Financial Markets And Empirical Regularities
An Introduction to Financial Econometrics

SAMSI Workshop 11/18/05
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Outline

I. Historical Perspective on Asset Prices
II. Predictability of Asset Returns
III. Asset Pricing Models
IV. Volatility Models
Essential Sources

- Cochrane, “Asset Pricing”
- Hamilton, “Time Series Analysis”
- Greene, “Econometric Analysis”
Types of Financial Assets:

Common: Stocks, Bonds, Commodities, Foreign Exchange,…

Exotic: Derivatives; Options, Futures,…
I. Historical Perspective on Asset Prices

A History of Asset Prices

Index Line Graph
Frequency: Monthly

Source: Campbell Harvey
I. Historical Perspective on Asset Prices

Definitions:

- **Simple Gross Return:**
  \[ R_t + 1 = \frac{P_t}{P_{t-1}}, \quad P_t = \text{price of asset at time } t. \]

- **Compound Gross Return:**
  \[ R_t(k) + 1 = (1 + R_t)(1 + R_{t-1}) \ldots (1 + R_{t-k+1}) \]

- **Annualized Return:**
  \[ \left[ \prod_{j=0}^{k-1} (1 + R_{t-j}) \right]^{1/k} - 1 \approx \frac{1}{k} \sum_{j=0}^{k-1} R_{t-j} \]

- **Continuously Compounded Return:**
  \[ r_t = \log(1 + R_t) = \log P_t - \log P_{t-1} \]
I. Historical Perspective on Asset Prices

Why Returns?
…..Because returns tend to be stationary.

Stationary: The joint distribution between two returns $x_t, x_{t-h}$ depends only on $h$ and NOT on $t$. 
I. Historical Perspective on Asset Prices

A History of Asset Returns

Rolling Line Graph
Interval: 120 Months

Source: Campbell Harvey
I. Historical Perspective on Asset Prices

Risk vs. Return
January 1926 - December 2004

Source: Campbell Harvey
I. Historical Perspective on Asset Prices

Source: Campbell Harvey
I. Historical Perspective on Asset Prices

Observation:

- There exists a Risk / Return Tradeoff
I. Historical Perspective on Asset Prices

Stats Reminder:
Consider a random variable \( x_t \)

- **Mean:**
  \[ \hat{\mu} \equiv \frac{1}{T} \sum_{t=1}^{T} x_t \]

- **Variance:**
  \[ \hat{\sigma}^2 \equiv \frac{1}{T} \sum_{t=1}^{T} (x_t - \hat{\mu})^2 \]

- **Skew:**
  \[ \hat{S} \equiv \frac{1}{T \hat{\sigma}^3} \sum_{t=1}^{T} (x_t - \hat{\mu})^3 \]

- **Kurtosis:**
  \[ \hat{K} \equiv \frac{1}{T \hat{\sigma}^4} \sum_{t=1}^{T} (x_t - \hat{\mu})^4 \]
I. Historical Perspective on Asset Prices

What is the Distribution of Returns?

- Normal?
## I. Historical Perspective on Asset Prices

### Table 1.1: Stock market returns, 1962 to 1994.

<table>
<thead>
<tr>
<th>Security</th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>Skewness</th>
<th>Excess Kurtosis</th>
<th>Minimum</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Panel A: Daily Returns</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Value-Weighted Index</td>
<td>0.044</td>
<td>0.82</td>
<td>-1.33</td>
<td>34.92</td>
<td>-18.10</td>
<td>8.87</td>
</tr>
<tr>
<td>Equal-Weighted Index</td>
<td>0.073</td>
<td>0.76</td>
<td>-0.93</td>
<td>26.03</td>
<td>-14.19</td>
<td>9.83</td>
</tr>
<tr>
<td>International Business Machines</td>
<td>0.039</td>
<td>1.42</td>
<td>-0.18</td>
<td>12.48</td>
<td>-22.96</td>
<td>11.72</td>
</tr>
<tr>
<td>General Signal Corp.</td>
<td>0.054</td>
<td>1.66</td>
<td>0.01</td>
<td>3.35</td>
<td>-13.46</td>
<td>9.43</td>
</tr>
<tr>
<td>Wrigley Co.</td>
<td>0.072</td>
<td>1.45</td>
<td>-0.00</td>
<td>11.03</td>
<td>-18.67</td>
<td>11.89</td>
</tr>
<tr>
<td>Interlake Corp.</td>
<td>0.043</td>
<td>2.16</td>
<td>0.72</td>
<td>12.35</td>
<td>-17.24</td>
<td>23.08</td>
</tr>
<tr>
<td>Raytech Corp.</td>
<td>0.050</td>
<td>3.39</td>
<td>2.25</td>
<td>59.40</td>
<td>-57.90</td>
<td>75.00</td>
</tr>
<tr>
<td>Ampco-Pittsburgh Corp.</td>
<td>0.053</td>
<td>2.41</td>
<td>0.66</td>
<td>5.02</td>
<td>-19.05</td>
<td>19.18</td>
</tr>
<tr>
<td>Energen Corp.</td>
<td>0.054</td>
<td>1.41</td>
<td>0.27</td>
<td>5.91</td>
<td>-12.82</td>
<td>11.11</td>
</tr>
<tr>
<td>General Host Corp.</td>
<td>0.070</td>
<td>2.79</td>
<td>0.74</td>
<td>6.18</td>
<td>-23.53</td>
<td>22.92</td>
</tr>
<tr>
<td>Garan Inc.</td>
<td>0.079</td>
<td>2.35</td>
<td>0.72</td>
<td>7.13</td>
<td>-16.67</td>
<td>19.07</td>
</tr>
<tr>
<td>Continental Materials Corp.</td>
<td>0.143</td>
<td>5.24</td>
<td>0.93</td>
<td>6.49</td>
<td>-26.92</td>
<td>50.00</td>
</tr>
<tr>
<td><strong>Panel B: Monthly Returns</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Value-Weighted Index</td>
<td>0.96</td>
<td>4.33</td>
<td>-0.29</td>
<td>2.42</td>
<td>-21.81</td>
<td>16.51</td>
</tr>
<tr>
<td>Equal-Weighted Index</td>
<td>1.25</td>
<td>5.77</td>
<td>0.07</td>
<td>4.14</td>
<td>-26.80</td>
<td>33.17</td>
</tr>
<tr>
<td>International Business Machines</td>
<td>0.81</td>
<td>6.18</td>
<td>-0.14</td>
<td>0.83</td>
<td>-26.19</td>
<td>18.95</td>
</tr>
<tr>
<td>General Signal Corp.</td>
<td>1.17</td>
<td>8.19</td>
<td>-0.02</td>
<td>1.87</td>
<td>-36.77</td>
<td>29.73</td>
</tr>
<tr>
<td>Wrigley Co.</td>
<td>1.51</td>
<td>6.68</td>
<td>0.30</td>
<td>1.31</td>
<td>-20.26</td>
<td>29.72</td>
</tr>
<tr>
<td>Interlake Corp.</td>
<td>0.86</td>
<td>9.38</td>
<td>0.67</td>
<td>4.09</td>
<td>-30.28</td>
<td>54.84</td>
</tr>
<tr>
<td>Raytech Corp.</td>
<td>0.83</td>
<td>14.88</td>
<td>2.73</td>
<td>22.70</td>
<td>-45.65</td>
<td>142.11</td>
</tr>
<tr>
<td>Ampco-Pittsburgh Corp.</td>
<td>1.06</td>
<td>10.64</td>
<td>0.77</td>
<td>2.04</td>
<td>-36.08</td>
<td>46.94</td>
</tr>
<tr>
<td>Energen Corp.</td>
<td>1.10</td>
<td>5.75</td>
<td>1.47</td>
<td>12.47</td>
<td>-24.61</td>
<td>48.36</td>
</tr>
<tr>
<td>General Host Corp.</td>
<td>1.33</td>
<td>11.67</td>
<td>0.35</td>
<td>1.11</td>
<td>-38.05</td>
<td>42.86</td>
</tr>
<tr>
<td>Garan Inc.</td>
<td>1.64</td>
<td>11.30</td>
<td>0.76</td>
<td>2.30</td>
<td>-35.48</td>
<td>51.60</td>
</tr>
<tr>
<td>Continental Materials Corp.</td>
<td>1.64</td>
<td>17.76</td>
<td>1.13</td>
<td>3.33</td>
<td>-58.09</td>
<td>84.78</td>
</tr>
</tbody>
</table>

Summary statistics for daily and monthly returns (in percent) of CRSP equal- and value-weighted stock indexes and ten individual securities continuously listed over the entire sample period from July 3, 1962 to December 30, 1994. Individual securities are selected to represent stocks in each size decile. Statistics are defined in (1.4.19)–(1.4.22).
I. Historical Perspective on Asset Prices

**Stylized Facts on the Distribution of Returns:**

1. Index Volatility < Stock Volatility
2. Negative Skewness
3. Excess Kurtosis
II. Predictability of Returns

Can We Predict Returns?

- Day Traders Say Yes

- Efficient Market Hypothesis Says No
II. Predictability of Returns

Efficient Market Hypothesis (EMH):

Fama (1970): “A market in which prices always ‘fully reflect’ available information is called ‘efficient’”.
II. Predictability of Returns

Efficient Market Hypothesis (EMH):

Fama (1970): “A market in which prices always ‘fully reflect’ available information is called ‘efficient’”.

Malkiel (1992): “…the market is said to be efficient with respect to an information set…”
II. Predictability of Returns

Probability Theory Reminder:

- The Information Set $F_t$ is a sigma field said to contain all of the relevant information up to time $t$. 
II. Predictability of Returns

**Probability Theory Reminder:**

- The Information Set $F_t$ is a sigma field said to contain all of the relevant information up to time $t$.

- Martingale: $E[x_{t+1} | F_t] = x_t$
II. Predictability of Returns

**EMH:**

**Weak Form Efficiency:**
- Information Set: Asset’s own history
- Test via Random Walk
II. Predictability of Returns

**EMH:**

**Weak Form Efficiency:**
- Information Set: Asset’s own history
- Test via Random Walk

**Semistrong Efficiency:**
- Information Set: Weak Form + Publicly available data
- Test via Event Studies
II. Predictability of Returns

**EMH:**

**Weak Form Efficiency:**
- Information Set: Asset’s own history
- Test via Random Walk

**Semistrong Efficiency:**
- Information Set: Weak Form + Publicly available data
- Test via Event Studies

**Strong Form Efficiency:**
- Information Set: Weak + Semi + Private Info
- Test via Performance Evaluation
Joint hypothesis problem: The EMH can’t be tested directly. Even if we reject the hypothesis of efficiency, this could either be because the market is truly inefficient, or because we have assumed an incorrect equilibrium model.
II. Predictability of Returns

Implication of Weak Form EMH:

- Expected Returns follow a martingale.
- Random Walk Hypothesis
II. Predictability of Returns

\[ P_t = \mu + P_{t-1} + \varepsilon_t \]

Random Walk 1: i.i.d Increments
Random Walk 2: Independent, Not Identical
Random Walk 3: Uncorrelated Increments
II. Predictability of Returns

Time Series Econometrics Reminder:

Auto Correlation

Correlation between two observations of the same series at different dates

AutoCovariance: \( \gamma(k) \equiv Cov(x_t, x_{t-k}) \)

AutoCorrelation: \( \rho(k) \equiv \frac{\gamma(k)}{\gamma(0)} \)
II. Predictability of Returns

Time Series Econometrics Reminder:

Auto Correlation

Correlation between two observations of the same series at different dates

AutoCovariance: \( \gamma(k) \equiv Cov(x_t, x_{t-k}) \)

AutoCorrelation: \( \rho(k) \equiv \frac{\gamma(k)}{\gamma(0)} \)

Box – Pierce Q Statistic: \( Q_m = T(T + 2) \sum_{k=1}^{m} \frac{\rho^2(k)}{T-k} \)
II. Predictability of Returns

### Table 2.4. Autocorrelation in daily, weekly, and monthly stock index returns.

<table>
<thead>
<tr>
<th>Sample Period</th>
<th>Sample Size</th>
<th>CRSP Value-Weighted Index</th>
<th>CRSP Equal-Weighted Index</th>
</tr>
</thead>
<tbody>
<tr>
<td>62:07-03-94:12:30</td>
<td>8,179</td>
<td>0.041 0.824 17.6 -0.7 0.1 -0.8 263.3 269.5</td>
<td></td>
</tr>
<tr>
<td>62:07-03-78:10:27</td>
<td>4,090</td>
<td>0.028 0.738 27.8 1.2 4.6 3.3 329.4 343.5</td>
<td></td>
</tr>
<tr>
<td>78:10:30-94:12:30</td>
<td>4,089</td>
<td>0.054 0.901 10.8 -2.2 -2.9 -3.5 69.5 72.1</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>CRSP Value-Weighted Index</td>
<td>CRSP Equal-Weighted Index</td>
</tr>
<tr>
<td>62:07-03-94:12:30</td>
<td>8,179</td>
<td>0.070 0.764 25.0 9.3 8.5 9.9 1,301.9 1,369.5</td>
<td></td>
</tr>
<tr>
<td>62:07-03-78:10:27</td>
<td>4,090</td>
<td>0.063 0.771 43.1 13.0 15.3 15.2 1,062.2 1,110.2</td>
<td></td>
</tr>
<tr>
<td>78:10:30-94:12:30</td>
<td>4,089</td>
<td>0.078 0.756 26.2 4.9 2.0 4.9 348.9 379.5</td>
<td></td>
</tr>
</tbody>
</table>

B. Weekly Returns

<table>
<thead>
<tr>
<th>Sample Period</th>
<th>Sample Size</th>
<th>CRSP Value-Weighted Index</th>
<th>CRSP Equal-Weighted Index</th>
</tr>
</thead>
<tbody>
<tr>
<td>62:07-10-94:12:27</td>
<td>1,695</td>
<td>0.196 2.093 1.5 -2.5 3.5 -0.7 8.8 36.7</td>
<td></td>
</tr>
<tr>
<td>62:07-10-78:10:03</td>
<td>848</td>
<td>0.144 1.994 5.6 -3.7 5.8 1.6 9.0 21.5</td>
<td></td>
</tr>
<tr>
<td>78:10:10-94:12:27</td>
<td>847</td>
<td>0.248 2.188 -2.0 -1.5 1.6 -3.3 5.3 25.2</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>CRSP Value-Weighted Index</td>
<td>CRSP Equal-Weighted Index</td>
</tr>
<tr>
<td>62:07-10-94:12:27</td>
<td>1,695</td>
<td>0.339 2.321 20.3 6.1 9.1 4.8 94.3 109.3</td>
<td></td>
</tr>
<tr>
<td>62:07-10-78:10:03</td>
<td>848</td>
<td>0.324 2.460 21.8 7.5 11.9 6.1 60.4 68.5</td>
<td></td>
</tr>
<tr>
<td>78:10:10-94:12:27</td>
<td>847</td>
<td>0.351 2.171 18.4 4.3 5.5 2.2 33.7 51.3</td>
<td></td>
</tr>
</tbody>
</table>

C. Monthly Returns

<table>
<thead>
<tr>
<th>Sample Period</th>
<th>Sample Size</th>
<th>CRSP Value-Weighted Index</th>
<th>CRSP Equal-Weighted Index</th>
</tr>
</thead>
<tbody>
<tr>
<td>62:07-03-94:12:30</td>
<td>390</td>
<td>0.861 4.336 4.3 -5.3 -1.3 -0.4 6.8 12.5</td>
<td></td>
</tr>
<tr>
<td>62:07-03-78:09:29</td>
<td>195</td>
<td>0.646 4.219 6.4 -3.8 7.3 6.2 3.9 9.7</td>
<td></td>
</tr>
<tr>
<td>78:10:31-94:12:30</td>
<td>195</td>
<td>1.076 4.450 1.3 -6.3 -8.3 -7.7 7.5 14.0</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>CRSP Value-Weighted Index</td>
<td>CRSP Equal-Weighted Index</td>
</tr>
<tr>
<td>62:07-03-94:12:30</td>
<td>390</td>
<td>1.077 5.749 17.1 -3.4 -3.3 -1.6 12.8 21.3</td>
<td></td>
</tr>
<tr>
<td>62:07-03-78:09:29</td>
<td>195</td>
<td>1.049 6.148 18.4 -2.5 4.4 4.4 7.5 12.6</td>
<td></td>
</tr>
<tr>
<td>78:10:31-94:12:30</td>
<td>195</td>
<td>1.105 5.336 15.0 -1.6 -12.4 -7.4 8.9 14.2</td>
<td></td>
</tr>
</tbody>
</table>

Autocorrelation coefficients (in percent) and Box-Pierce $Q$-statistics for CRSP daily, weekly, and monthly value- and equal-weighted return indexes for the sample period from July 3, 1962 to December 30, 1994 and subperiods.
III. Asset Pricing Models

- Capital Asset Pricing Model (CAPM)
  Sharpe (1964) and Lintner (1965)

- Consumption Based-CAPM

- Inter-temporal-CAPM

- Arbitrage Pricing Theory
  Ross (1976)
III. Asset Pricing Models - CAPM

**CAPM:** Differences in excess returns across assets are due to differences in the *riskiness* of each asset.

**Beta:** Measure of *riskiness*

Market Model: \( (R_i - R_f) = \beta (R_m - R_f) + u \)

⇒ There exists a linear Risk/Reward Tradeoff

OLS ⇒ \( \beta = \frac{Cov[(R_i - R_f), (R_m - R_f)]}{Var(R_m - R_f)} \)
III. Asset Pricing Models - CAPM

**CAPM Pricing Equation:**

\[ E[R_i] = R_f + \beta_{im} (E[R_m] - R_f) \]
III. Asset Pricing Models - CAPM

**CAPM Pricing Equation:**

\[ E[R_i] = R_f + \beta_{im} (E[R_m] - R_f) \]

Testable Implications:

1) Only beta is required to price assets.
2) There is a linear risk/return relationship.
III. Asset Pricing Models - CAPM

Testing the CAPM:

Fama & MacBeth (1973)

Step 1: For each time period estimate the cross-sectional regression:

\[(R_i - R_f) = \theta_0 + \theta_1 \beta_{im} + \theta_2 \beta_{im}^2 + \theta_3 s_i + u_i, \text{ where } s \text{ is the standard deviation of } u.\]

Step 2: Aggregate parameter estimates over time such that \(\theta_k = E[\theta_k] \forall k = 0,1,2,3\)
Fama & MacBeth cont’d

There is an errors-in-variables (EIV) problem because the $\beta$'s must first be estimated.

**Fama & MacBeth solution**: Sort stocks into portfolios.

**Shanken solution**: correct variance of estimators post estimation.

\[ H_0 : \theta_0 = \theta_1 = \theta_2 = \theta_3 = 0 \]

$\Rightarrow \beta$ is the only factor necessary to price assets & there is a positive, linear risk-return tradeoff.
### III. Asset Pricing Models - CAPM

#### TABLE 3

**Summary Results for the Regression**

\[
R_p = \hat{\gamma}_0t + \hat{\gamma}_1t\hat{\beta}_p + \hat{\gamma}_2t\hat{\beta}_p^2 + \hat{\gamma}_3t\hat{\beta}_p + \hat{\eta}_{pt}
\]

| Period | \(\hat{\gamma}_0\) | \(\hat{\gamma}_1\) | \(\hat{\gamma}_2\) | \(\hat{\gamma}_3\) | \(\hat{\gamma}_0 - R_f\) | \(s(\hat{\gamma}_0)\) | \(s(\hat{\gamma}_1)\) | \(s(\hat{\gamma}_2)\) | \(s(\hat{\gamma}_3)\) | \(\rho(\hat{\gamma}_0)\) | \(\rho(\hat{\gamma}_1)\) | \(\rho(\hat{\gamma}_2)\) | \(\rho(\hat{\gamma}_3)\) | \(t(\hat{\gamma}_0)\) | \(t(\hat{\gamma}_1)\) | \(t(\hat{\gamma}_2)\) | \(t(\hat{\gamma}_3)\) | \(t(\hat{\gamma}_0 - R_f)\) | \(s(\hat{\gamma})\) |
|--------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| Panel C |
| 1935-6/68 | .0054 | .0072 | ... | .0198 | .0041 | .052 | .065 | ... | .868 | .04 | ... | .04 | 2.10 | 2.20 | ... | .46 | 1.39 | .32 | .31 |
| 1945-6/68 | .0017 | .0104 | ... | .0841 | .0015 | .073 | .083 | ... | .921 | .00 | ... | .08 | .26 | .14 | ... | .05 | .24 | .32 | .31 |
| 1946-55 | .0110 | .0075 | ... | -.132 | .0100 | .032 | .056 | ... | .609 | .08 | ... | .20 | 3.78 | 1.47 | ... | 1.89 | .34 | .32 |
| 1956-6/68 | .0042 | .0041 | ... | .0833 | .0016 | .040 | .032 | ... | .844 | .12 | .08 | ... | .03 | 1.28 | .96 | ... | .79 | .30 | .30 | .29 |
| 1935-40 | ... | .0036 | .0119 | ... | -.0370 | .0035 | .082 | .105 | ... | .744 | .03 | ... | .18 | .37 | .97 | ... | .19 | .36 | .25 | .30 |
| 1941-45 | ... | -.0006 | .0083 | ... | .2053 | -.0000 | .061 | .032 | ... | 1.091 | .07 | ... | .02 | -0.8 | 1.15 | ... | 1.46 | .11 | .41 | .30 |
| 1946-50 | ... | .0069 | .0081 | ... | -.020 | .0062 | .034 | .065 | ... | .504 | .14 | .06 | ... | -.02 | 1.56 | .95 | ... | -1.41 | 1.40 | .42 | .33 |
| 1951-55 | ... | .0150 | .0069 | ... | -.1135 | .0138 | .029 | .043 | ... | .702 | .06 | ... | -.32 | 4.05 | 1.24 | ... | -1.31 | 3.72 | .27 | .29 |
| 1956-60 | ... | .0127 | -.0081 | ... | .5728 | .0107 | .057 | .043 | ... | 1.164 | .15 | .15 | ... | .21 | 2.68 | -1.40 | ... | .48 | 2.16 | .26 | .30 |
| 1961-6/68 | ... | -.0014 | .0122 | ... | .0370 | -.0044 | .042 | .033 | ... | .830 | .10 | .00 | ... | -.19 | -3.2 | 2.12 | ... | .64 | -.98 | .33 | .27 |

Panel D:

| Period | \(\hat{\gamma}_0\) | \(\hat{\gamma}_1\) | \(\hat{\gamma}_2\) | \(\hat{\gamma}_3\) | \(\hat{\gamma}_0 - R_f\) | \(s(\hat{\gamma}_0)\) | \(s(\hat{\gamma}_1)\) | \(s(\hat{\gamma}_2)\) | \(s(\hat{\gamma}_3)\) | \(\rho(\hat{\gamma}_0)\) | \(\rho(\hat{\gamma}_1)\) | \(\rho(\hat{\gamma}_2)\) | \(\rho(\hat{\gamma}_3)\) | \(t(\hat{\gamma}_0)\) | \(t(\hat{\gamma}_1)\) | \(t(\hat{\gamma}_2)\) | \(t(\hat{\gamma}_3)\) | \(t(\hat{\gamma}_0 - R_f)\) | \(s(\hat{\gamma})\) |
|--------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| 1935-6/68 | .0020 | .0114 | -.0026 | .0516 | .0008 | .075 | .123 | .060 | .292 | -.09 | -.09 | .12 | .10 | .55 | .155 | -.86 | 1.11 | .10 | .34 | .31 |
| 1945-6/68 | .0011 | .0118 | -.0009 | .0817 | .0010 | .103 | .145 | .079 | 1.003 | -.10 | -.23 | -.24 | -.15 | .13 | .94 | -.14 | .94 | .11 | .34 | .31 |
| 1946-50 | ... | .0017 | .0209 | -.0076 | .0738 | .0002 | .042 | .095 | .358 | .619 | -.10 | -.00 | .01 | -.20 | .44 | 2.29 | -2.16 | .67 | .10 | .36 | .32 |
| 1956-6/68 | .0031 | .0034 | -.0000 | .0966 | .0003 | .065 | .121 | .053 | 1.061 | .12 | .03 | .01 | -.08 | .59 | .34 | -.00 | 1.11 | .10 | .32 | .29 |
| 1935-40 | ... | .0009 | .0156 | -.0029 | .0225 | .0008 | .112 | .171 | .085 | .826 | -.16 | -.23 | -.26 | -.12 | .07 | .18 | -.29 | .03 | .06 | .26 | .30 |
| 1941-45 | ... | .0015 | .0073 | .0014 | .1767 | .0043 | .032 | .092 | .092 | 1.181 | -.18 | -.21 | -.22 | -.18 | .12 | .12 | .15 | 1.16 | .10 | .43 | .61 |
| 1946-50 | ... | .0011 | .0141 | .0040 | -.0313 | .0004 | .047 | .105 | .042 | .590 | -.10 | .03 | .01 | -.12 | .18 | 1.63 | -.73 | .41 | .07 | .44 | .43 |
| 1951-55 | ... | .0023 | .0277 | -.0112 | -.0443 | .0011 | .037 | .083 | .035 | .71 | -.11 | -.13 | -.01 | -.28 | .48 | 2.13 | -2.34 | -.33 | .13 | .29 | .30 |
| 1956-60 | ... | -.0103 | -.0047 | -.0020 | .0979 | .0063 | .049 | .075 | .632 | 1.286 | -.16 | .19 | -.01 | .02 | 1.63 | -.47 | -.49 | .59 | 1.11 | .28 | .30 |
| 1961-6/68 | ... | -.0017 | .0088 | .0013 | .0957 | -.0046 | .073 | .144 | .066 | .887 | .10 | .00 | .01 | -.15 | -.21 | .38 | .19 | 1.02 | -.60 | .35 | .29 |

36
III. Asset Pricing Models - CAPM

CAPM Anomalies:

- Small Firm effect (Keim `81)
- P/E Ratio Effect (Ball `78 and Basu `83)
- Book-to-Market Effect (Stattman `80 and Rosenberg `85)
- Momentum Effect (Jegadeesh and Titman `93)

⇒ Is Beta Dead?
## III. Asset Pricing Models - CAPM

Table III

Average Slopes (t-Statistics) from Month-by-Month Regressions of Stock Returns on $\beta$, Size, Book-to-Market Equity, Leverage, and E/P; July 1963 to December 1990

Stocks are assigned the post-ranking $\beta$ of the size-$\beta$ portfolio they are in at the end of June of year $t$ (Table I). BE is the book value of common equity plus balance-sheet deferred taxes, $A$ is total book assets, and $E$ is earnings (income before extraordinary items, plus income-statement deferred taxes, minus preferred dividends). BE, $A$, and $E$ are for each firm's latest fiscal year ending in calendar year $t - 1$. The accounting ratios are measured using market equity $ME$ in December of year $t - 1$. Firm size $\ln(ME)$ is measured in June of year $t$. In the regressions, these values of the explanatory variables for individual stocks are matched with CRSP returns for the months from July of year $t$ to June of year $t + 1$. The gap between the accounting data and the returns ensures that the accounting data are available prior to the returns. If earnings are positive, $E(+) / P$ is the ratio of total earnings to market equity and $E / P$ dummy is 0. If earnings are negative, $E(+) / P$ is 0 and $E / P$ dummy is 1.

The average slope is the time-series average of the monthly regression slopes for July 1963 to December 1990, and the $t$ statistic is the average slope divided by its time-series standard error.

On average, there are 2277 stocks in the monthly regressions. To avoid giving extreme observations heavy weight in the regressions, the smallest and largest 0.5% of the observations on $E(+) / P$, $BE / ME$, $A / ME$, and $A / BE$ are set equal to the next largest or smallest values of the ratios (the 0.005 and 0.995 fractiles). This has no effect on inferences.

<table>
<thead>
<tr>
<th>$\beta$</th>
<th>$\ln(ME)$</th>
<th>$\ln(BE/ME)$</th>
<th>$\ln(A/ME)$</th>
<th>$\ln(A/B)$</th>
<th>$E/P$ Dummy</th>
<th>$E(+) / P$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.15</td>
<td>(0.46)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-0.15</td>
<td>(2.58)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-0.37</td>
<td>(1.21)</td>
<td></td>
<td>-0.17</td>
<td>(3.41)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.50</td>
<td>(5.71)</td>
<td>0.50</td>
<td>-0.07</td>
<td>(5.69)</td>
<td>-5.34</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.57</td>
<td>4.72</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(2.28)</td>
<td>(4.57)</td>
</tr>
<tr>
<td>-0.11</td>
<td>(1.99)</td>
<td>0.35</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-0.11</td>
<td>(2.08)</td>
<td>0.35</td>
<td>-0.50</td>
<td>(4.32)</td>
<td>-4.56</td>
<td></td>
</tr>
<tr>
<td>-0.16</td>
<td>(3.06)</td>
<td>-0.14</td>
<td>0.06</td>
<td>(0.38)</td>
<td>2.99</td>
<td></td>
</tr>
<tr>
<td>-0.13</td>
<td>(2.47)</td>
<td>0.33</td>
<td>0.08</td>
<td>(0.90)</td>
<td>0.87</td>
<td></td>
</tr>
<tr>
<td>-0.13</td>
<td>(2.47)</td>
<td>0.32</td>
<td>-0.46</td>
<td>(2.48)</td>
<td>1.15</td>
<td></td>
</tr>
</tbody>
</table>
Consumption Based Asset Pricing Model

\[ U(C_t) = \frac{C_t^{1-\gamma}}{1-\gamma} \]

\[
\text{Max} \sum_{t=0}^{\infty} \beta^t U(C_t) \text{ subject to } C_t + \sum_{i=0}^{N} P_{it} Q_{it} \leq \sum_{i=0}^{N} P_{it} Q_{it-1} + W_t
\]

\[ \Rightarrow E_t [\beta (\frac{C_{t+1}}{C_t})^{-\gamma} R_{t+1}] - 1 = 0 \]

\( U \) : Utility Function
\( C \) : Consumption
\( \gamma \) : Coefficient of Risk Aversion
\( \beta \) : Intertemporal Rate of Substitution
\( P \) : Value of asset
\( Q \) : Amount of assets owned
\( N \) : Number of assets in the economy
\( W \) : Real Labor Income
III. Asset Pricing Models – Consumption Based Models

GMM Estimation

\[ \theta = [\gamma, \beta] \]
\[ e_t = [\beta \left( \frac{C_t}{C_{t-1}} \right)^{-\gamma} R_t ] - 1 \]
\[ z_t : \text{Vector of instruments at time } t \]
\[ m(\theta) = \frac{1}{T} Z'e \]
III. Asset Pricing Models – Consumption Based Models

GMM Estimation

\[ \theta = [\gamma, \beta] \]

\[ e_t = \left[ \beta \left( \frac{C_t}{C_{t-1}} \right)^{-\gamma} R_t \right] \]

\( z_t \): Vector of instruments at time \( t \)

\[ m(\theta) = \frac{1}{T} Z'e \]

\[ J_t = \text{Min } m(\theta)'Wm(\theta) \text{ where } W = S^{-1} \]

\[ S = \frac{1}{T} \sum_{t=1}^{T} [e_t e_t' \otimes z_t z_t'] \]

\[ TJ_t \approx \chi^2_{q-p}; q = \dim(e) \& p = \dim(z) \]
IV. Volatility Models

Fig. 1a. Monthly percent standard deviations of the returns to the Standard & Poor’s composite portfolio, $\sigma_{rt}$, estimated from returns for days $t$ within the month $t$, $r_{it}$, 1928–84.
IV. Volatility Models

ARCH–Autoregressive Conditional Heteroscedasticity

Engle (‘82) & French, Schwert, Stambaugh (‘87)

\[
ARCH(q): \sigma_t^2 = \omega + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2
\]

\[
GARCH(p, q): \sigma_t^2 = \omega + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2
\]
### IV. Volatility Models

#### Table 2

Autoregressive conditional heteroskedasticity (ARCH) models for daily excess holding period returns to the Standard & Poor’s composite portfolio.\(^a\)

\[
(R_{it} - R_{jt}) = \alpha + \epsilon_t - \theta \epsilon_{t-1} \quad (5c)
\]

\[
\sigma_t^2 = a + b \left( \sum_{i=1}^{22} \frac{\epsilon_{t-i-1}^2}{22} \right) \quad (5d)
\]

\[
\sigma_t^2 = a + b \sigma_{t-1}^2 + c_1 \epsilon_{t-1}^2 + c_2 \epsilon_{t-2}^2 \quad (5e)
\]

<table>
<thead>
<tr>
<th>ARCH model equations</th>
<th>$\alpha \times 10^3$</th>
<th>$a \times 10^3$</th>
<th>$b$</th>
<th>$c_1$</th>
<th>$c_2$</th>
<th>$\theta$</th>
<th>$\chi^2$ test for stability</th>
</tr>
</thead>
<tbody>
<tr>
<td>ARCH (5c), (5d)</td>
<td>0.265</td>
<td>1.006</td>
<td>0.938</td>
<td></td>
<td></td>
<td>-0.142</td>
<td>92.7</td>
</tr>
<tr>
<td></td>
<td>(0.061)</td>
<td>(0.048)</td>
<td>(0.012)</td>
<td></td>
<td></td>
<td>(0.007)</td>
<td></td>
</tr>
<tr>
<td>GARCH (5c), (5e)</td>
<td>0.324</td>
<td>0.062</td>
<td>0.919</td>
<td>0.121</td>
<td>-0.044</td>
<td>-0.157</td>
<td>86.7</td>
</tr>
<tr>
<td></td>
<td>(0.063)</td>
<td>(0.005)</td>
<td>(0.002)</td>
<td>(0.007)</td>
<td>(0.007)</td>
<td>(0.007)</td>
<td></td>
</tr>
</tbody>
</table>

(A) January 1928 to December 1984, $T = 15,369$

(B) January 1928 to December 1952, $T = 7,326$

(C) January 1953 to December 1984, $T = 8,043$

---

\(^a\) $(R_{it} - R_{jt})$ is the daily excess holding period return to the Standard & Poor’s composite portfolio (the percentage price change minus the yield on a short-term default-free government bond). Non-linear optimization techniques are used to calculate maximum likelihood estimates. Asymptotic standard errors are in parentheses under the coefficient estimates. The $\chi^2$ test statistic is distributed $\chi^2_{1}$ for the ARCH model (5d) and $\chi^2_{2}$ for the generalized ARCH or GARCH model (5e) under the hypothesis that the parameters are equal in the subperiods.
Financial Econometrics is the Application of Econometric Methods to Financial Markets

- **Econometrics**: Time Series, GMM, ARCH,..
- **Finance**:
  - Returns not Predictable?
  - Volatility is Predictable?