

# Financial Markets And Empirical Regularities

An Introduction to Financial Econometrics

SAMSI Workshop 11/18/05

Mike Aguilar – UNC at Chapel Hill

[www.unc.edu/~maguilar](http://www.unc.edu/~maguilar)

# Outline

- I. Historical Perspective on Asset Prices
- II. Predictability of Asset Returns
- III. Asset Pricing Models
- IV. Volatility Models

# Essential Sources

- Campbell, Lo, & MacKinlay, “The Econometrics of Financial Markets”
- Cochrane, “Asset Pricing”
- Huang & Litzenberger, “Foundations for Financial Economics”
- Hamilton, “Time Series Analysis”
- Greene, “Econometric Analysis”

# I. Historical Perspective on Asset Prices

## **Types of Financial Assets:**

Common: Stocks, Bonds, Commodities,  
Foreign Exchange,...

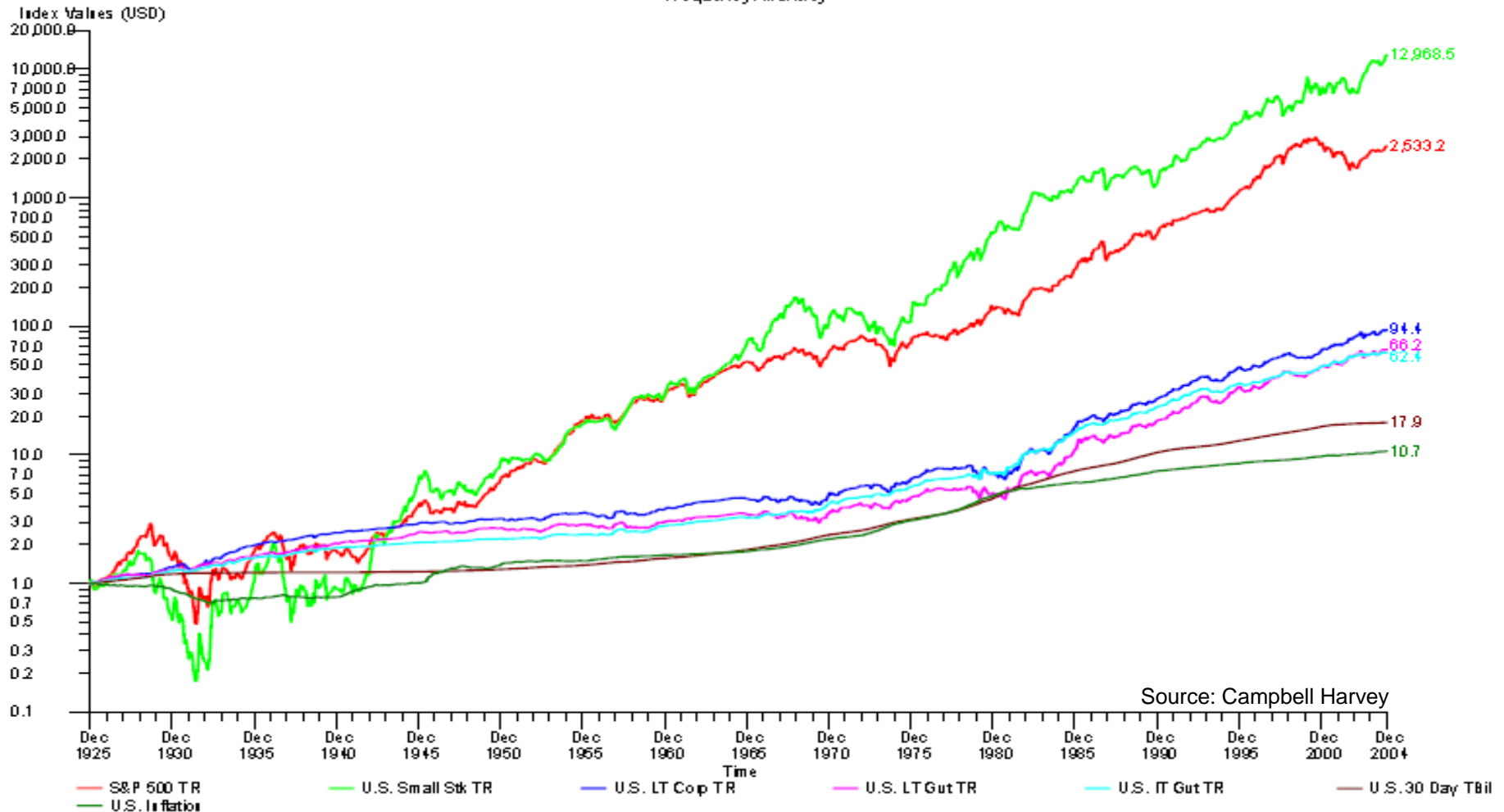
Exotic: Derivatives; Options, Futures,...

# I. Historical Perspective on Asset Prices

## A History of Asset Prices

### Index Line Graph

Frequency: Monthly



# I. Historical Perspective on Asset Prices

## Definitions:

- Simple Gross Return:  $R_t + 1 = \frac{P_t}{P_{t-1}}$ ,  $P_t$  = price of asset at time  $t$ .
- Compound Gross Return:  $R_t(k) + 1 = (1 + R_t) * (1 + R_{t-1}) * \dots * (1 + R_{t-k+1})$
- Annualized Return:  $\left[ \prod_{j=0}^{k-1} (1 + R_{t-j}) \right]^{1/k} - 1 \approx \frac{1}{k} \sum_{j=0}^{k-1} R_{t-j}$
- Continuously Compounded Return:  $r_t = \log(1 + R_t) = \log P_t - \log P_{t-1}$

# I. Historical Perspective on Asset Prices

## Why Returns?

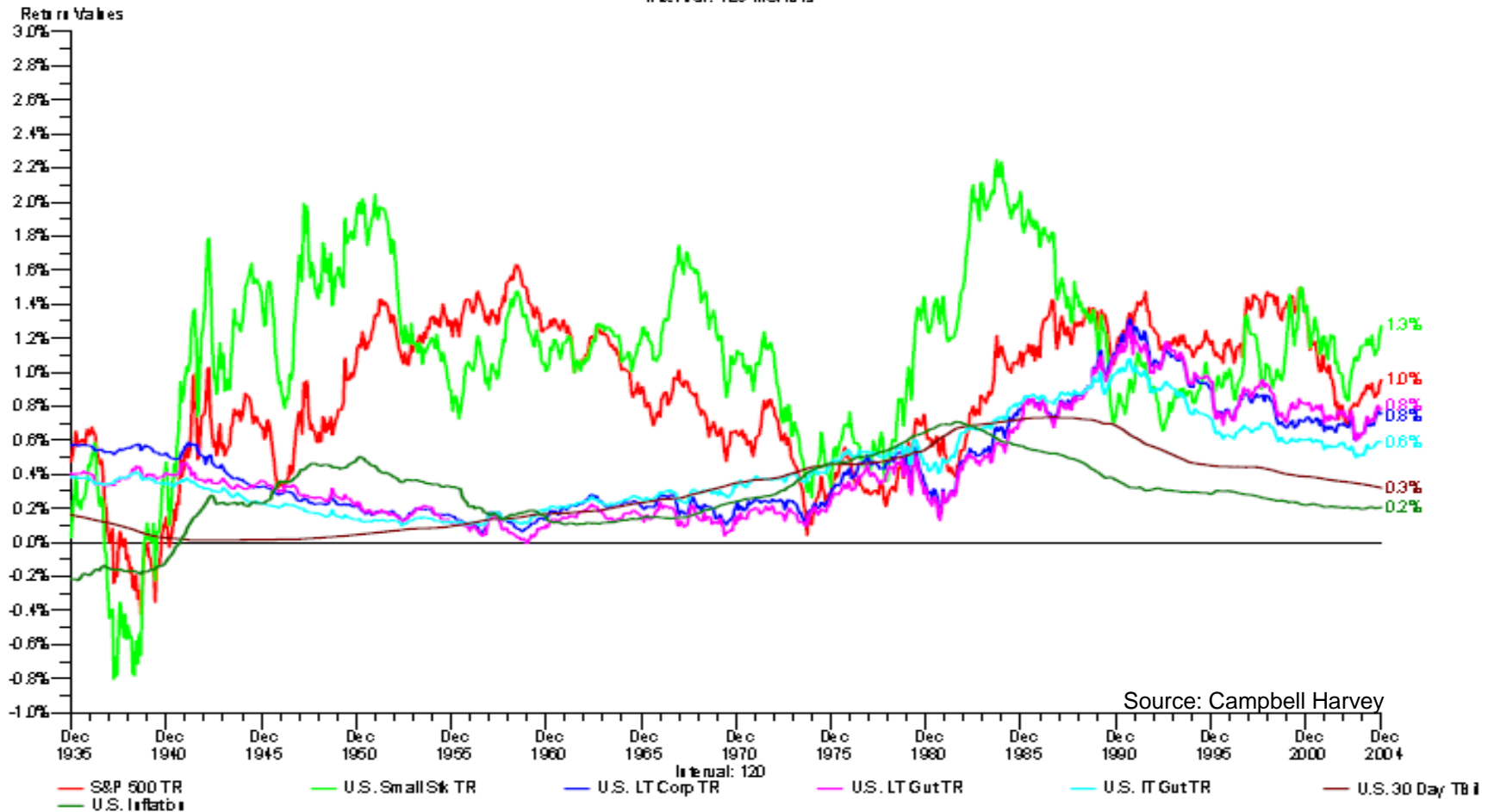
.....Because returns tend to be stationary.

Stationary: The joint distribution between two returns  $x_t, x_{t-h}$  depends only on  $h$  and NOT on  $t$ .

# I. Historical Perspective on Asset Prices

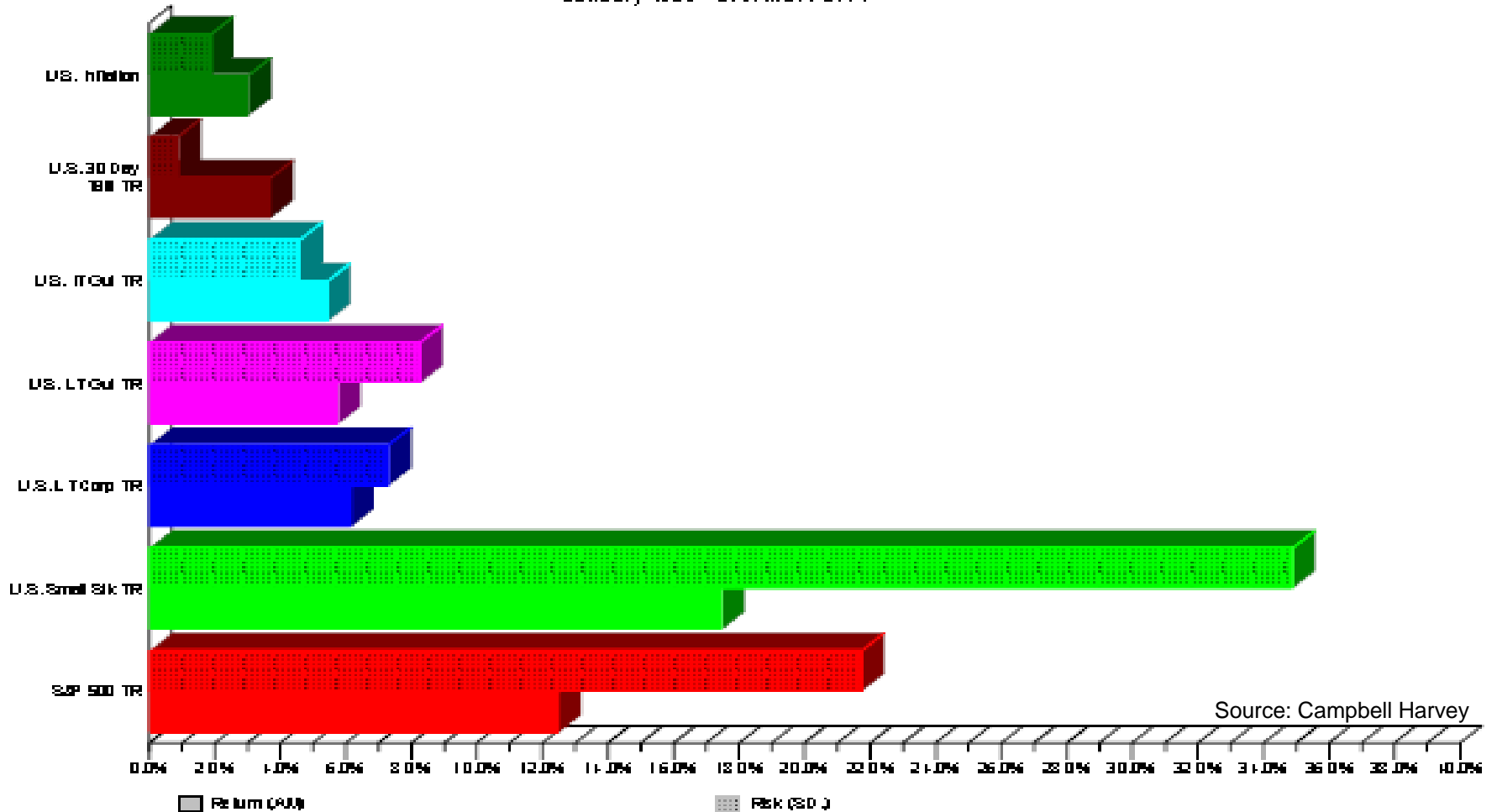
## A History of Asset Returns

Rolling Line Graph  
Interval: 120 Months



# I. Historical Perspective on Asset Prices

**Risk vs. Return**  
January 1926 - December 2004

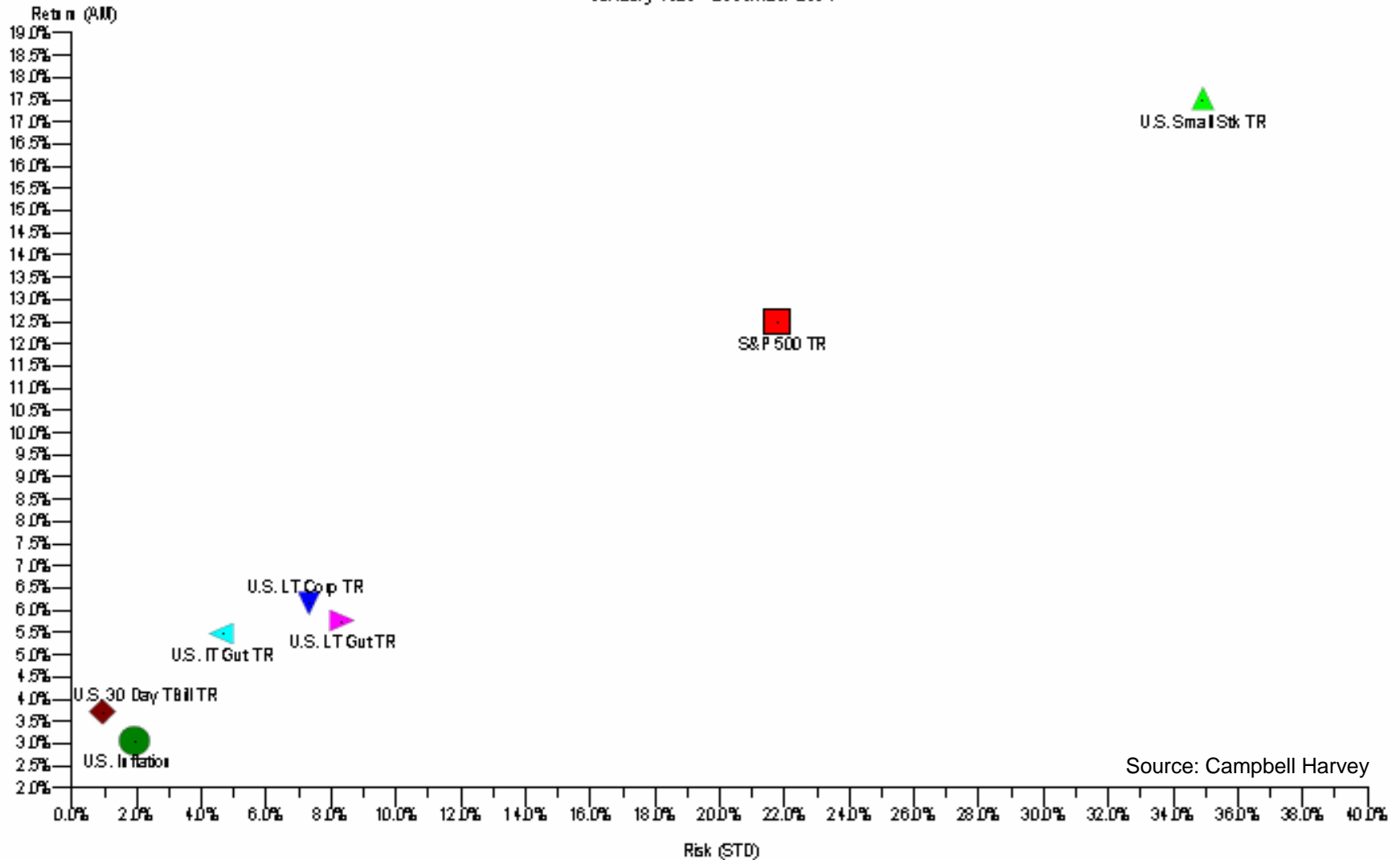


Source: Campbell Harvey

# I. Historical Perspective on Asset Prices

## Risk vs. Return

January 1926 - December 2004



Source: Campbell Harvey

# I. Historical Perspective on Asset Prices

## **Observation:**

- There exists a Risk / Return Tradeoff

# I. Historical Perspective on Asset Prices

## Stats Reminder:

Consider a random variable  $x_t$

- Mean:

$$\hat{\mu} \equiv \frac{1}{T} \sum_{t=1}^T x_t$$

- Variance:

$$\hat{\sigma}^2 \equiv \frac{1}{T} \sum_{t=1}^T (x_t - \hat{\mu})^2$$

- Skew:

$$\hat{S} \equiv \frac{1}{T \hat{\sigma}^3} \sum_{t=1}^T (x_t - \hat{\mu})^3$$

- Kurtosis:

$$\hat{K} \equiv \frac{1}{T \hat{\sigma}^4} \sum_{t=1}^T (x_t - \hat{\mu})^4$$

# I. Historical Perspective on Asset Prices

## What is the Distribution of Returns?

➤ Normal?

# I. Historical Perspective on Asset Prices

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*Table 1.1. Stock market returns, 1962 to 1994.*

Security	Mean	Standard Deviation	Skewness	Excess Kurtosis	Minimum	Maximum
<i>Panel A: Daily Returns</i>						
Value-Weighted Index	0.044	0.82	-1.33	34.92	-18.10	8.87
Equal-Weighted Index	0.073	0.76	-0.93	26.03	-14.19	9.83
International Business Machines	0.039	1.42	-0.18	12.48	-22.96	11.72
General Signal Corp.	0.054	1.66	0.01	3.35	-13.46	9.43
Wrigley Co.	0.072	1.45	-0.00	11.03	-18.67	11.89
Interlake Corp.	0.043	2.16	0.72	12.35	-17.24	23.08
Raytech Corp.	0.050	3.39	2.25	59.40	-57.90	75.00
Ampco-Pittsburgh Corp.	0.053	2.41	0.66	5.02	-19.05	19.18
Energen Corp.	0.054	1.41	0.27	5.91	-12.82	11.11
General Host Corp.	0.070	2.79	0.74	6.18	-23.53	22.92
Garan Inc.	0.079	2.35	0.72	7.13	-16.67	19.07
Continental Materials Corp.	0.143	5.24	0.93	6.49	-26.92	50.00
<i>Panel B: Monthly Returns</i>						
Value-Weighted Index	0.96	4.33	-0.29	2.42	-21.81	16.51
Equal-Weighted Index	1.25	5.77	0.07	4.14	-26.80	33.17
International Business Machines	0.81	6.18	-0.14	0.83	-26.19	18.95
General Signal Corp.	1.17	8.19	-0.02	1.87	-36.77	29.73
Wrigley Co.	1.51	6.68	0.30	1.31	-20.26	29.72
Interlake Corp.	0.86	9.38	0.67	4.09	-30.28	54.84
Raytech Corp.	0.83	14.88	2.73	22.70	-45.65	142.11
Ampco-Pittsburgh Corp.	1.06	10.64	0.77	2.04	-36.08	46.94
Energen Corp.	1.10	5.75	1.47	12.47	-24.61	48.36
General Host Corp.	1.33	11.67	0.35	1.11	-38.05	42.86
Garan Inc.	1.64	11.30	0.76	2.30	-35.48	51.60
Continental Materials Corp.	1.64	17.76	1.13	3.33	-58.09	84.78

Summary statistics for daily and monthly returns (in percent) of CRSP equal- and value-weighted stock indexes and ten individual securities continuously listed over the entire sample period from July 3, 1962 to December 30, 1994. Individual securities are selected to represent stocks in each size decile. Statistics are defined in (1.4.19)–(1.4.22).

## **Stylized Facts on the Distribution of Returns:**

1. Index Volatility < Stock Volatility
2. Negative Skewness
3. Excess Kurtosis

## II. Predictability of Returns

### Can We Predict Returns?

- Day Traders Say Yes
- Efficient Market Hypothesis Says No

### **Efficient Market Hypothesis (EMH):**

Fama (1970): “A market in which prices always ‘fully reflect’ available information is called ‘efficient’”.

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Malkiel (1992): “...the market is said to be efficient with respect to an *information set*...”

### Probability Theory Reminder:

- The Information Set  $F_t$  is a sigma field said to contain all of the relevant information up to time  $t$ .

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- Martingale:  $E[x_{t+1} | F_t] = x_t$

## II. Predictability of Returns

**EMH:**

Weak Form Efficiency:

Information Set: Asset's own history

Test via Random Walk

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### **EMH:**

#### Weak Form Efficiency:

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#### Semistrong Efficiency:

Information Set: Weak Form + Publicly available data

Test via Event Studies

## II. Predictability of Returns

### **EMH:**

#### Weak Form Efficiency:

Information Set: Asset's own history

Test via Random Walk

#### Semistrong Efficiency:

Information Set: Weak Form + Publicly available data

Test via Event Studies

#### Strong Form Efficiency:

Information Set: Weak + Semi + Private Info

Test via Performance Evaluation

**Joint hypothesis problem:** The EMH can't be tested directly. Even if we reject the hypothesis of efficiency, this could either be because the market is truly inefficient, or because we have assumed an incorrect equilibrium model.

### **Implication of Weak Form EMH:**

- Expected Returns follow a martingale.
- Random Walk Hypothesis

## II. Predictability of Returns

$$P_t = \mu + P_{t-1} + \varepsilon_t$$

**Random Walk 1: i.i.d Increments**

**Random Walk 2: Independent, Not Identical**

**Random Walk 3: Uncorrelated Increments**

## Time Series Econometrics Reminder:

### Auto Correlation

Correlation between two observations of the same series at different dates

AutoCovariance :  $\gamma(k) \equiv \text{Cov}(x_t, x_{t-k})$

AutoCorrelation :  $\rho(k) \equiv \frac{\gamma(k)}{\gamma(0)}$

# Time Series Econometrics Reminder:

## Auto Correlation

Correlation between two observations of the same series at different dates

AutoCovariance :  $\gamma(k) \equiv Cov(x_t, x_{t-k})$

AutoCorrelation :  $\rho(k) \equiv \frac{\gamma(k)}{\gamma(0)}$

Box – Pierce Q Statistic :  $Q_m = T(T + 2) \sum_{k=1}^m \frac{\rho^2(k)}{T-k}$

## II. Predictability of Returns

Table 2.4. Autocorrelation in daily, weekly, and monthly stock index returns.

Sample Period	Sample Size	Mean	SD	$\hat{\rho}_1$	$\hat{\rho}_2$	$\hat{\rho}_3$	$\hat{\rho}_4$	$\hat{Q}_5$	$\hat{Q}_{10}$
A. Daily Returns									
CRSP Value-Weighted Index									
62:07:03–94:12:30	8,179	0.041	0.824	17.6	-0.7	0.1	-0.8	263.3	269.5
62:07:03–78:10:27	4,090	0.028	0.738	27.8	1.2	4.6	3.3	329.4	343.5
78:10:30–94:12:30	4,089	0.054	0.901	10.8	-2.2	-2.9	-3.5	69.5	72.1
CRSP Equal-Weighted Index									
62:07:03–94:12:30	8,179	0.070	0.764	35.0	9.3	8.5	9.9	1,301.9	1,369.5
62:07:03–78:10:27	4,090	0.063	0.771	43.1	13.0	15.3	15.2	1,062.2	1,110.2
78:10:30–94:12:30	4,089	0.078	0.756	26.2	4.9	2.0	4.9	348.9	379.5
B. Weekly Returns									
CRSP Value-Weighted Index									
62:07:10–94:12:27	1,695	0.196	2.093	1.5	-2.5	3.5	-0.7	8.8	36.7
62:07:10–78:10:03	848	0.144	1.994	5.6	-3.7	5.8	1.6	9.0	21.5
78:10:10–94:12:27	847	0.248	2.188	-2.0	-1.5	1.6	-3.3	5.3	25.2
CRSP Equal-Weighted Index									
62:07:10–94:12:27	1,695	0.339	2.321	20.3	6.1	9.1	4.8	94.3	109.3
62:07:10–78:10:03	848	0.324	2.460	21.8	7.5	11.9	6.1	60.4	68.5
78:10:10–94:12:27	847	0.354	2.174	18.4	4.3	5.5	2.2	33.7	51.3
C. Monthly Returns									
CRSP Value-Weighted Index									
62:07:31–94:12:30	390	0.861	4.336	4.3	-5.3	-1.3	-0.4	6.8	12.5
62:07:31–78:09:29	195	0.646	4.219	6.4	-3.8	7.3	6.2	3.9	9.7
78:10:31–94:12:30	195	1.076	4.450	1.3	-6.3	-8.3	-7.7	7.5	14.0
CRSP Equal-Weighted Index									
62:07:31–94:12:30	390	1.077	5.749	17.1	-3.4	-3.3	-1.6	12.8	21.3
62:07:31–78:09:29	195	1.049	6.148	18.4	-2.5	4.4	2.4	7.5	12.6
78:10:31–94:12:30	195	1.105	5.336	15.0	-1.6	-12.4	-7.4	8.9	14.2

Autocorrelation coefficients (in percent) and Box-Pierce  $Q$ -statistics for CRSP daily, weekly, and monthly value- and equal-weighted return indexes for the sample period from July 3, 1962 to December 30, 1994 and subperiods.

### III. Asset Pricing Models

- Capital Asset Pricing Model (CAPM)

Sharpe (1964) and Lintner (1965)

- Consumption Based-CAPM

- Inter-temporal-CAPM

- Arbitrage Pricing Theory

Ross (1976)

### III. Asset Pricing Models - CAPM

**CAPM:** Differences in excess returns across assets are due to differences in the *riskiness* of each asset.

**Beta:** Measure of *riskiness*

$$\text{Market Model: } (R_i - R_f) = \beta(R_m - R_f) + u$$

⇒ There exists a linear Risk/Reward Tradeoff

$$\text{OLS } \Rightarrow \beta = \frac{\text{Cov}[(R_i - R_f), (R_m - R_f)]}{\text{Var}(R_m - R_f)}$$

## CAPM Pricing Equation:

$$E[R_i] = R_f + \beta_{im} (E[R_m] - R_f)$$

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$$E[R_i] = R_f + \beta_{im} (E[R_m] - R_f)$$

Testable Implications:

- 1) Only beta is required to price assets.
- 2) There is a linear risk/return relationship.

## Testing the CAPM:

Fama & MacBeth (1973)

Step 1: For each time period estimate the cross-sectional regression:

$(R_i - R_f) = \theta_0 + \theta_1 \beta_{im} + \theta_2 \beta_{im}^2 + \theta_3 s_i + u_i$ , where  $s$  is the standard deviation of  $u$ .

Step 2: Aggregate parameter estimates over time such that  $\theta_k = E[\theta_k] \forall k = 0,1,2,3$

### III. Asset Pricing Models - CAPM

Fama & MacBeth cont'd

There is an errors - in - variables (EIV) problem because the  $\beta$ 's must first be estimated.

Fama & MacBeth solution: Sort stocks into portfolios.

Shanken solution: correct variance of estimators post estimation.

$$H_0 : \theta_0 = \theta_1 = \theta_2 = \theta_3 = 0$$

$\Rightarrow \beta$  is the only factor necessary to price assets  
& there is a positive, linear risk - return tradeoff.

# III. Asset Pricing Models - CAPM

TABLE 3

SUMMARY RESULTS FOR THE REGRESSION

$$R_p = \hat{\gamma}_0 r + \hat{\gamma}_1 t \hat{\beta}_p + \hat{\gamma}_2 t \hat{\beta}_p^2 + \hat{\gamma}_3 t \bar{s}_p(\hat{\epsilon}_i) + \hat{\eta}_{pt}$$

PERIOD	STATISTIC																			
	$\hat{\gamma}_0$	$\hat{\gamma}_1$	$\hat{\gamma}_2$	$\hat{\gamma}_3$	$\hat{\gamma}_0 - R_f$	$s(\hat{\gamma}_0)$	$s(\hat{\gamma}_1)$	$s(\hat{\gamma}_2)$	$s(\hat{\gamma}_3)$	$\rho_0(\hat{\gamma}_0 - R_f)$	$\rho_H(\hat{\gamma}_1)$	$\rho_0(\hat{\gamma}_2)$	$\rho_0(\hat{\gamma}_3)$	$t(\hat{\gamma}_0)$	$t(\hat{\gamma}_1)$	$t(\hat{\gamma}_2)$	$t(\hat{\gamma}_3)$	$t(\hat{\gamma}_0 - R_f)$	$\bar{r}^2$	$s(r^2)$
Panel C:																				
1935-6/68 ..	.0054	.0072	...	.0198	.0041	.052	.065	...	.868	.04	-.12	...	-.04	2.10	2.20	...	.46	1.59	.32	.31
1935-45 ....	.0017	.0104	...	.0841	.0015	.073	.083	...	.921	-.00	-.26	...	-.08	.26	1.41	...	1.05	.24	.32	.31
1946-55 ....	.0110	.0075	...	-.1052	.0100	.032	.056	...	.609	.08	.02	...	-.20	3.78	1.47	...	-1.89	3.46	.34	.32
1956-6/68 ..	.0042	.0041	...	.0633	.0016	.040	.052	...	.984	.12	.08	...	.03	1.28	.96	...	.79	.50	.30	.29
1935-40 ....	.0036	.0119	...	-.0170	.0035	.082	.105	...	.744	-.03	-.26	...	-.18	.37	.97	...	-.19	.36	.25	.30
1941-45 ....	-.0006	.0085	...	.2053	-.0009	.061	.052	...	1.091	.07	-.29	...	-.02	-.08	1.25	...	1.46	-.11	.41	.30
1946-50 ....	.0069	.0081	...	-.0920	.0062	.034	.066	...	.504	.14	.06	...	-.02	1.56	.95	...	-1.41	1.40	.42	.33
1951-55 ....	.0150	.0069	...	-.1185	.0138	.029	.043	...	.702	.06	-.18	...	-.32	4.05	1.24	...	-1.31	3.72	.27	.29
1956-60 ....	.0127	-.0081	...	.0728	.0107	.037	.045	...	1.164	.15	.15	...	.21	2.68	-1.40	...	.48	2.26	.26	.30
1961-6/68 ..	-.0014	.0122	...	.0570	-.0044	.042	.055	...	.850	.10	.00	...	-.19	-.32	2.12	...	.64	-.98	.33	.27
Panel D:																				
1935-6/68 ..	.0020	.0114	-.0026	.0516	.0008	.075	.123	.060	.929	-.09	-.09	-.12	-.10	.55	1.85	-.86	1.11	.20	.34	.31
1935-45 ....	.0011	.0118	-.0009	.0817	.0010	.103	.146	.079	1.003	-.20	-.23	-.24	-.15	.13	.94	-.14	.94	.11	.34	.31
1946-55 ....	.0017	.0209	-.0076	-.0378	.0008	.042	.096	.038	.619	-.10	-.00	-.01	-.20	.44	2.39	-2.16	-.67	.20	.36	.32
1956-6/68 ..	.0031	.0034	-.0000	.0966	.0005	.065	.122	.055	1.061	.12	.03	.01	-.05	.59	.34	-.00	1.11	.10	.32	.29
1935-40 ....	.0009	.0156	-.0029	.0025	.0008	.112	.171	.085	.826	-.16	-.23	-.26	-.12	.07	.78	-.29	.03	.06	.26	.30
1941-45 ....	.0015	.0073	.0014	.1767	.0012	.092	.109	.072	1.181	-.28	-.21	-.22	-.18	.12	.52	.15	1.16	.10	.43	.31
1946-50 ....	.0011	.0141	-.0040	-.0313	.0004	.047	.106	.042	.590	-.10	.03	-.01	-.12	.18	1.03	-.73	-.41	.07	.44	.33
1951-55 ....	.0023	.0277	-.0112	-.0443	.0011	.037	.085	.034	.651	-.11	-.13	-.01	-.28	.48	2.53	-2.54	-.53	.23	.29	.30
1956-60 ....	.0103	-.0047	-.0020	.0979	.0083	.049	.078	.032	1.286	-.16	.19	-.01	.02	1.63	-.47	-.49	.59	1.31	.28	.30
1961-6/68 ..	-.0017	.0088	.0013	.0957	-.0046	.073	.144	.066	.887	.20	.00	.01	-.15	-.21	.58	.19	1.02	-.60	.35	.29

## CAPM Anomalies:

- Small Firm effect (Keim `81)
- P/E Ratio Effect (Ball `78 and Basu `83)
- Book-to-Market Effect (Stattman `80 and Rosenberg `85)
- Momentum Effect (Jegadeesh and Titman `93)

⇒ Is Beta Dead?

### III. Asset Pricing Models - CAPM

Table III

**Average Slopes (*t*-Statistics) from Month-by-Month Regressions of Stock Returns on  $\beta$ , Size, Book-to-Market Equity, Leverage, and E/P: July 1963 to December 1990**

Stocks are assigned the post-ranking  $\beta$  of the size- $\beta$  portfolio they are in at the end of June of year *t* (Table I). BE is the book value of common equity plus balance-sheet deferred taxes, A is total book assets, and E is earnings (income before extraordinary items, plus income-statement deferred taxes, minus preferred dividends). BE, A, and E are for each firm's latest fiscal year ending in calendar year *t* - 1. The accounting ratios are measured using market equity ME in December of year *t* - 1. Firm size ln(ME) is measured in June of year *t*. In the regressions, these values of the explanatory variables for individual stocks are matched with CRSP returns for the months from July of year *t* to June of year *t* + 1. The gap between the accounting data and the returns ensures that the accounting data are available prior to the returns. If earnings are positive, E(+)/P is the ratio of total earnings to market equity and E/P dummy is 0. If earnings are negative, E(+)/P is 0 and E/P dummy is 1.

The average slope is the time-series average of the monthly regression slopes for July 1963 to December 1990, and the *t*-statistic is the average slope divided by its time-series standard error.

On average, there are 2267 stocks in the monthly regressions. To avoid giving extreme observations heavy weight in the regressions, the smallest and largest 0.5% of the observations on E(+)/P, BE/ME, A/ME, and A/BE are set equal to the next largest or smallest values of the ratios (the 0.005 and 0.995 fractiles). This has no effect on inferences.

$\beta$	ln(ME)	ln(BE/ME)	ln(A/ME)	ln(A/BE)	E/P Dummy	E(+)/P
0.15 (0.46)						
	-0.15 (-2.58)					
-0.37 (-1.21)	-0.17 (-3.41)					
		0.50 (5.71)				
			0.50 (5.69)	-0.57 (-5.34)		
					0.57 (2.28)	4.72 (4.57)
	-0.11 (-1.99)	0.35 (4.44)				
	-0.11 (-2.06)		0.35 (4.32)	-0.50 (-4.56)		
	-0.16 (-3.06)				0.06 (0.38)	2.99 (3.04)
	-0.13 (-2.47)	0.33 (4.46)			-0.14 (-0.90)	0.87 (1.23)
	-0.13 (-2.47)		0.32 (4.28)	-0.46 (-4.45)	-0.08 (-0.56)	1.15 (1.57)

# Consumption Based Asset Pricing Model

$$U(C_t) = \frac{C_t^{1-\gamma}}{1-\gamma}$$

$$\text{Max} \sum_{t=0}^{\infty} \beta^t U(C_t) \text{ subject to } C_t + \sum_{i=0}^N P_{it} Q_{it} \leq \sum_{i=0}^N P_{it} Q_{it-1} + W_t$$

$$\Rightarrow E_t \left[ \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\gamma} R_{t+1} \right] - 1 = 0$$

$U$  : Utility Function

$C$  : Consumption

$\gamma$  : Coefficient of Risk Aversion

$\beta$  : Intertemporal Rate of Substitution

$P$  : Value of asset

$Q$  : Amount of assets owned

$N$  : Number of assets in the economy

$W$  : Real Labor Income

## GMM Estimation

$$\theta = [\gamma, \beta]$$

$$e_t = [\beta \left(\frac{c_t}{c_{t-1}}\right)^{-\gamma} R_t] - 1$$

$z_t$  : Vector of instruments at time t

$$m(\theta) = \frac{1}{T} Z' e$$

## GMM Estimation

$$\theta = [\gamma, \beta]$$

$$e_t = [\beta(\frac{C_t}{C_{t-1}})^{-\gamma} R_t]$$

$z_t$  : Vector of instruments at time t

$$m(\theta) = \frac{1}{T} Z' e$$

$$J_t = \text{Min } m(\theta)' W m(\theta) \text{ where } W = S^{-1}$$

$$S = \frac{1}{T} \sum_{t=1}^T [e_t e_t' \otimes z_t z_t']$$

$$TJ_t \approx \chi_{q-p}^2; q = \dim(e) \text{ \& } p = \dim(z)$$

# IV. Volatility Models

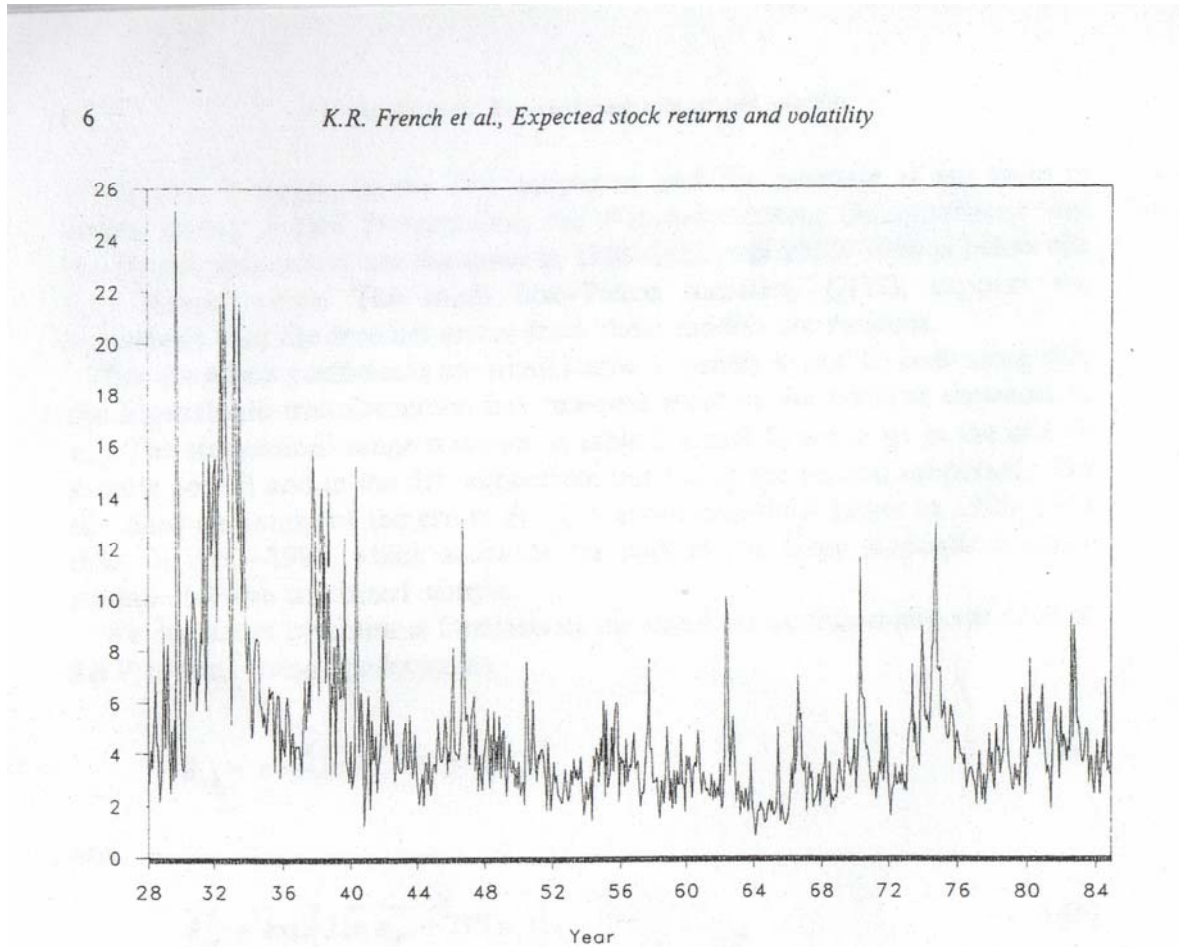


Fig. 1a. Monthly percent standard deviations of the returns to the Standard & Poor's composite portfolio,  $\sigma_{mt}$ , estimated from returns for days  $i$  within the month  $t$ ,  $r_{it}$ , 1928–84.

# ARCH–Autoregressive Conditional Heteroscedasticity

Engle (ˆ82) & French, Schwert, Stambaugh (ˆ87)

$$ARCH(q) : \sigma_t^2 = \omega + \sum_{i=1}^q \alpha_i \varepsilon_{t-i}^2$$

$$GARCH(p, q) : \sigma_t^2 = \omega + \sum_{i=1}^q \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^p \beta_j \sigma_{t-j}^2$$

# IV. Volatility Models

Table 2

Autoregressive conditional heteroskedasticity (ARCH) models for daily excess holding period returns to the Standard & Poor's composite portfolio.<sup>a</sup>

$$(R_{mt} - R_{ft}) = \alpha + \varepsilon_t - \theta \varepsilon_{t-1} \quad (5c)$$

$$\sigma_t^2 = a + b \left( \sum_{i=1}^{22} \frac{\varepsilon_{t-i}^2}{22} \right) \quad (5d)$$

$$\sigma_t^2 = a + b\sigma_{t-1}^2 + c_1\varepsilon_{t-1}^2 + c_2\varepsilon_{t-2}^2 \quad (5e)$$

ARCH model equations	$\alpha \times 10^3$	$a \times 10^5$	$b$	$c_1$	$c_2$	$\theta$	$\chi^2$ test for stability
<i>(A) January 1928 to December 1984, T = 15,369</i>							
ARCH (5c), (5d)	0.265 (0.061)	1.006 (0.048)	0.938 (0.012)			-0.142 (0.007)	92.7
GARCH (5c), (5e)	0.324 (0.063)	0.062 (0.005)	0.919 (0.002)	0.121 (0.007)	-0.044 (0.007)	-0.157 (0.007)	86.7
<i>(B) January 1928 to December 1952, T = 7,326</i>							
ARCH (5c), (5d)	0.405 (0.111)	1.678 (0.094)	0.924 (0.015)			-0.080 (0.010)	
GARCH (5c), (5e)	0.496 (0.111)	0.149 (0.013)	0.898 (0.004)	0.106 (0.009)	-0.012 (0.009)	-0.090 (0.012)	
<i>(C) January 1953 to December 1984, T = 8,043</i>							
ARCH (5c), (5d)	0.218 (0.076)	0.947 (0.069)	0.856 (0.023)			-0.194 (0.010)	
GARCH (5c), (5e)	0.257 (0.080)	0.052 (0.008)	0.922 (0.004)	0.130 (0.010)	-0.060 (0.010)	-0.211 (0.012)	

<sup>a</sup> $(R_{mt} - R_{ft})$  is the daily excess holding period return to the Standard & Poor's composite portfolio (the percentage price change minus the yield on a short-term default-free government bond). Non-linear optimization techniques are used to calculate maximum likelihood estimates. Asymptotic standard errors are in parentheses under the coefficient estimates. The  $\chi^2$  test statistic is distributed  $\chi_4^2$  for the ARCH model (5d) and  $\chi_6^2$  for the generalized ARCH or GARCH model (5e) under the hypothesis that the parameters are equal in the subperiods.

# Financial Econometrics is the Application of Econometric Methods to Financial Markets

- Econometrics: Time Series, GMM, ARCH, ..
- Finance:
  - Returns not Predictable?
  - Volatility is Predictable?