.79	0.65	0.57	0.08	0.72	0.91

**Table 7: Variance Decomposition.** This table reports the contributions to yield variance of (i) all macro variables through both the physical and market price of risk channels, and (ii) all variables through the market price of risk channel only. The contributions through the market price of risk channel are calculated by subtracting the forecast error variance predicted by the expectations hypothesis from the total forecast error variance implied by the model. The bond maturities are in the columns and the forecast horizons are in the rows.

**Panel A: U.S.-U.K.**

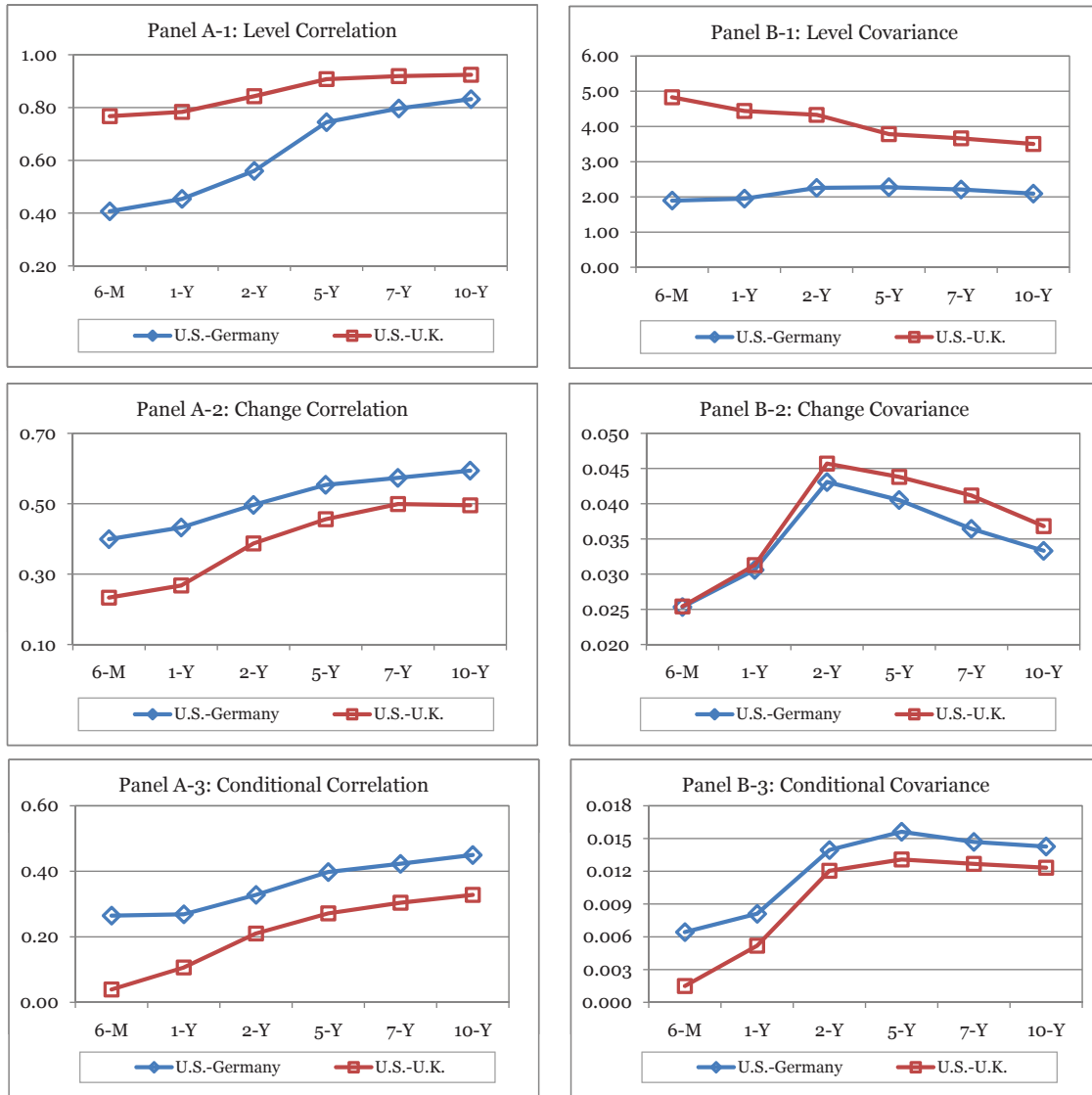
Maturity	Horizon	Contribution Through Physical Dynamics			Contribution Through Market Price of Risk								
		$\pi^W$	$g^W$	Total	$\pi^W$	$g^W$	Total						
6-Month	3 months	0.81	0.20	0.51	-0.05	0.00	1.47	-0.11	-0.11	-0.29	0.03	0.00	-0.47
	1 year	0.51	0.28	0.33	-0.03	0.02	1.11	0.02	-0.10	-0.04	0.01	0.00	-0.11
	3 years	0.32	0.53	0.21	-0.02	0.01	1.05	0.05	-0.10	-0.01	0.00	0.00	-0.05
	5 years	0.31	0.57	0.18	-0.02	0.01	1.05	0.04	-0.09	-0.01	0.00	0.00	-0.05
5-Year	3 months	0.05	0.46	0.08	-0.02	0.01	0.58	0.81	-0.41	-0.01	0.03	0.00	0.42
	1 year	0.02	0.40	0.05	-0.01	0.01	0.46	0.80	-0.34	0.04	0.04	0.00	0.54
	3 years	0.03	0.44	0.03	-0.01	0.00	0.50	0.78	-0.29	-0.03	0.04	0.00	0.50
	5 years	0.08	0.45	0.03	-0.01	0.00	0.56	0.71	-0.27	-0.04	0.04	0.00	0.44
10-Year	3 months	0.00	0.16	0.02	-0.01	0.00	0.17	0.93	-0.13	0.01	0.02	-0.01	0.83
	1 year	0.00	0.15	0.01	0.00	0.00	0.16	0.96	-0.12	-0.03	0.05	-0.02	0.84
	3 years	0.02	0.16	0.01	0.00	0.00	0.18	1.02	-0.10	-0.15	0.06	-0.02	0.82
	5 years	0.04	0.16	0.01	0.00	0.00	0.21	1.00	-0.09	-0.17	0.06	-0.01	0.79

**Table 8: Covariance Decomposition.** This table reports detailed decompositions of covariance of yields in two countries for the same maturity. Factors include world inflation ( $\pi^W$ ), IP growth ( $g^W$ ), and the principal components of U.S. yields ( $P^{US,1}, P^{US,2}, P^{US,3}$ ). The contribution of each factor to yield covariance is further decomposed into the part that works through the factor physical dynamics and the part that works through the market price of risk. The latter is calculated by subtracting the former (predicted by the expectations hypothesis) from the total yield covariance implied by the model. Panels A and B report the yield covariance decompositions for the U.S.-U.K. pair and for the U.S.-Germany pair, respectively.

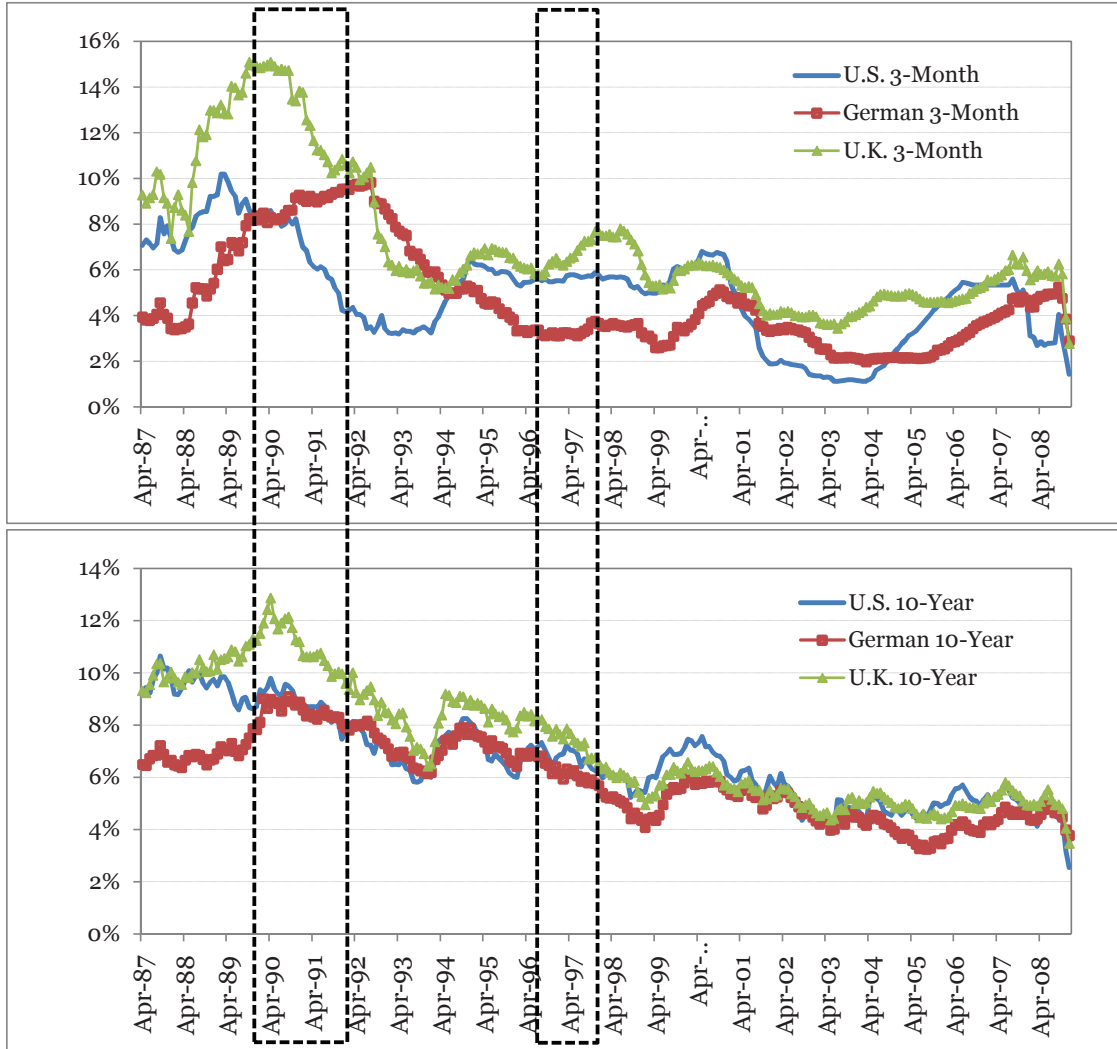
**Panel B: U.S.-Germany**

Maturity	Horizon	Contribution Through Physical Dynamics					Contribution Through Market Price of Risk						
		$\pi^W$	$g^W$	$P^{W,1}$	$P^{W,2}$	$P^{W,3}$	Total	$\pi^W$	$g^W$	$P^{W,1}$	$P^{W,2}$	$P^{W,3}$	Total
6-Month	3 months	0.97	0.02	0.12	-0.01	0.00	1.10	-0.04	0.00	-0.07	0.01	0.00	-0.10
	1 year	0.80	0.03	0.12	0.00	0.02	0.97	0.03	0.00	-0.01	0.00	0.00	0.03
	3 years	0.60	0.11	0.10	0.01	0.04	0.86	0.12	0.00	0.01	0.00	0.00	0.14
	5 years	0.50	0.17	0.10	0.02	0.05	0.84	0.14	0.01	0.01	0.00	0.01	0.16
5-Year	3 months	0.13	0.11	0.03	0.01	0.02	0.30	0.75	-0.03	0.02	-0.02	-0.02	0.70
	1 year	0.04	0.10	0.02	0.01	0.02	0.20	0.79	-0.04	0.09	-0.02	-0.02	0.80
	3 years	-0.06	0.12	0.01	0.01	0.02	0.11	0.85	0.01	0.09	-0.04	-0.01	0.89
	5 years	-0.07	0.12	0.01	0.02	0.03	0.10	0.83	0.05	0.07	-0.04	-0.02	0.90
10-Year	3 months	0.00	0.06	0.00	0.01	0.01	0.09	0.88	0.03	0.04	-0.02	-0.02	0.91
	1 year	-0.02	0.06	0.00	0.01	0.01	0.06	0.89	0.02	0.10	-0.04	-0.03	0.94
	3 years	-0.04	0.06	0.00	0.01	0.01	0.04	0.92	0.07	0.09	-0.08	-0.04	0.96
	5 years	-0.04	0.06	0.00	0.01	0.01	0.04	0.91	0.12	0.07	-0.09	-0.05	0.96

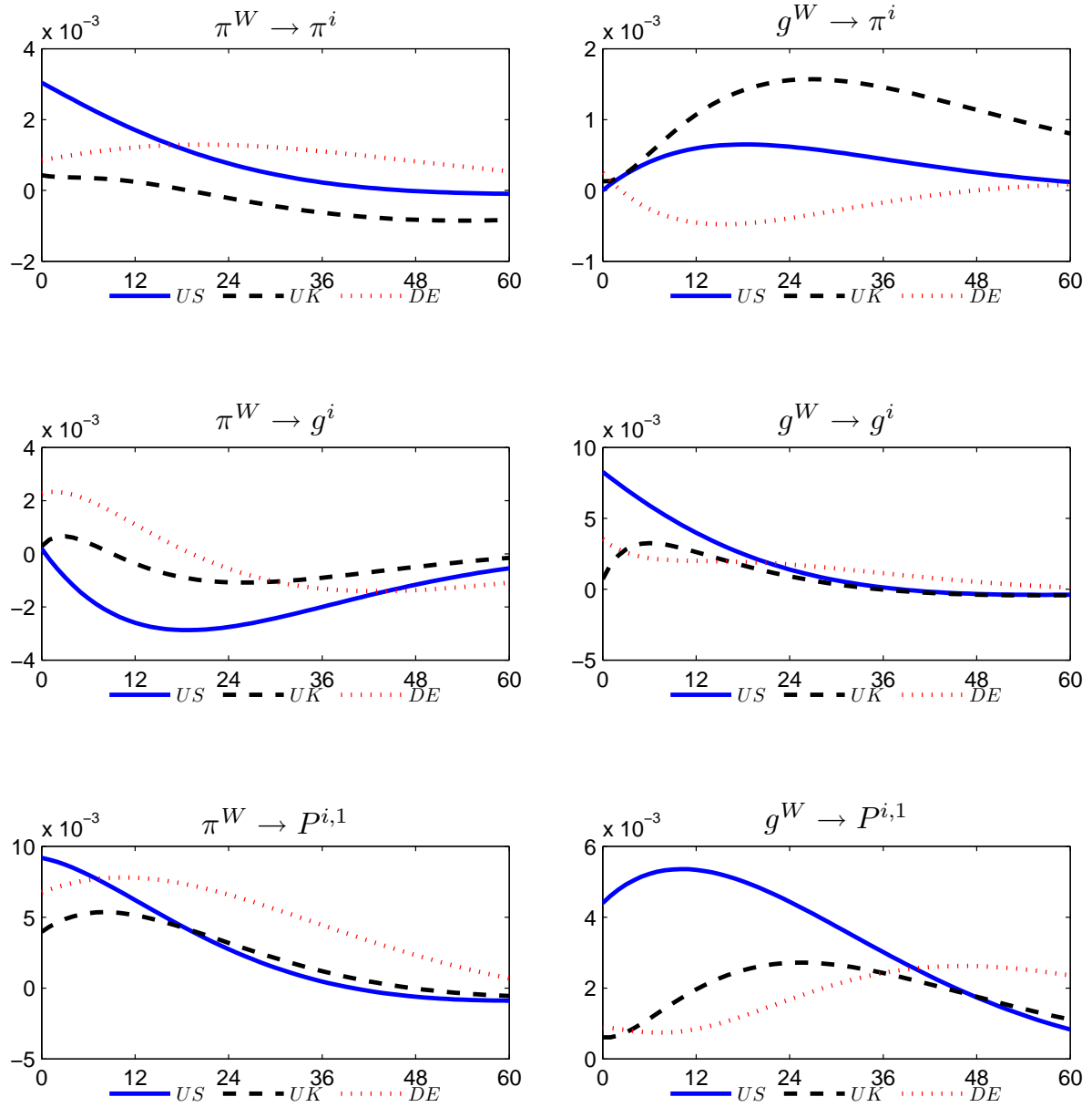
**Table 8, cont'd: Covariance Decomposition.**



**Figure 1: Cross-Country Correlations and Covariances.** This figure plots cross-country correlations and covariances for the interest rates and bond yields in two pairs of countries: (a) the U.S. and Germany, and (b) the U.S. and the U.K. The frequency is monthly. The sample period is 1987:04-2008:03. The correlations and covariances are calculated based on: (i) the level, (ii) the monthly change, and (iii) the forecast errors based on a linear model with state variables and bond yields up to 12 lags. The diamond markers and blue lines are for the U.S.-Germany pair, and the square markers and red lines are for the U.S.-U.K. pair. Panel A reports the correlations. Panel B reports the covariances.

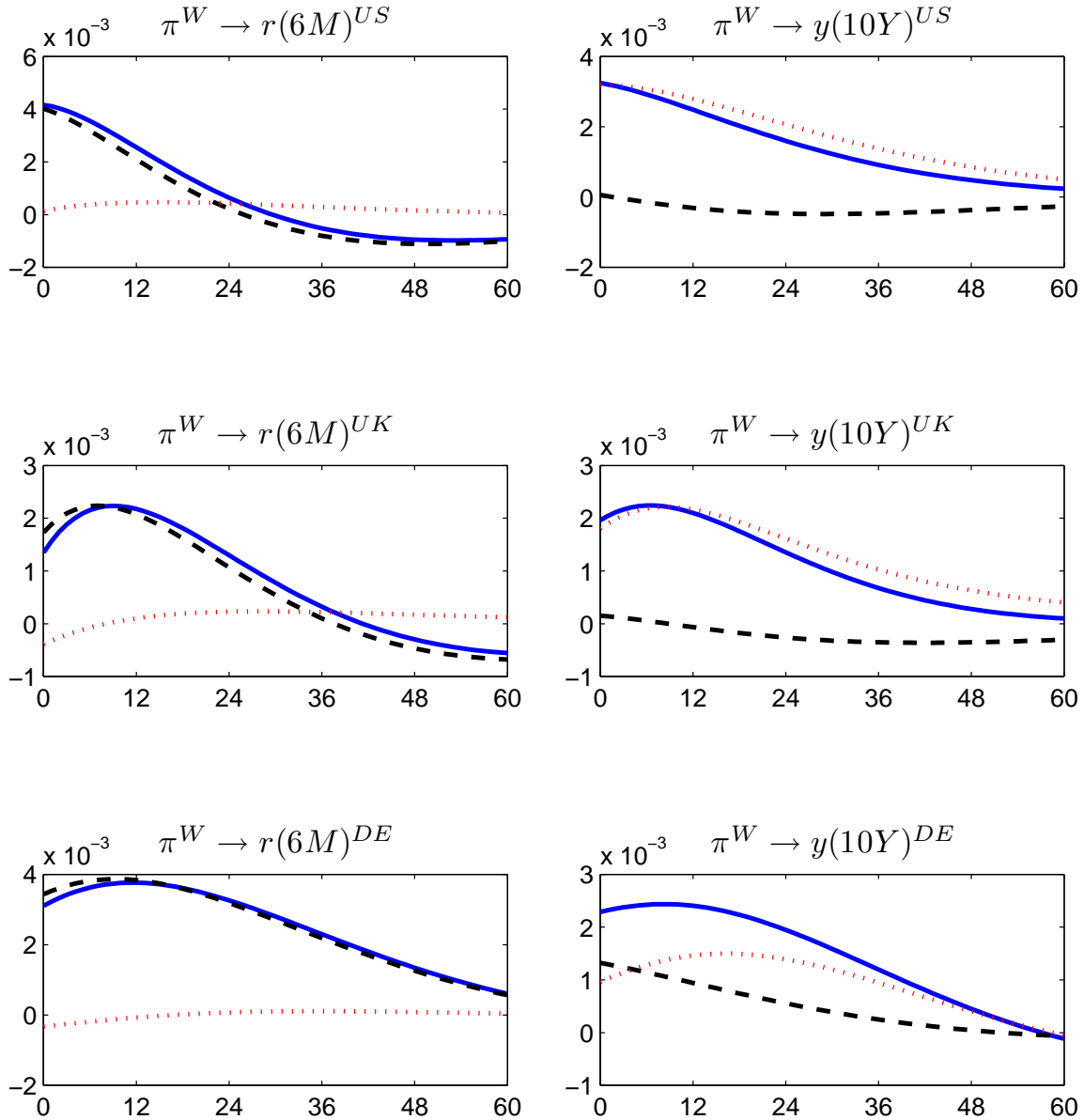


**Figure 2: Time Series of 3-Month Rates and 10-Year Yields.** This figure plots the time series of 3-month LIBOR rates and 10-year swap-implied zero-coupon yields for the U.S., Germany, and the U.K. for the period from 1987:04 to 2008:03. The blue solid line represents the U.S. The red solid line with square markers represent Germany. The green solid line with triangle markers represent the U.K. The dashed rectangular boxes identify two periods during which the monetary policies among these countries diverged. Panel A presents the 3-month LIBOR rates. Panel B presents the 10-year swap-implied zero-coupon yields.



**Figure 3: Impulse Responses of Macroeconomic Variables and the Level Factor.** This figure plots the impulse responses of inflation ( $\pi^{US}$ ,  $\pi^{UK}$ , and  $\pi^{DE}$ ), IP growth ( $g^{US}$ ,  $g^{UK}$ , and  $g^{DE}$ ) and the first principal component of yields, or the level factor, ( $P^{US,1}$ ,  $P^{UK,1}$ , and  $P^{DE,1}$ ) to a Choleski one standard deviation innovation in the world inflation and IP growth ( $\pi^W$  and  $g^W$ ).

Panel A-1: World Inflation on Yields



**Figure 4: Impulse Responses of Yields.** This figure plots the impulse responses of interest rates and swap-implied zero-coupon yields to a Choleski one standard deviation innovation in each of the following state variables: world inflation ( $\pi^W$ ) and local inflations ( $\pi^{UK}$  and  $\pi^{DE}$ ) in Panels A-1 and A-2, world IP growth ( $g^W$ ) and local IP growths ( $g^{UK}$  and  $g^{DE}$ ) in Panels B-1 and B-2, and the U.S. level factor ( $P^{US,1}$ ) and the local level factors ( $P^{UK,1}$  and  $P^{DE,1}$ ) in Panels C-1 and C-2. The blue solid line represents the total responses. The black dashed line represents the physical responses. The dotted red line represents the market price of risk responses.

Panel A-2: Local Inflation on Yields

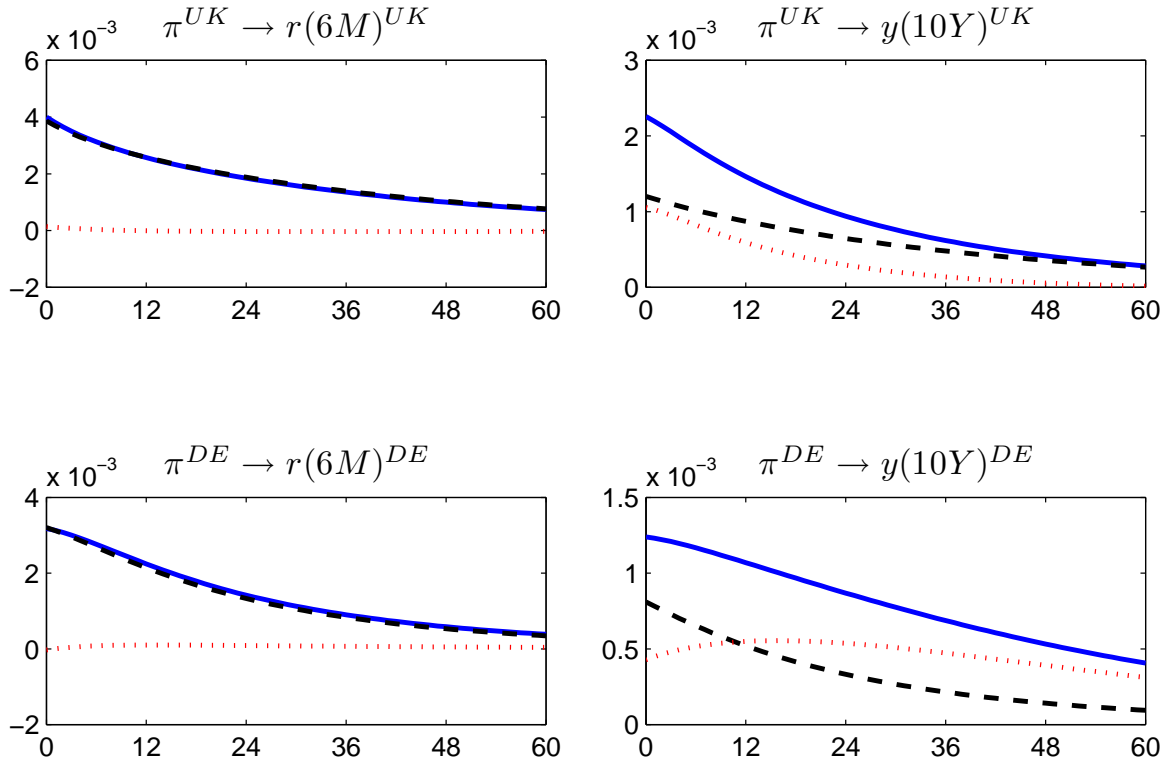


Figure 4, cont'd: Impulse Responses of Yields.

Panel B-1: World IP Growth on Yields

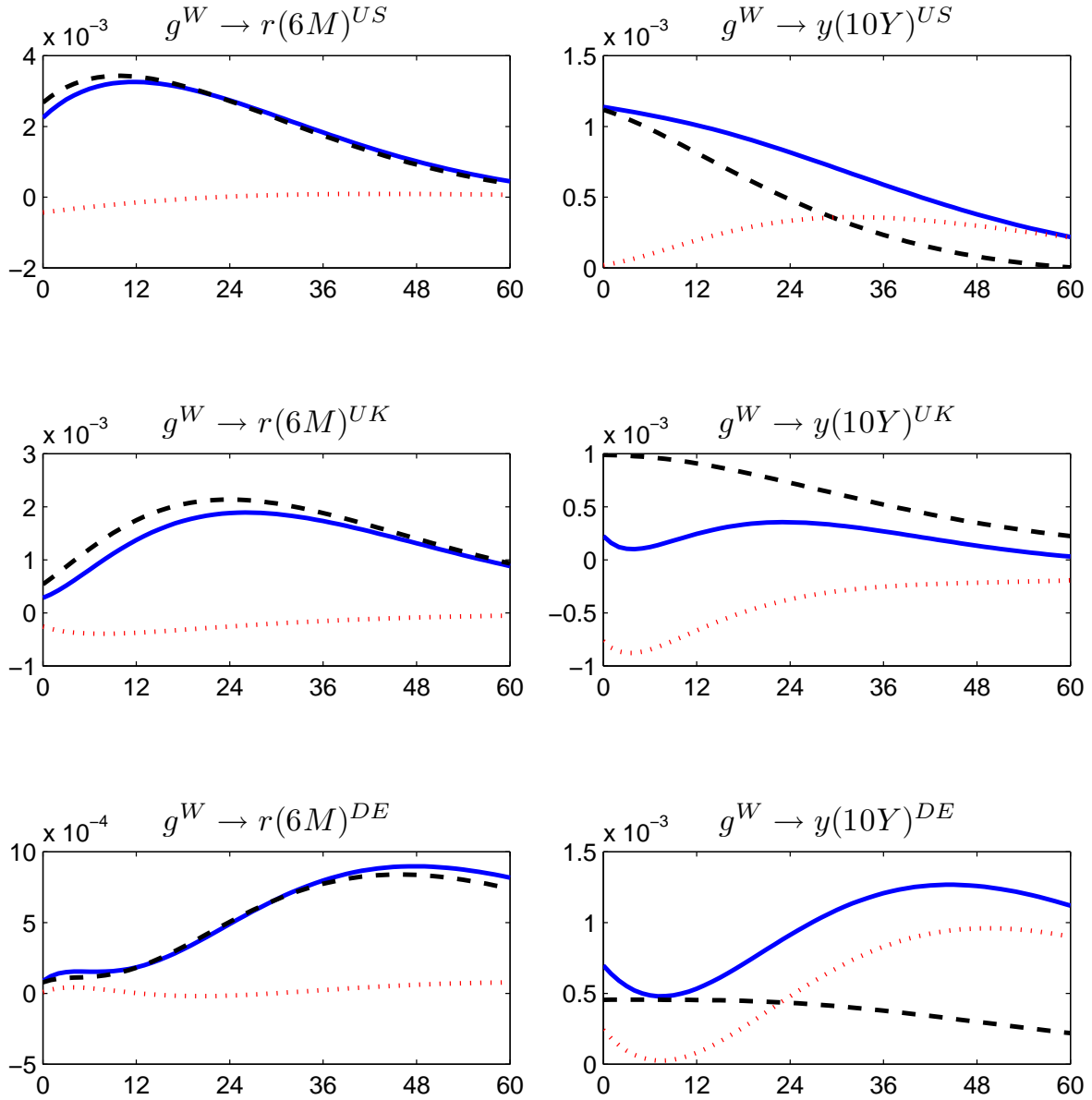


Figure 4, cont'd: Impulse Responses of Yields.

Panel B-2: Local IP Growths on Yields

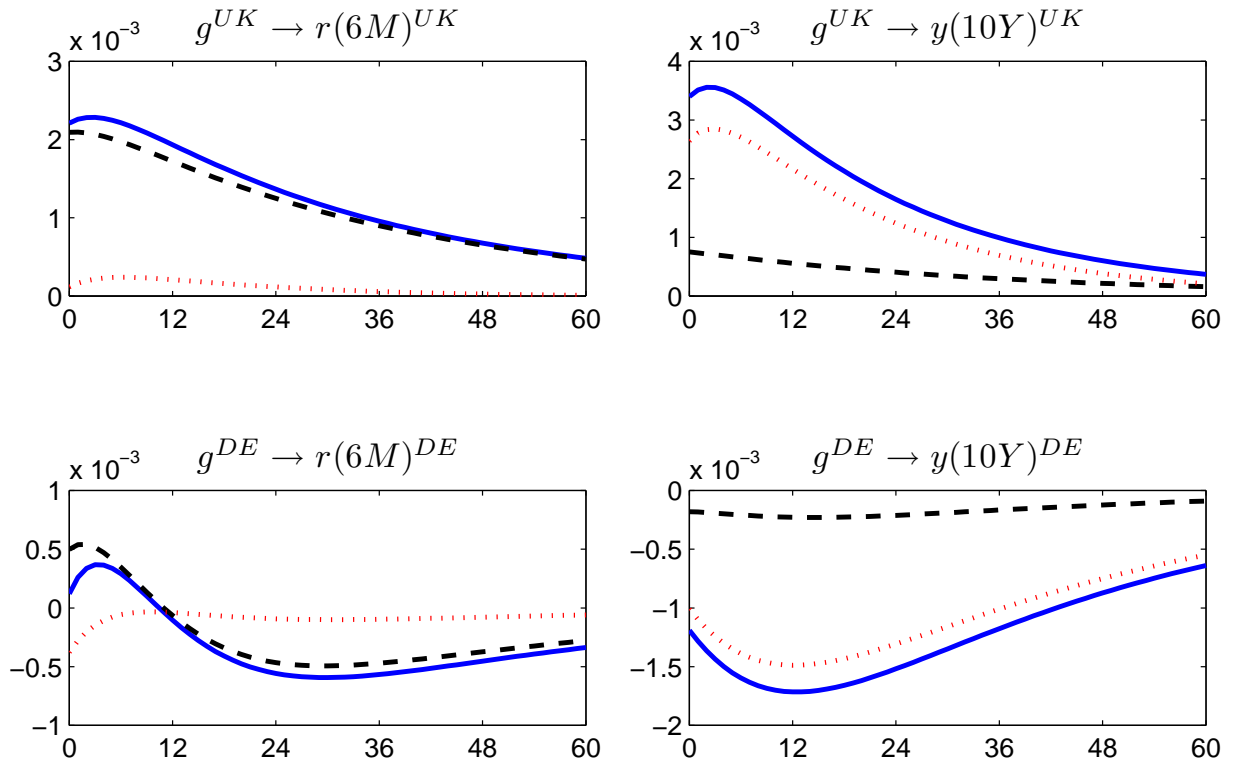


Figure 4, cont'd: Impulse Responses of Yields.

Panel C-1: U.S. Level Factor on Yields

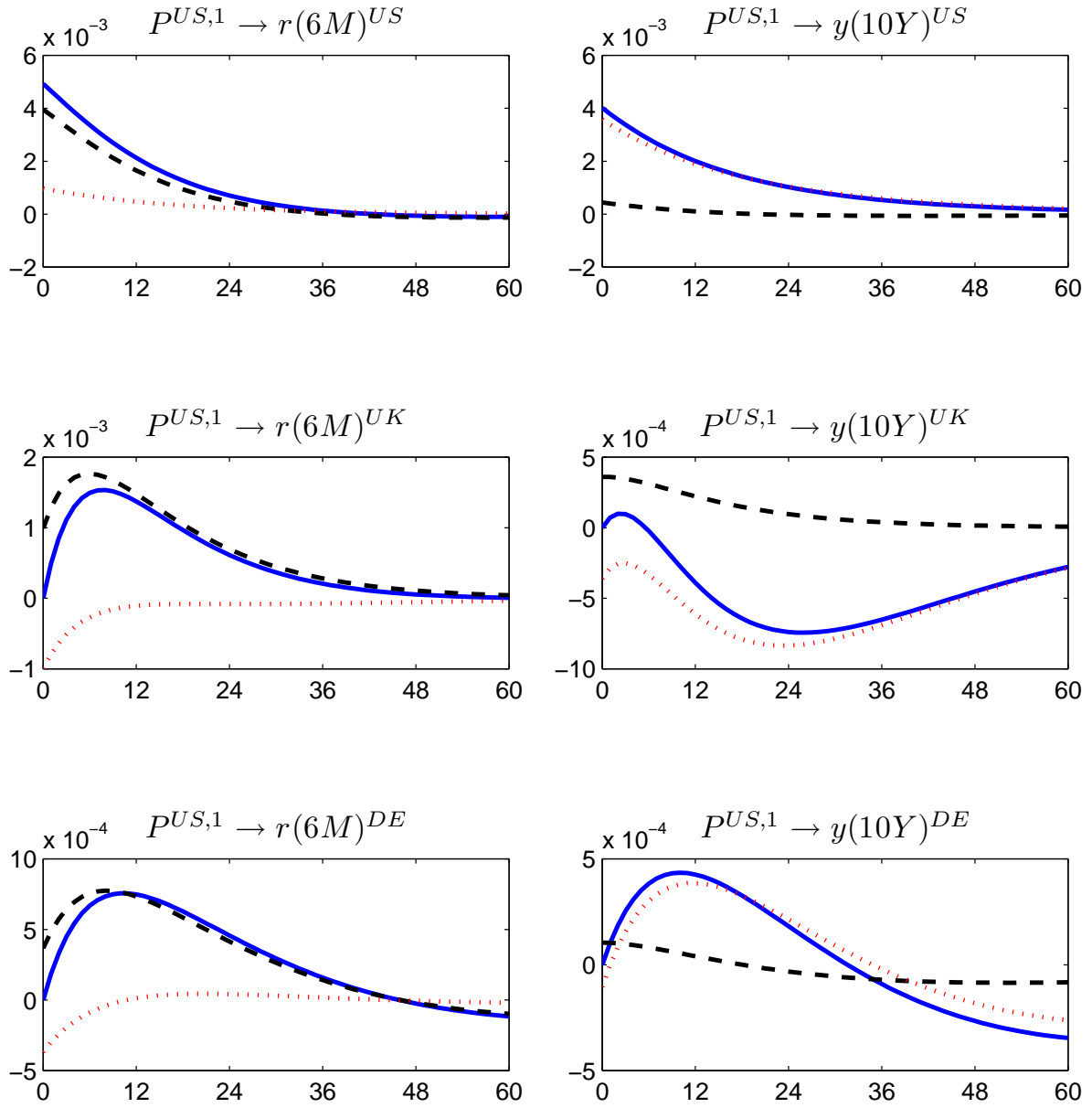


Figure 4, cont'd: Impulse Responses of Yields.

Panel C-2: Local Level Factors on Yields

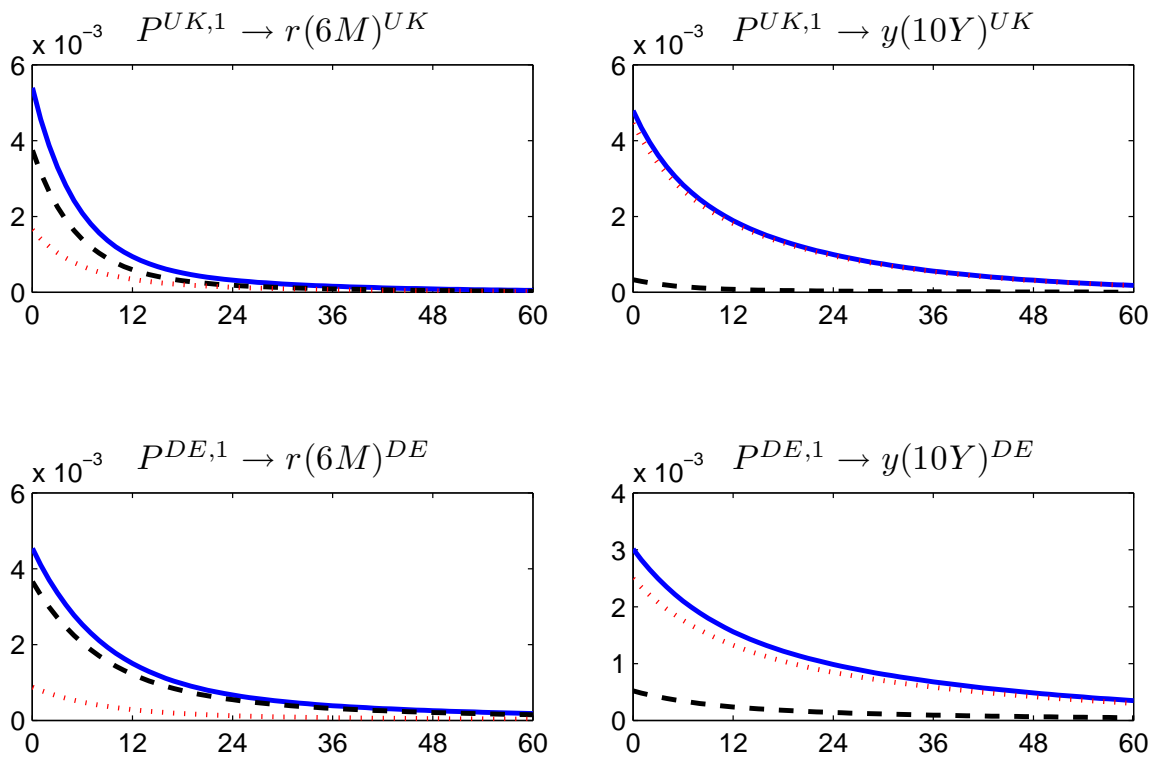


Figure 4, cont'd: Impulse Responses of Yields.