Predatory Delays in Patent Litigation

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October 27, 2009

Abstract

This paper explores theoretical motives for predatory patent holder behavior. We develop a signaling model where the timing of litigation against an initial act of infringement transmits information about a patent holder's private expectations of litigation awards. We find a tension between two reputation effects: deterrence and luring. Patent holders with sufficiently high award expectations will exhibit predatory behavior by delaying litigation to lure infringers. At the same time, other patent holders will litigate quickly to deter additional acts of infringement. We demonstrate that the predatory behavior in our model is not prohibited by current patent law.

1 Introduction

In 2007 a jury awarded the plaintiff Alcatel-Lucent $1.5 billion for the violation of two patents by the defendant Microsoft, who allegedly used unlicensed audio technology in their Windows operating systems beginning in 1997. After siding with Alcatel-Lucent, the court based their award on 0.5% of the retail value of all the computers sold with the unlicensed technology. We notice that Alcatel-Lucent waited until after the release of Windows XP in 2001 to pursue litigation in 2002, well after the first unlicensed use of the patented technology in 1997. So the question remains: why did Alcatel-Lucent wait so long to initiate litigation?

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One theory, supported by our model, is that Alcatel-Lucent had an incentive to delay litigation until after Microsoft committed a subsequent act of infringement with the release of Windows XP. In this sense, Alcatel-Lucent may have foreseen the potential for a larger award in the future by noting that sales were accumulating during the delay, and that these sales would probably be used to calculate awards. Therefore, if Alcatel-Lucent had initiated litigation based on earlier versions of Windows, Microsoft may have avoided incorporating the patented technology into future products altogether, and thus eliminated the potential for Alcatel-Lucent to extract any royalty for subsequent use.

The example above suggests that the timing of litigation can affect the likelihood of additional acts of infringement. We develop a signaling model where the timing of litigation against an initial act of infringement transmits information about a patent holder's private expectations of litigation awards. We analyze the impact of this information transmission on patent holder and infringer behavior.

We find many cases where patent holder behavior is impacted by two reputation effects: deterrence and luring. Patent holders with sufficiently high award expectations will utilize luring by delaying litigation.[1] At the same time, other patent holders will utilize deterrence by litigating quickly before additional acts of infringement can be committed. Thus we find that the probability of preemptive litigation is increasing in the patent holder's award expectation up to a point, after which the probability of preemptive litigation occurring is decreasing until it reaches zero. From this analysis we learn how litigation timing is affected by issues of reputation.[2]

We show that the incentives for predatory patent holder behavior will impact the game when both of the following criteria are met: 1) at least one patent holder type expects litigation to return greater expected profits than the next best alternative to litigation. 2)

[1] Litigation Awards can be up to three times damages plus legal costs. Furthermore, since awards for patent infringement cannot fall below the courts definition of a "reasonable royalty", a patent holder who is confident in the validly of his patent, as well as his ability to prove infringement, should view an expected licensing fee as an approximate baseline for awards.

[2] In our model, a patent holder that desires deterrence is analogous to a "weak" incumbent acting "tough" in the classic entrant incumbent game modeled in Kreps and Wilson (1982). However the potential incentives for luring provide a new twist on an old game. That is, we also see tough types acting weak.

The infringer's equilibrium entry decision is conditioned on their observations of any previous litigation. (1) requires that at least one patent holder type has an incentive for predatory behavior. (2) requires that the there is an opportunity to benefit from reputations impact on the infringer's future entry decision.

We show that our model generates a unique perfect Bayesian equilibrium for each parameter setting. The existence property of the model, combined with the intuitive nature of resulting equilibrium demonstrates the model's ability to provide insightful results for a wide range of settings. Furthermore, the uniqueness property show that our conclusions are based on the only equilibrium that exists within any setting, thereby showing that our conclusions do not depend on any selection criteria for the equilibrium to analyze.

The predatory behavior found in our model is not necessarily prohibited under current patent law. The current statute of limitations requires that litigation be brought within 6 years of an initial act of infringement.[3] In many cases, opportunities for the infringer to commit additional acts of infringement exist well within this time frame.[4] However, we show that policy makers can affect the magnitude of reputational distortions without changing the statute of limitations. To this end, we demonstrate how the magnitude of reputation's effect on behavior is linked to the prior distribution of expected awards, noting that policy makers have some control over these distributions.

Our research is related to other work that focuses on the strategic behavior of patent holders in the face of potential infringement. Crampes and Langinier (2002) model patent holders who must expend effort to monitor for infringement. They calculate the optimal effort level, and find that increasing awards may not necessarily decrease the probability of infringement. We include a similar notion of patent holder effort. Michael J. Meurer (1989) models the nature of settlement offers to a potential infringer. Meurer analyzes how a patent holder's private expectations of their patent's validity can lead to a particular settlement, litigation, or inaction outcome. Our model also assumes that patent holders have private information about their patent. However, we expand on Meurer's notion of private information by includ-

[3] Pincus (1991)

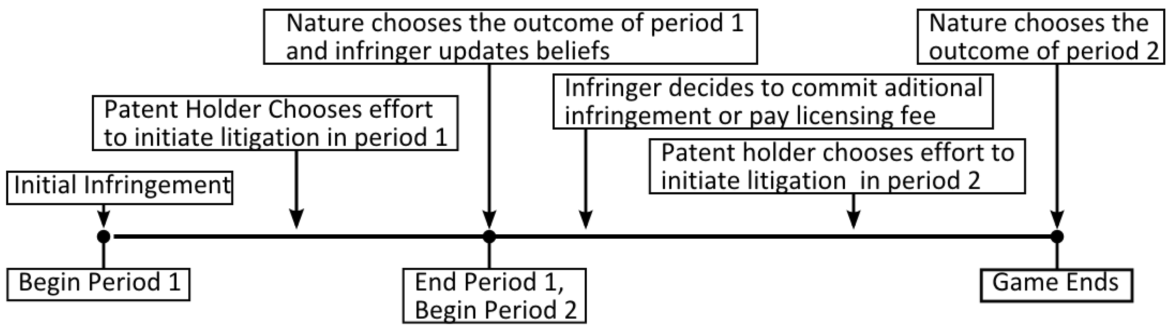
[4] Note that any future production or sales of an infringing product after the initial act of infringement will constitute an additional act of infringement in our model.

ing any element of private information that affects expected awards. Certainly, information about validity will affect a patent holder's expected award. Therefore, our interpretation of private information includes, but is not limited to, issues of validity. Neither Crampes and Langinier nor Meurer, considers a repeated interaction between a patent holder and a sequence of infringers. Therefore, issues concerning information revelation and reputation effects are not present.

The closest to modeling reputation effects in patent litigation is Choi (1998), who examines the implications of information transmission on the strategic behavior of an endogenous sequence of infringers. While there are some similarities in our models, there are also some important differences: In his model all patent holders are ex-ante identical. Therefore, it is impossible to account for the different reputational incentives of patent holders who differ in their private expectations of litigation awards. Choi also assumes that information is transmitted the moment litigation begins; whereas in our model, litigation is ongoing, such that it does not fully reveal private information to other potential infringers. The distinctions lead us to different conclusions about why patent holders might want to delay litigation. Whereas, Choi finds that patent holders will delay litigation because they are afraid of revealing weakness, we find that patent holders may delay litigation to avoid revealing strength.

The rest of this paper is presented in the following order: The second section presents the model preliminaries, and defines equilibrium strategies and beliefs. The third section presents the resulting equilibrium properties of the model. The fourth section discusses the properties of the model as they relate to reputation, potential policy, and signaling games in general. The fifth section states our conclusions. The sixth section is an appendix containing the proofs of all Lemmas and Propositions.

Figure 1: A timeline of the game.



2 Model

Consider a two period model where a patent holder faces off against an initial act of infringement committed at the beginning of period 1.[5] Furthermore, the patent holder foresees the potential of an additional act of infringement in period 2.[6] Assume patent holders have private information about their expected awards from any litigation. In the resulting signaling game, signals about the patent holder's private information will be transmitted by the outcome of period 1 to the potential infringer before they are called to make an entry decision in period 2. The timing of this game is summarized in figure 1.

Assume patent holders vary in their ex-ante expected awards from any ongoing litigation that has been successfully initiated, such that awards are correlated with private information θ for each type $\theta \in \Theta \subset R$.[7]

Furthermore, let $\phi_0(\cdot)$ denote the infringer's infringer's prior beliefs about the patent

[5] First period infringement is assumed for simplicity. Endogenous first period entry will not change the general intuition of our results.

[6] This model depicts what happens when the potential infringer is the same as the initial infringer (See the Microsoft example in our introduction). If instead we want to analyze what happens when the potential infringer is another firm, we can make minor modifications to our model that will not affect the intuition of our results.

[7] We allow for types $\theta < 0$ for generality. $\theta < 0$ might be representative of a patent holder with invalid property rights who might expect a countersuit.

holder's private information in period 1. Such that $\phi_0(\cdot)$ assigns each type θ a prior probability density, denoted as $\phi_0(\theta)$.

Let $\gamma > 0$ be an exogenous scalar that captures the relative size of the potential act of infringement to the initial act, such that

1. If litigation is initiated successfully against either act *separately*, the patent holder would expect award θ for the initial act of infringement, and $\theta\gamma$ for the potential act of infringement.

2. If litigation is initiated successfully against both acts *jointly*, the expected award becomes $(\theta + \theta\gamma)$.

The game begins when a patent holder faces an initial act of infringement in period 1. In order to successfully initiate litigation, the patent holder must expend effort denoted e_1. Based on the patent holder's effort, nature determines the the probability $p(e_1)$ that the patent holder successfully finds arguments sufficient to bring the case to court in period 1. We assume $e_1 \in R_+$, and $p(e_1)$ is twice continuously differentiable such that $p'(e_1) > 0$, $p''(e_1) < 0$, and $p(0) = 0$.[8] Therefore, based on his effort, the patent holder creates litigation in period 1 with probability $p(e_1)$, with complementary probability $(1 - p(e_1))$ no litigation exists at the beginning of period 2.

Any litigation initiated in period 1 is on-going at the beginning of period two. Therefore, the only signal observed by the infringer in period 2 is the history $h_1 \in \{L, \not{L}\}$, where $\{L\}$ denotes the **existence,** and $\{\not{L}\}$ denotes the **absence,** of ongoing litigation from period 1.[9]

Therefore, we denote the infringer's period 2 entry action as being conditional on observing h_1. let $\sigma_2(h_1) \in [0, 1]$ denote the conditional entry probability, as mixed strategies over

[8] It is important to keep in mind that failure to initiate litigation does not mean that a patent holder loses in court. In our model, failure to initiate litigation in period 1 is interpreted as a failure to find arguments sufficient to bring the case to court in period 1.

[9] We defend our assumption about the observable history as follows: litigation is usually a long process. Protracted legal battles can delay information transmission beyond the onset of a trial. For example, in the case of Polaroid V. Kodak, it took 14 years for the patent infringement suit to be resolved. Therefore, the infringer will likely have other opportunities to commit additional acts of infringement before they can observe the final award.

$\{Out, In\}$. If the infringer decides not to commit additional acts of infringement, they pay an exogenous licensing fee F to the patent holder for additional use.[10] On the other hand, if they commit additional acts of infringement, they avoid the licensing fee.

After the potential infringer's decision in period two, there are four possible histories facing a patent holder in period 2, stemming from the two possible histories of period 1 $\{L, \not{L}\}$, and the two possible actions of the infringer in period 2 $\{Out, In\}$ (See Figure 2). Therefore, the expected award for successfully initiated litigation in period 2 will be conditional on a patent holder's type (θ) and the set of observable histories $h_2 \in \{h_1, a_2(h_1)\}$. Here we summarize the four possible histories and provide the corresponding expected player profits.

Let $E^{PH}\left[\Pi_\theta^{PH} \mid h_2\right]$ denote a type θ patent holder's expected profits upon reaching history h_2. Furthermore, let $E^{IN}\left[\Pi^{IN} \mid h_2, \theta\right]$ denote the infringer's expected profits upon reaching history h_2, conditional on facing a patent holder of type θ.[11] The four potential histories facing the patent holder in period 2 are defined as follows:

History $h_2 = \{L, Out\}$: The infringer decides not to commit an additional act of infringement in the presence of ongoing litigation from period 1. The infringer pays an exogenous licensing fee (F) to the patent holder for additional use, and awaits the outcome of the ongoing litigation from round 1.

$$E^{PH}\left[\Pi_\theta^{PH} \mid L, Out\right] = \theta + F - e_1 \quad (1)$$

$$E^{IN}\left[\Pi^{IN} \mid L, Out, \theta\right] = -\theta - F \quad (2)$$

[10] The implications of an endogenous licensing fee are examined in the discussion section. We show that with common sense assumptions, allowing for the endogenous licensing will not change the intuition of our results.

[11] Note that at this point in the game, the infringer will not necessarily know what type of patent holder it faces. Thus, the infringer's equilibrium action will depend on its beliefs. Beliefs are detailed in section 2.3.

History $h_2 = \{L, In\}$: The infringer commits an additional act of infringement in the presence of ongoing litigation from period 1. The infringer avoids paying the licensing fee. However, the patent holder can then initiate litigation against the additional act of infringement separately, with certainty, and without any additional cost of effort.

$$E^{PH}\left[\Pi_{\theta}^{PH} \mid L, In\right] = \theta + \theta\gamma - e_1 \tag{3}$$

$$E^{IN}\left[\Pi^{IN} \mid L, In, \theta\right] = -\theta - \theta\gamma \tag{4}$$

History $h_2 = \{\not{L}, In\}$: the infringer commits an additional act of infringement in the absence of ongoing litigation from period 1. The infringer avoids paying the licensing fee. However, the patent holder can expend effort (e_2) to try and initiate litigation against both the initial and additional acts of infringement jointly.

$$E^{PH}\left[\Pi_{\theta}^{PH} \mid \not{L}, In\right] = (\theta\gamma + \theta)\, p\left(e_2^*\left(\theta \mid In\right)\right) - e_2^*(\theta \mid In) - e_1 \tag{5}$$

$$E^{IN}\left[\Pi^{IN} \mid \not{L}, In, \theta\right] = -\left(\theta\gamma + \theta\right) p\left(e_2^*\left(\theta \mid In\right)\right), \tag{6}$$

$$\text{Where } e_2^*\left(\theta \mid In\right) = \underset{e_2 \in R_+}{\operatorname{argmax}}\left\{\left(\theta\gamma + \theta\right) p\left(e_2\right) - e_2\right\}$$

History $h_2 = \{\not{L}, Out\}$: The infringer decides not to commit an additional act of infringement in the absence of litigation from period 1. The infringer pays a licensing fee for additional use of the patent, but the patent holder can still expend effort (e_2) to litigate the initial act of infringement from period one.

$$E^{PH}\left[\Pi_{\theta}^{PH} \mid \not{L}, Out\right] = F + \theta p\left(e_2^*(\theta \mid Out)\right) - e_2^*(\theta \mid Out) - e_1 \tag{7}$$

$$E^{IN}\left[\Pi^{IN} \mid \not{L}, Out, \theta\right] = -F - \theta p\left(e_2^*\left(\theta \mid Out\right)\right), \tag{8}$$

$$\text{Where } e_2^*\left(\theta \mid Out\right) = \underset{e_2 \in R_+}{\operatorname{argmax}}\left\{\theta p\left(e_2\right) - e_2\right\}$$

All payoffs are realized at the end of the second period. We now define a reduced form of a perfect Bayesian equilibrium of our model.

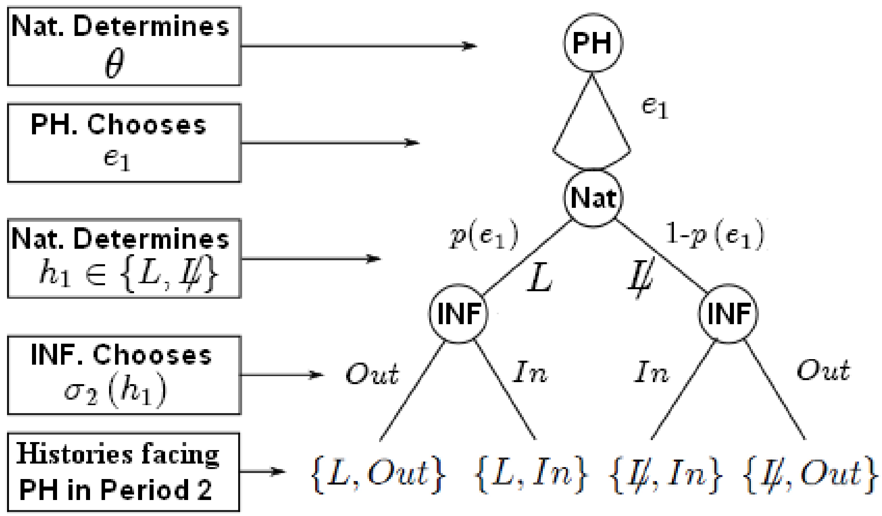


Figure 2: An overview of the game.

The reduced form of a **Perfect Bayesian Equilibrium** will consist of[12]

1. Each patent holder type's effort levels: $\{e_1^*(\theta)\}\ \forall\theta \in \Theta$

2. The infringer's entry probabilities: $\sigma_2^*(L), \sigma_2^*(\not{L})$.

3. The infringer's updated beliefs: $\{\phi(\cdot \mid L), \phi(\cdot \mid \not{L})\}$.

Such that:

- Given $\sigma_2^*(L)$ and $\sigma_2^*(\not{L})$, $e_1^*(\theta)$ maximizes the expected profits of each type θ.

- Given $\phi(\cdot \mid h_1)$, $\sigma_2^*(h_1)$ is profit maximizing for the infringer for all $h_1 \in \{L, \not{L}\}$.

- The infringer's updated beliefs $\phi(\cdot \mid h_1)$ are consistent with $\{e_1^*(\theta)\ \forall\ \theta \in \Theta\}$ for all $h_1 \in \{L, \not{L}\}$.[13]

[12] We omit $e_2^*(\theta \mid Out)$, $e_2^*(\theta \mid In)$ from the reduced form because they are unaffected by beliefs. See (7) and (5).

[13] Perfection is satisfied when there is at least one type of patent holder with positive litigation expectations because both possible signals will occur with positive probability in equilibrium. In this case, there will be no off equilibrium histories to generate off equilibrium beliefs. Furthermore, when no patent holder type has a positive expectation of litigation awards, we can assume that the infringer always enters in period 2 because no posterior beliefs over the support of initial beliefs could deter additional infringement.

2.1 Solving for Equilibrium

In the signaling game we model the infringer observes a noisy signal of the patent holder's award expectation based on their understanding of each patent holder type's first period optimization problem. Therefore, while an infringer does not observe the patent holder's first period effort, they can still update their beliefs in a manner consistent with the equilibrium effort of the patent holder. This consistency results in a useful property: that a change in equilibrium entry probability will always affect the equilibrium effort level of at least one patent holder type. Therefore any potential change in equilibrium infringer entry probability will change their expected profits. Therefore, the resulting PBE of the game will always be unique. Thus we prove that our findings about predatory behavior generated by the model are general, and not the result of a selective choice of the equilibria to analyze.

The general procedure for identifying equilibria is guess and verify, which we briefly summarize as follows:

1. Guess a set of infringer best responses

$$\{\hat{\sigma}_2^*(L), \hat{\sigma}_2^*(\not{L})\} \tag{9}$$

2. Given $\{\hat{\sigma}_2^*(L), \hat{\sigma}_2^*(\not{L})\}$, calculate each type patent holder's optimal effort.

$$\{\hat{e}_1^*(\theta)\,\forall\,\theta \in \Theta\} = \{e_1^*(\theta \mid \hat{\sigma}_2^*(L), \hat{\sigma}_2^*(\not{L}))\,\forall\,\theta \in \Theta\} \tag{10}$$

3. Given $\{\hat{e}_1^*(\theta)\,\forall\,\theta \in \Theta\}$, calculate the infringer's updated beliefs

$$\hat{\phi}(\theta \mid L) = \phi(\theta \mid L, \hat{e}_1^*(\theta)\,\forall\,\theta \in \Theta) \tag{11}$$

$$\hat{\phi}(\theta \mid \not{L}) = \phi(\theta \mid \not{L}, \hat{e}_1^*(\theta)\,\forall\,\theta \in \Theta) \tag{12}$$

4. Using updated beliefs, verify that best responses are indeed best responses

$$\hat{\sigma}_2^*(L) = \sigma_2^*(L \mid \{\hat{e}_1^*(\theta)\,\forall\,\theta \in \Theta\}) \tag{13}$$

$$\hat{\sigma}_2^*(\not{L}) = \sigma_2^*(\not{L} \mid \{\hat{e}_1^*(\theta)\,\forall\,\theta \in \Theta\}) \tag{14}$$

In the following we sections we will describe the relationships between optimal patent holder effort, the infringer's beliefs, and the infringer's best responses.

2.2 Updating Beliefs

Before making an entry decision in period 2, the infringer will update its beliefs about the distribution of patent holder types based on the observable outcome of period 1, its prior beliefs $\phi_0(\cdot)$, and the set of equilibrium effort levels for each patent holder type $\{e_1^*(\theta)\,\forall\,\theta\in\Theta\}$. Note that effort is not observable; however, within any equilibrium, we show below that each patent holder type has a unique optimal amount of effort. Therefore, $\{e_1^*(\theta)\,\forall\,\theta\in\Theta\}$ can be used to update beliefs in equilibrium, because the infringer will know the optimization problem faced by each patent holder type. Here we define the infringer's updated beliefs as the set of density functions $\{\phi(\cdot\mid h_1)\ \ \forall h_1\in\{L,\not{L}\}\}$.

Definition 1 *The infringer's updated beliefs, is the mapping*

$$\phi:\{\Theta,h_1\}\longrightarrow[0,1] \tag{15}$$

such that, upon observing h_1

$$\phi(\theta\mid L) = \frac{p(e_1^*(\theta))\,\phi_0(\theta)}{\int_{\theta\in\Theta} p(e_1^*(\theta))\,\phi_0(\theta)\,d\theta} \tag{16}$$

$$\phi(\theta\mid \not{L}) = \frac{(1-p(e_1^*(\theta)))\,\phi_0(\theta)}{\int_{\theta\in\Theta}(1-p(e_1^*(\theta)))\,\phi_0(\theta)\,d\theta} \tag{17}$$

where $p(e_1^*(\theta))$ *is the probability a type* θ *patent holder creates litigation in period 1 given its equilibrium first period effort* $e_1^*(\theta)$, *and* $\phi_0(\theta)$ *is the prior density of a type* θ *in nature.*[14]

2.3 The Infringer's Entry Decision

There are only two subgames facing the potential infringer stemming from the two observable outcomes in period one: the existence of litigation, or lack thereof, denoted respectively as the history $h_1\in\{L,\not{L}\}$. This outcome will allow the infringer to update their beliefs from

[14] Off-equilibrium beliefs only become an issue when there is a zero probability of any type of patent holder creating litigation in period 1. In these cases, no patent holder can benefit from creating litigation in period 1. This necessarily implies that the infringer's period 2 entry decision is unaffected by any outcome of period 1. Therefore, off equilibrium beliefs will never affect the behavior of the infringer in period 2.

$\phi_0(\cdot)$ to $\phi(\cdot \mid h_1)$. Therefore the potential infringer's expected payoffs from infringement become conditional on the outcome of round 1, and their entry decision.

To see if the potential infringer is better when choosing $\{In\}$, we must compare it to their reservation value from choosing $\{Out\}$. Note, that an infringer's expected profits from entry and their expected reservation value is conditional on the observed the first period outcome. Here we define their net expected payoffs for each observable first period history. Notice that the infringer's expected payoffs consist of its expected profits within the corresponding second period histories (see (2), (4), (6), and (8)), weighted by their updated beliefs about the patent holder type.

$$E^{IN}_{net}\left[\Pi^{IN} \mid h_1\right] = \int\limits_{\theta\in\Theta} \left[E^{IN}(\Pi^{IN} \mid h_1, In, \theta) - E^{IN}(\Pi^{IN} \mid h_1, Out, \theta)\right]\phi(\theta \mid h_1)d\theta \tag{18}$$

We now define an infringer best response for each observed history form period 1, as a function of the net expected payoffs from entry.

Definition 2 *The best response for the infringer, given an observed first period history* h_1, *is the mapping*

$$\sigma_2^* : \{L, \not{L}\} \longrightarrow [0,1] \tag{19}$$

such that

$$\begin{aligned} \sigma_2^*(h_1) &= 1, && \text{if } E^{IN}_{net}\left[\Pi^{IN} \mid h_1\right] > 0 \\ \sigma_2^*(h_1) &\in (0,1), && \text{if } E^{IN}_{net}\left[\Pi^{IN} \mid h_1\right] = 0 \\ \sigma_2^*(h_1) &= 0, && \text{if } E^{IN}_{net}\left[\Pi^{IN} \mid h_1\right] < 0 \end{aligned} \tag{20}$$

where $\sigma_2^*(h_1)$ *is their probability of committing the additional act of infringement conditional on observing the first period history* h_1.

2.4 The Patent Holder's Optimization Problem

The patent holder's optimization problem depends on the equilibrium best responses of the infringer to the first period outcome. Therefore, any set of best responses $\{\sigma_2^*(L), \sigma_2^*(\not{L})\}$ will lead to a unique choice of effort$(e_1^*(\theta \mid \sigma_2^*(L), \sigma_2^*(\not{L})))$ for each patent holder of type θ to maximize. Hence for each patent holder type $\theta \in \Theta$

$$e_1^*(\theta \mid \sigma_2^*(L), \sigma_2^*(\not{L})) = \underset{e_1 \in R_+}{\arg\max} \left\{ \begin{array}{l} p(e_1)\sigma_2^*(L)\left(E^{PH}\left[\Pi_\theta^{PH} \mid L, In\right]\right) + \\ \left[1-p(e_1)\right]\sigma_2^*(\not{L})\left(E^{PH}\left[\Pi_\theta^{PH} \mid \not{L}, In\right]\right) + \\ p(e_1)\left[1-\sigma_2^*(L)\right]\left(E^{PH}\left[\Pi_\theta^{PH} \mid L, Out\right]\right) + \\ \left[1-p(e_1)\right]\left[1-\sigma_2^*(\not{L})\right]\left(E^{PH}\left[\Pi_\theta^{PH} \mid \not{L}, Out\right]\right) \end{array} \right\} . \tag{21}$$

[15]

As we see from the patent holder's profit maximization problem above, the patent holder considers the likelihood of reaching any history given its choice of effort in period 1 and the set of equilibrium infringer best responses $\{\sigma_2^*(L), \sigma_2^*(\not{L})\}$. A patent holder must also consider its expected profits given that it reaches a particular history, given in (1) (3) (5) and (7).[16]

The patent holder's optimal first period effort is based on the following first order condition. Here we present this condition with respect to e_1.

$$p'(e_1)\psi(\theta) \leq 1, \tag{22}$$

where $\psi(\theta)$ is defined as

$$\psi(\theta) = \left[\begin{array}{l} E^{PH}\left[\Pi_\theta^{PH} \mid L, Out\right] - E^{PH}\left[\Pi_\theta^{PH} \mid \not{L}, Out\right] + \\ \sigma_2^*(L)\left[E^{PH}\left[\Pi_\theta^{PH} \mid L, In\right] - E^{PH}\left[\Pi_\theta^{PH} \mid L, Out\right]\right] + \\ \sigma_2^*(\not{L})\left[E^{PH}\left[\Pi_\theta^{PH} \mid \not{L}, Out\right] - E^{PH}\left[\Pi_\theta^{PH} \mid \not{L}, In\right]\right] \end{array} \right], \tag{23}$$

and the corresponding second order condition is

$$p''(e_1)\psi(\theta) < 0. \tag{24}$$

Notice that a given set of infringer best responses $\{\sigma_2^*(L), \sigma_2^*(\not{L})\}$ is sufficient to identify the optimal first period effort for each patent holder type. Notice that the first order condition holds with equality for a given set of best responses $\{\sigma_2^*(L), \sigma_2^*(\not{L})\}$ whenever

[16]Note that the patent holders expected profits, given that they reach a particular subgame, are ex-ante predetermined (as illustrated in descriptions of the period 2 subgames above). Therefore we do not restate these profit maximizing period two choices of effort here because they are static with respect to any potential equilibrium.

$\psi(\theta) > 0$, because $p(e_1)$ is concave and twice continuously differentiable, and thus the second order condition is also satisfied. Furthermore, for all types θ such that $\psi(\theta) \leq 0$, we know $e_1^*(\theta \mid \sigma_2^*(L), \sigma_2^*(\not{L})) = 0$ because the first order condition will never bind. Therefore, for a given set of best responses each patent holder type has only one level of effort that maximizes its expected profits. This property is essential for showing that any equilibrium is unique.

3 Results

3.1 Existence and Uniqueness of Equilibrium

In this section, we show that any parameterization of the model, with any distribution of types $\Theta \subset R$, and any twice continuously differentiable function $p(\cdot)$ such that $p'(e) > 0$, $p''(e) < 0$, $p(0) = 0$, has a unique Perfect Bayesian equilibrium.

First, we prove in Lemma 1 that the patent holder's choice of first period effort is a continuously differentiable function of the patent holder's type θ, and the infringer's conditional equilibrium entry probabilities: $\sigma_2^*(L)$ and $\sigma_2^*(\not{L})$. This property is reliant on allowing the patent holder's choice of e_1 to be continuous. This will allow us to use the appropriate derivatives in our proofs of the subsequent Lemmas and Propositions.

Lemma 1 *For all $\theta \in R$, $e_1^*(\theta \mid \sigma_2^*(L), \sigma_2^*(\not{L}))$ is continuously differentiable in $\sigma_2^*(L)$, $\sigma_2^*(\not{L})$, and θ.*

Next, Lemmas 2 and 3 show how a change in the infringer's equilibrium conditional entry probability affects the equilibrium relative first period effort of each patent holder type. These lemmas will help us prove lemmas 4 and 5, and assist us in our analysis patent holder effort in the discussion section.

Lemma 2 *As $\sigma_2^*(\not{L})$ increases, the change in first period patent holder effort is a weakly decreasing function of patent holder type θ, such that*

$$\frac{\partial^2 e_1^*(\theta \mid \sigma_2^*(L), \sigma_2^*(\not{L}))}{\partial \sigma_2^*(\not{L})\, \partial\theta} \leq 0, \ \forall\theta \tag{25}$$

Lemma 3 *As $\sigma_2^*(L)$ increases, the change in equilibrium first period effort is a weakly increasing function of type θ.*

$$\frac{\partial^2 e_1^*(\theta \mid \sigma_2^*(L), \sigma_2^*(\not{L}))}{\partial \sigma_2^*(L)\, \partial \theta} \geq 0, \ \forall \theta \tag{26}$$

Next, in Lemmas 4 and 5 we show how a change in the infringer's equilibrium conditional entry probability (either $\sigma_2^*(L)$ or $\sigma_2^*(\not{L})$) affects its expected profits from entry after any first period history (h_1), via its affect on equilibrium patent holder effort shown in Lemmas 2 and 3.

Lemma 4 *The infringer's expected profits from entry, after observing the existence of litigation in period 1 $(h_1 = L)$, is*

(4.1) *monotonically increasing in $\sigma_2^*(\not{L})$*

(4.2) *monotonically decreasing in $\sigma_2^*(L)$*

Lemma 5 *The infringer's expected profits from entry after observing the absence of litigation $(h_1 = \not{L})$ in period 1, is*

(5.1) *monotonically decreasing in $\sigma_2^*(\not{L})$*

(5.2) *monotonically increasing in $\sigma_2^*(L)$.*

Next, Lemma 6 will show how the probability of the infringer committing additional acts of infringement must be greater under the absence of litigation than it is under the existence of litigation. This assures us that in all equilibria the existence of litigation will cause more deterrence than the absence of litigation.

Lemma 6 *In any Bayesian equilibrium $\sigma_2^*(L) > 0$ only if $\sigma_2^*(\not{L}) = 1$.*

We now present Proposition 1, which summarizes the existence and uniqueness properties of equilibria in the model. It states that for any setting of the model there will exist one, and only one equilibrium.

Proposition 1 *For any setting of the parameters and any continuous function $p(\cdot)$ under our assumptions, and for any possible prior beliefs $\phi_0(\cdot)$ and any type space $\Theta \subset R$, there exists a unique perfect Bayesian equilibrium.*

The existence property of the model is proven by construction in the appendix. The proof of existence is outlined as follows: We begin by showing the procedure for checking for equilibrium in pure strategy infringer best responses using the method outlined in (14)-(19).

$$\{\sigma_2^*(L), \sigma_2^*(\not{L})\} \in \{\{Out, Out\}, \{Out, In\}, \{In, In\}\},$$

When there is no equilibrium in pure strategies, we use the following Lemmas, to show that there must be an equilibrium in mixed entry strategies. In other words, if

$$\{\sigma_2^*(L), \sigma_2^*(\not{L})\} \notin \{\{Out, Out\}, \{Out, In\}, \{In, In\}\} \tag{27}$$

then

$$\{\sigma_2^*(L), \sigma_2^*(\not{L})\} \in \{Out, \sigma_2^*\} \text{ or } \{\sigma_2^*, In\} \tag{28}$$

where $\sigma_2^* \in (0,1)$.

The uniqueness property of equilibrium in our model is derived from Lemmas 4 and 5; because, even when the infringer is indifferent to entry under a certain history, and therefore playing a equilibrium mixed entry strategy $\sigma_2^*(h_1)$, deviating and playing $\tilde{\sigma}_2^*(h_1) \neq \sigma_2^*(h_1)$ will disrupt the equilibria, by changing the equilibrium first period patent holder efforts, which will in turn affect the infringer's expected profits from entry under any history.

For example, suppose infringer responses $\{\sigma_2^*(L), \sigma_2^*(\not{L})\} = \left\{Out, \frac{1}{2}\right\}$ are in equilibrium. From $\sigma_2^*(\not{L}) = \frac{1}{2}$, we know his expected profits from entry given $\sigma_2^*(\not{L}) = \frac{1}{2}$, after observing $\{\not{L}\}$ are zero. Therefore, by lemma 5.1, if the infringer tries to enter with lower probability $\left(\hat{\sigma}_2^*(\not{L}) < \frac{1}{2}\right)$ upon observing $\{\not{L}\}$, his expected profit from entry upon observing $\not{L}$ will become positive, thus forcing $\hat{\sigma}_2^*(\not{L}) = 1$, contradicting our supposition that $\left(\hat{\sigma}_2^*(\not{L}) < \frac{1}{2}\right)$. Furthermore, if the infringer tries enter with higher probability $\left(\hat{\sigma}_2^*(\not{L}) > \frac{1}{2}\right)$ then by lemma 5.1, his expected profits from entry upon observing $\not{L}$ will become negative, thus forcing $\hat{\sigma}_2^*(\not{L}) = 0$, thereby contradicting our supposition that $\hat{\sigma}_2^*(\not{L}) > \frac{1}{2}$.

By Lemma 6 we know that the only remaining potential alternative equilibria can lie in $\{\{\hat{\sigma}_2^*(L), 1\} \mid \hat{\sigma}_2^*(L) > 0\}$. However we have already shown that $\{\hat{\sigma}_2^*(L), \hat{\sigma}_2^*(\not{L})\} = \{0,1\}$ will cause the infringer to expect negative profits from entry upon observing $\{\not{L}\}$. Therefore by lemma 6 we know that the infringer's expected profits from entry upon observing $\{L\}$ must also be weakly negative when $\{\hat{\sigma}_2^*(L), \hat{\sigma}_2^*(\not{L})\} = \{0,1\}$. Furthermore, by lemma 4.2

the infringer's expected profits from entry, conditional on observing $\{L\}$ are continuously decreasing in $\hat{\sigma}_2^*(L)$. Therefore there can be no alternative equilibrium such that $\hat{\sigma}_2^*(L) > 0$. Thus we have shown that if $\{\sigma_2^*(L), \sigma_2^*(\not{L})\} = \left\{Out, \frac{1}{2}\right\}$ is an equilibrium, there can be no other equilibrium $\{\hat{\sigma}_2^*(L), \hat{\sigma}_2^*(\not{L})\} \neq \{\sigma_2^*(L), \sigma_2^*(\not{L})\}$.

3.2 Infringer Best Response Properties

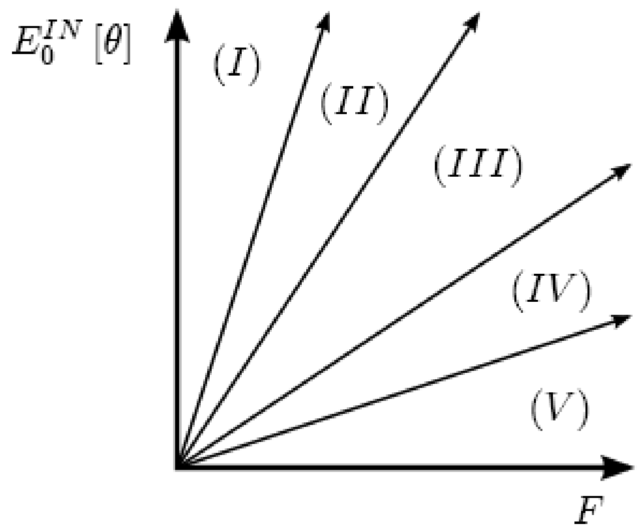
In any equilibrium, the existence of litigation in period 1 will weakly decrease the likelihood of infringement in period 2. This property exists because litigation is positively correlated with the patent holder's private expectation of the likely award in period 2, and thus negatively correlated with the infringer's expected profits from additional infringement. This notion is formalized in Corollary 1 which stems directly from Lemma 6.

Corollary 1 *In any Bayesian equilibrium* $\sigma_2^*(\not{L}) \geq \sigma_2^*(L)$.

From Corollary 1, we see that litigation must always be a signal that causes deterrence ($\sigma_2^*(L) < \sigma_2^*(\not{L})$). Imagine if we assumed instead that $\sigma_2^*(L) > \sigma_2^*(\not{L})$. In this case, litigation would be more likely to encourage entry than the absence of litigation, implying that patent holders optimal first period effort would all be monotonically increasing in their type. As a result, this would imply necessarily that $\int [\phi(\theta \mid L)\theta\gamma]\, d\theta > \int [\phi(\theta \mid \not{L})\theta\gamma]\, d\theta$. This in turn would cause the infringer's expected profit from entry under existing litigation (24) to be less than its expected profits under the absence of litigation (23), which contradicts the hypothetical $\sigma_2^*(L) > \sigma_2^*(\not{L})$.

Here we classify equilibria in terms of the entry behavior of potential infringers. This classification system will aid us in the discussion of our results.

Figure 3: The the resulting class of equilibrium for an infringer's ex-ante expectation of awards $\left(E_0^{IN}[\theta]\right)$ and exogenous licensing fee (F).



Class (I): Infringer always chooses $\{Out\}$.

Class (II): Infringer chooses $\{Out\}$ when observing $\{L\}$ and mixes over $\{In, Out\}$ when observing $\{\not{L}\}$.

Class (III): Infringer chooses $\{Out\}$ when observing $\{L\}$ and chooses $\{In\}$ when observing $\{\not{L}\}$.

Class(IV): Infringer mixes over $\{In, Out\}$ when observing $\{L\}$, and chooses $\{In\}$ when observing $\{\not{L}\}$.

Class (V): Infringer always chooses $\{In\}$.

3.3 The Existence of Reputational Effects

In this section, we determine the regions of the parameter space where we find reputation effects. Furthermore, we show that these effects exist whenever the potential infringer's entry behavior is affected by the first period outcome. To this end we define what we refer as a non-trivial equilibrium. This will allow us to focus on equilibria where reputations are a factor. After all, any equilibrium where the infringer's period 2 entry action is unaffected by the first period history is trivial in terms of any reputation effects, because the infringer's entry behavior is unaffected by any signal it receives.

Definition 3 *We define a Perfect Bayesian equilibrium in our model as* ***non-trivial*** *if* $\{\sigma_2^*(L), \sigma_2^*(\not{L})\} \notin \{\{Out, Out\}, \{In, In\}\}$.

Here we state the following proposition to show that non-trivial equilibria exist within a non-empty region of the parameter space.

Proposition 2 *There exists a non-empty region of the parameter space, prior beliefs* $\phi_0(\theta)$, *and type spaces* $\Theta \subset R$, *such that the equilibrium is non-trivial.*

From the proof of Proposition 2 (in the Appendix), we see that the equilibrium will depend on the size of ex-ante expected awards $E_0^{IN}[\theta] = \int_{\theta \in \Theta} \theta \phi_0(\theta) \, d\theta$ relative to the size exogenous licensing fee F. Consider Figure 3. Here we see that the existence of non trivial equilibria (Regions II,III,IV) relies on the ex-ante expected award being neither too large in comparison to the licensing fee (Region I), nor too small (Region V).

3.4 The Properties of Optimal Effort

In this section we uncover the relationship between the incentives for reputational effects and a patent holder's first period effort. To this end, we use Propositions 4 and 5 to show the thresholds in the space of private information where one effect (deterrence or luring) begins to dominate the other.

Proposition 3 *In any non-trivial equilibrium, there exists* $\overline{\theta} \in R$ *such that first period effort is*

(3.1) *increasing in* θ, *if and only if* $\theta < \overline{\theta}$

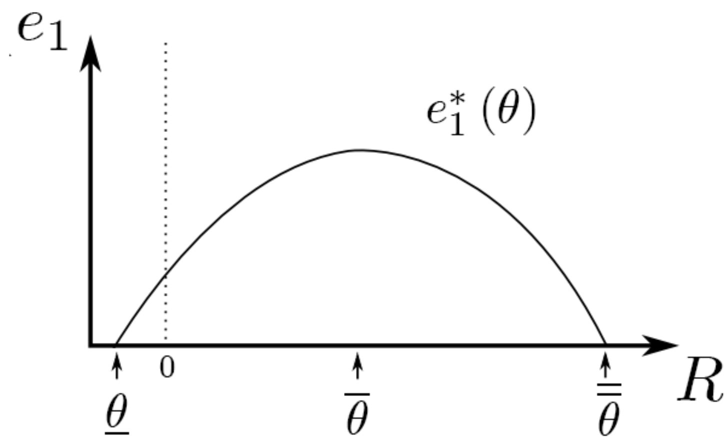
(3.2) *decreasing in* θ, *if and only if* $\theta > \overline{\theta}$.

Proposition 4 *In any non-trivial equilibrium, there exists* $\overline{\overline{\theta}} \in R$ *and* $\underline{\theta} \in R$ *such that*

(4.1) $e_1^*(\theta) = 0$, *if* $\theta < \underline{\theta}$

(4.2) $e_1^*(\theta) = 0$, *if* $\theta > \overline{\overline{\theta}}$.

Figure 4: Optimal patent holder effort in any non-trivial equilibria.



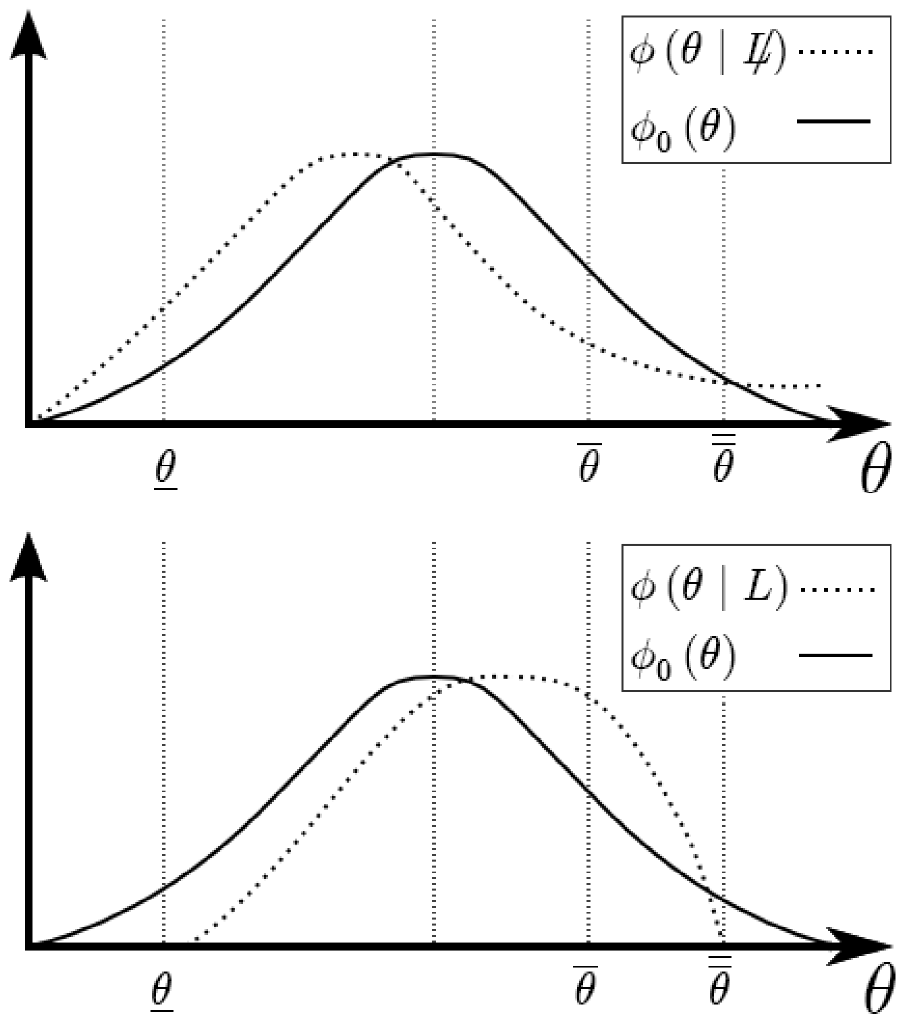
Propositions 3 and 4 give us thresholds in the patent holder type space. These thresholds ($\underline{\theta}, \overline{\theta}$, and $\overline{\overline{\theta}}$) are used in Figure 4 to compare each patent holder type's first period effort. We see here that first period effort is non-monotonic in type. This property occurs because, for all types θ greater than $\overline{\theta}$, the benefits of deterrence are diminishing, relative to the benefits of luring, as we increase θ.

Note that for all patent holder types $\theta \in \left[\overline{\theta}, \overline{\overline{\theta}}\right]$, the patent holder is still choosing positive effort levels in period 1. Therefore, if a patent holder of this type succeeds in initiating litigation in period 1, they will still go to trial. Furthermore, all patent holder types $\theta \geq \overline{\overline{\theta}}$ are expending zero effort in period 1, even though they have the highest award expectations, because they do not want to deter entry under in period 2 under any circumstance. This behavior may be characteristic of what Herald (2008) refers to as a Patent Troll: A patent holder that relies on litigation to extract revenue from potential licensors, who obscures their own property rights to encourage infringers.[17]

Corollary 2 *The probability that litigation occurs in period 1, is weakly increasing in* θ*, if and only if* $\theta < \overline{\theta}$.

[17] An endeavor to find observable statistics that identify potential patent trolls post litigation merits consideration.

Figure 5: An illustration of the impact of the observed first period history on beliefs.



Corollary 2 is a direct result of Proposition 2 and corollary 1. When a patent holder is benefiting from the deterrence effect, increasing its expected award makes it more likely that he will create litigation in period 1. On the other hand, when a patent holder is benefitting from the luring effect, increasing its expected award makes it less likely that he will create litigation in period 1. This non-monotonic property is illustrated by the optimal effort depicted in Figure 4. Note that some patent holders are so interested in luring the potential infringer, that the probability that they create first period litigation is zero (types $\theta > \bar{\bar{\theta}}$).

Consider figure 5. This figure shows the change in beliefs, for each first period history, with respect to the thresholds defined in Propositions 3 and 4. We see in Figure 5 that equilibria exist such that the absence of litigation from period 1 causes the infringer to enter, even when it assigns a higher probability density to some patent holder types with the highest award expectations.[18] At the same time, the existence of litigation will deter entry, even though the probability of patent holder having the highest award expectation goes to zero, because all patent holder types greater than $\bar{\bar{\theta}}$ will never create litigation in period 1.

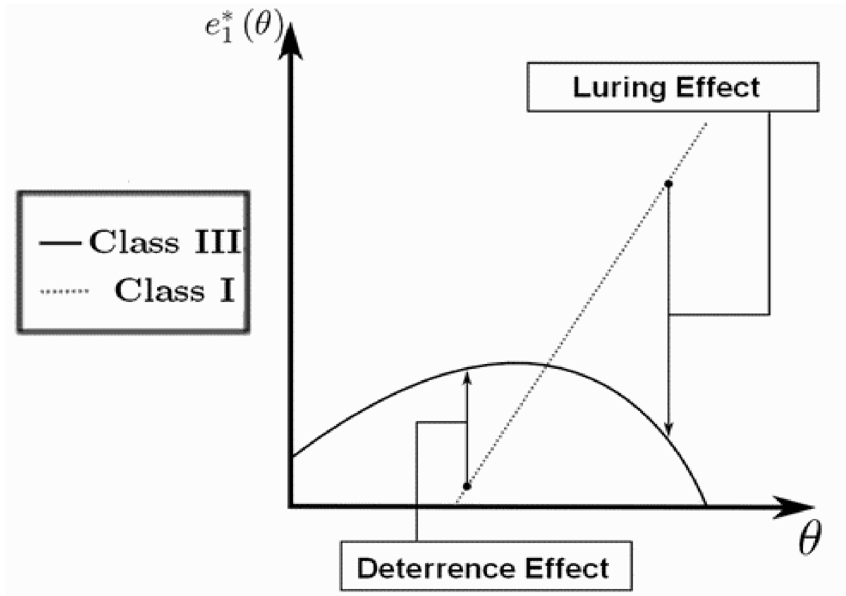
Corollary 3 *The magnitude of reputation's effect on patent holder effort is*
(C3.1) *increasing as* $\sigma_2^*(\not{L}) \to 1$
(C3.2) *decreasing as* $\sigma_2^*(L) \to 1$

In any trivial equilibrium, the infringer's entry action is unconditional; therefore, patent holders can not benefit from either deterrence or luring effects, because the infringer's action is unaffected by the signal they observe. Thus, we can analyze how the conditionality of the infringer's entry decision affects the patent holder's incentives to deceive, by observing the relative changes in patent holder effort as we move from $\{\sigma_2^*(L), \sigma_2^*(\not{L})\} = \{Out, In\}$ (the dotted line in Figure 6) to $\{\sigma_2^*(L), \sigma_2^*(\not{L})\} = \{Out, Out\}$ (the solid line in Figure 6). The relative changes in patent holder effort are a result of Lemma 3, which shows the change in patent holder effort across types, as we increase $\sigma_2^*(\not{L})$.

As we see in Figure 6, when moving from a conditional entry equilibrium (Class III) to an unconditional entry equilibrium (Class I), the optimal effort in period 1 becomes

[18] This effect on beliefs is analogous to a poker game, where a player passes on the opportunity to raise the pot. Opposing players might assign a higher probability to them having a weak hand, and at the same time assign higher probability to having a really strong hand.

Figure 6: Reputation's effect on patent holder effort.



steadily increasing in type, because no type wants to expend more or less effort for the sake of any reputation effect. Furthermore, we see for all types greater than $\overline{\theta}$, patent holders are spending less effort when infringer's entry decision is conditional, than if it were unconditional. These types are doing so because the benefits from luring are outweighing the benefits from deterrence. At the same time, we see that below $\overline{\theta}$ in the type space, patent holders are spending more effort. These types are doing so because the benefits from deterrence are outweighing the benefits from luring.

4 Discussion

4.1 Implications of Award Distributions

The existence non-trivial equilibria depends on both the expected award from litigation $E_0^{IN}[\theta] = \int_{\theta\in\Theta} \theta\phi_0(\theta)\, d\theta$ and the variance of the expected award. Distributions, with relatively low means ($E_0^{IN}[\theta]$) and long and thin upper tails create the ideal setting for predatory behavior because most patent holder types will try and create deterrence. In these settings, the absence of litigation in period 1 will be very enticing to a potential infringer. Thus, predatory patent holders will be able to use the luring effect to good advantage, by delaying litigation. On the other hand, consider a distribution with two symmetrically distributed types: a patent holder type with extremely large award expectations and those who expect zero awards from litigation. In this setting, patent holders with extremely large award expectations are effectively unable to use the luring effect, because the there is no one else for the zero expectation patent holders to pool with (to create deterrence). For this reason, when there are only two types, we know that the first period effort of the low expectation type must be less than the first period effort of the high expectation type. This type of setting will usually result in equilibria with minimal opportunities for deterrence and luring (either Class II or Class IV depending on the prior density of the two types).

We have shown that for any nontrivial equilibrium that there exists a type $\overline{\theta} \in R$ such that for all $\theta > \overline{\theta}$, $\frac{\partial e_1^*(\theta)}{\partial \theta} < 0$. However to show that $\overline{\theta} \in \Theta$, requires a specific assumptions about the distribution. For example, under certain parameter settings it is possible that the patent holder with the largest award expectation is threatening to a potential infringer, but doesn't want to encourage entry. In these cases, we see reputational behavior analogous to the findings in Kreps and Wilson (1981), where every patent holder has an incentive to use deterrence.

4.2 Policy

A policy maker might wish to control the impact of reputation in patent litigation. Furthermore, we have shown how these incentives are linked to the distributions of awards. Therefore, since policy makers can affect the distribution through the average size of potential awards and the requirements for recovering those awards, we conclude they have some control over reputational behavior.

For example, allowing for awards in excess of 3 times damages in special circumstances might stretch out the upper tail of an existing distribution of expected awards. From Proposition 3 we know that this will weakly increase the likelihood of predatory behavior (Luring). On the other hand, relaxing the requirements for receiving modest awards may shift the bottom part of the distribution towards the middle. From corollary 3 we see that this will weakly decrease the amount of anti-competitive behavior (Using deterrence when perfect information about the patent holder's property rights would lead to entry).

In general, vagueness about a patent holder's property rights will broaden the distribution of possible expected awards. In this sense, the reputational distortions present in our model may represent a cost associated with a lack of clarity in patent law. At this point we do not conjecture about the actual costs to society of any reputational behavior. These questions lay beyond the scope of our model in its current form. However, with the proper additions to our model we believe we can specifically address policy issues. This may present an interesting avenue for research in both theoretical and empirical analysis.

4.3 Endogenous Licensing Fee

To this point we have assumed that the licensing fee is exogenous for simplicity. If we were to allow the licensing fee to be endogenous, we would expect it to be a function of the observed history and the patent holder's private information.

We show that if we apply the following common sense restrictions on the endogenous nature of the licensing fee, we do not change the general intuition of our original model.

Assume the licensing fee $F(\theta, h_1)$ is a twice continuously differentiable function of θ and h_1 such that:

$$\frac{\partial F(\theta, h_1)}{\partial \theta} > 0, \tag{29}$$

$$F(\theta, L) > F(\theta, \not{L}) \tag{30}$$

$$F(\theta, L) > \int_{\theta \in \Theta} (\theta\gamma)\, \phi_0(\theta)\, d\theta \tag{31}$$

$$F(\theta, \not{L}) < \int_{\theta \in \Theta} p\left(e_2^*(\theta, In)\right)(\theta\gamma)\, \phi_0(\theta)\, d\theta \tag{32}$$

The intuition of these restrictions is as follows.(29) we require that any licensing fee secured by the PH is increasing in their expected award. (30) The licensing fee gained under the threat of current litigation must be greater than the licensing fee obtained without ongoing litigation. (31) The licensing fee agreed upon in the presence of litigation is greater than the expected litigation award for the additional act of infringement given initial beliefs. (32) The licensing fee agreed upon in the absence of litigation is less than the expected litigation award for the additional act of infringement given initial beliefs.

4.4 Symbiosis Between Types

In our model we see an interesting relationship between patent holder types on the relative extremum of the spectrum of award expectations. In non-trivial equilibrium, patent holders types with exceptionally large award expectations, who are interested in luring the infringer to commit an additional act of infringement, benefit from the existence of patent holder types with small award expectations because it increases the likelihood of entry in the second period. At the same time, patent holders with relatively small award expectations, who are interested in deterring second period entry, benefit from the existence of patent holder types with large award expectations because it decreases the likelihood of entry in the second period. The mutually beneficial relationship we see in this setting is what we might expect in situations where the predator can also become the prey.

4.5 Repeated Signaling Games

In most signaling games with reputation, the receiver's payoffs resulting from a signal are independent of the signal sender's type. This assumption prevents the realization of a receiver's payoff from revealing the sender's type at the end of a stage game. In our model the payoff resulting from a particular signal is dependent on type. However, the signals in our model will not reveal type at the end of a stage game, because the assumed delay between the signal reception and the realization of the receiver's resulting payoffs.

The uniqueness property of any equilibrium in our model stems from the inter-temporal circularity between equilibrium beliefs and prior equilibrium actions. The signal receiver's equilibrium strategy in the second round affects the signal sender's equilibrium strategy in the first round. Thus the signal receiver's equilibrium action can also affect its resulting posterior beliefs, and therefore affect its expected profits from any action after receiving a certain signal. Therefore, the indifference over actions for the signal receiver is only generated by a unique mixed entry strategy.[19] In this sense, the circularity removes a degree of freedom from the construction of a potential equilibrium. As a result, one and only one perfect Bayesian equilibrium exists for each setting.

[19] See the example following the statement of Proposition 1 for the intuition behind this claim.

5 Conclusions

We have developed a two period model where litigation timing affects the endogenous entry behavior of potential infringers. Our model adds to the existing literature by considering the timing of litigation in the enforcement of patents and addressing the delay of information revelation inherent in the long litigation process. Furthermore, we have uncovered theoretical motives that can explain why patent holders with really high expectations of litigation awards might strategically delay litigation. This finding could potentially be used by firms found guilty of infringement to argue for a reduction in awards whenever there is a noticeable delay between an initial act of infringement and the beginning of litigation. Also, we provide a mechanism for policy makers to affect reputational behavior.

With regards to signaling games in general, our model presents an environment where there are two opposing reputational effects. Patent holders with weak property rights might want to appear strong. At the same time, patent holders with strong property rights might want to appear weak. Similar phenomenon may exist in other signaling games that have the potential for conflicting reputational effects. In this paper the high types actually benefit from their type being underestimated. One could imagine many other situations like this. For example, a ratchet effect model where perhaps intermediate types want to show off to keep their jobs but high types want to avoid showing off initially to avoid a ratcheting up of the employer's demands.

6 Appendix

Lemma 1. For all $\theta \in R$, $e_1^*(\theta \mid \sigma_2^*(L), \sigma_2^*(\not{L}))$ is continuously differentiable in $\sigma_2^*(L)$, $\sigma_2^*(\not{L})$, and θ.

Proof: Within each history of period 2, each patent holder type's expected profits are fixed as indicated by (1) (3) (5) (7).

$$\begin{aligned} E^{PH}\left[\Pi_\theta^{PH} \mid L, Out\right] &= \theta + F - e_1 \\ E^{PH}\left[\Pi_\theta^{PH} \mid L, In\right] &= \theta + \theta\gamma - e_1 \\ E^{PH}\left[\Pi_\theta^{PH} \mid \not{L}, In\right] &= \theta(\gamma+1)\, p\left(e_2^*(\theta \mid In)\right) - e_2^*(\theta \mid In) - e_1 \\ E^{PH}\left[\Pi_\theta^{PH} \mid \not{L}, Out\right] &= F + \theta p\left(e_2^*(\theta \mid Out)\right) - e_2^*(\theta \mid Out) - e_1 \\ \text{where } e_2^*(\theta \mid In) &= \underset{e_2 \in R_+}{\operatorname{argmax}} \left\{\theta(\gamma+1)\, p(e_2) - e_2\right\} \\ \text{where } e_2^*(\theta \mid Out) &= \underset{e_2 \in R_+}{\operatorname{argmax}} \left\{\theta p(e_2) - e_2\right\} \end{aligned}$$

Therefore, the first order condition of a patent holder's profit maximization problem is given in (21) by,

$$p'(e_1)\left[\begin{array}{l} E^{PH}\left[\Pi_\theta^{PH} \mid L, Out\right] - E^{PH}\left[\Pi_\theta^{PH} \mid \not{L}, Out\right] + \\ \sigma_2^*(L)\left(E^{PH}\left[\Pi_\theta^{PH} \mid L, In\right] - E^{PH}\left[\Pi_\theta^{PH} \mid L, Out\right]\right) + \\ \sigma_2^*(\not{L})\left(E^{PH}\left[\Pi_\theta^{PH} \mid \not{L}, Out\right] - E^{PH}\left[\Pi_\theta^{PH} \mid \not{L}, In\right]\right) \end{array}\right] \leq 1 \tag{33}$$

by (22), this condition holds with equality for any positive amount of optimal effort. Therefore, since $p'(e_1)$ is continuously differentiable and $p''(e_1) < 0$ for all $e_1 \in R_+$, the inverse function theorem proves $p'(e_1)$ has an inverse $g(\cdot) = [p'(e_1)]^{-1}$ which is also continuously differentiable over the interval $p'(R_+)$ such that for all z in the domain of $g(\cdot)$, $g'(z) = \frac{1}{p''(g(z))}$.
Recall,

$$\psi(\theta) = \left[\begin{array}{l} E^{PH}\left[\Pi_\theta^{PH} \mid L, Out\right] - E^{PH}\left[\Pi_\theta^{PH} \mid \not{L}, Out\right] + \\ \sigma_2^*(L)\left(E^{PH}\left[\Pi_\theta^{PH} \mid L, In\right] - E^{PH}\left[\Pi_\theta^{PH} \mid L, Out\right]\right) + \\ \sigma_2^*(\not{L})\left(E^{PH}\left[\Pi_\theta^{PH} \mid \not{L}, Out\right] - E^{PH}\left[\Pi_\theta^{PH} \mid \not{L}, In\right]\right) \end{array}\right] \tag{34}$$

such that the first order condition is written

$$p'(e_1)\psi(\theta) = 1 \tag{35}$$

Therefore the inverse $g(\cdot)$ has domain $\left(\frac{1}{\psi(\theta)}\right)$

$$g\left(p'(e_1)\right) = g\left(\frac{1}{\psi(\theta)}\right) \tag{36}$$

Thus we have

$$e_1^*\left(\theta \mid \sigma_2^*(L), \sigma_2^*(\not{L})\right) = g\left(\frac{1}{\psi(\theta)}\right) \tag{37}$$

By the inverse function theorem, and our assumption that $p\prime\prime(e_1) < 0$, we have

$$g'\left(\frac{1}{\psi(\theta)}\right) = \frac{1}{p\prime\prime\left(g\left(\frac{1}{\psi(\theta)}\right)\right)} < 0 \tag{38}$$

$$\frac{\partial g\left(\frac{1}{\psi(\theta)}\right)}{\partial \psi(\theta)} = g'\left(\frac{1}{\psi(\theta)}\right)\frac{\partial\left(\frac{1}{\psi(\theta)}\right)}{\partial \psi(\theta)} \tag{39}$$

$$= g'\left(\frac{1}{\psi(\theta)}\right)\frac{\partial}{\partial \psi(\theta)}\left[\psi(\theta)\right]^{-1} \tag{40}$$

Therefore, since

$$g'\left(\frac{1}{\psi(\theta)}\right) = \frac{1}{p\prime\prime\left(g\left(\frac{1}{\psi(\theta)}\right)\right)} < 0 \tag{41}$$

$$\frac{\partial g\left(\frac{1}{\psi(\theta)}\right)}{\partial \psi(\theta)} = -g'\left(\frac{1}{\psi(\theta)}\right)\left[\psi(\theta)\right]^{-2} \tag{42}$$

$$> 0 \tag{43}$$

Therefore, since $p(e_1)$ is concave, $g\left(\frac{1}{\psi(\theta)}\right)$ is an increasing function of $\psi(\theta) > 0$.
Therefore, the sign of the derivative of optimal effort $g\left(\frac{1}{\psi(\theta)}\right)$ with respect to variable $x \in \{\theta, \sigma_2^*(L), \sigma_2^*(\not{L})\}$ is such that

$$\frac{\partial g\left(\frac{1}{\psi(\theta)}\right)}{\partial x} > 0 \text{ if } \frac{\partial}{\partial x}\left[\psi(\theta)\right] > 0 \tag{44}$$

$$\frac{\partial g\left(\frac{1}{\psi(\theta)}\right)}{\partial x} < 0 \text{ if } \frac{\partial}{\partial x}\left[\psi(\theta)\right] < 0 \tag{45}$$

Furthermore we find,

$$\frac{\partial \psi(\theta)}{\partial \sigma_2^*(L)} = [\theta\gamma - F] \tag{46}$$

$$\frac{\partial \psi(\theta)}{\partial \sigma_2^*(\not L)} = \left[F + \theta p\left(e_2^*(\theta \mid Out)\right) - e_2^*(\theta \mid Out) - \theta(\gamma+1)\, p\left(e_2^*(\theta \mid In)\right) + e_2^*(\theta \mid In)\right] \quad (47)$$

Using the envelope theorem, because of maximizers $e_2^*(\theta \mid In)$ and $e_2^*(\theta \mid Out)$, we find

$$\frac{\partial \psi(\theta)}{\partial \theta} = \left[1 + \sigma_2^*(L)\,\gamma + \sigma_2^*(\not L)\left(e_2^*(\theta \mid Out) - (\gamma+1)\, p\left(e_2^*(\theta \mid In)\right)\right) - e_2^*(\theta \mid Out)\right] \quad (48)$$

Lemma 2: As $\sigma_2^*(\not L)$ increases, the change in first period patent holder effort is a weakly decreasing function of patent holder type θ, such that

$$\frac{\partial^2 e_1^*\left(\theta \mid \sigma_2^*(L), \sigma_2^*(\not L)\right)}{\partial \sigma_2^*(\not L)\, \partial \theta} \leq 0, \forall \theta \quad (49)$$

Proof: From Lemma 1, we know that $\frac{\partial e_1^*\left(\theta \mid \sigma_2^*(L), \sigma_2^*(\not L)\right)}{\partial \sigma_2^*(\not L)} < 0$ if and only if $\frac{\partial \psi(\theta)}{\partial \sigma_2^*(\not L)} < 0$. Furthermore, from Lemma 1

$$\frac{\partial \psi(\theta)}{\partial \sigma_2^*(\not L)} = \quad (50)$$

$$\left[F + \theta p\left(e_2^*(\theta \mid Out)\right) - e_2^*(\theta \mid Out) - \theta(\gamma+1)\, p\left(e_2^*(\theta \mid In)\right) + e_2^*(\theta \mid In)\right] \quad (51)$$

Therefore, using the envelope theorem because of second period optimal efforts for subgames 3 and 4 ($e_2^*(\theta \mid Out)$ and $e_2^*(\theta \mid In)$) given in (5) and (7), we conclude that

$$\frac{\partial \psi(\theta)}{\partial \sigma_2^*(\not L)\, \partial \theta} = \quad (52)$$

$$\frac{\partial\left[F + \theta p\left(e_2^*(\theta \mid Out)\right) - e_2^*(\theta \mid Out) - \theta(\gamma+1)\, p\left(e_2^*(\theta \mid In)\right) + e_2^*(\theta \mid In)\right]}{\partial \theta} \quad (53)$$

$$= p\left(e_2^*(\theta \mid Out)\right) - (\gamma+1)\, p\left(e_2^*(\theta \mid In)\right)) \quad (54)$$

$$< 0, \forall \theta > 0 \quad (55)$$

$$= 0, \forall \theta \leq 0 \quad (56)$$

Therefore,

$$\frac{\partial^2 \psi(\theta)}{\partial \sigma_2^*(\not L)\, \partial \theta} < 0, \forall \theta > 0 \quad (57)$$

$$\frac{\partial^2 \psi(\theta)}{\partial \sigma_2^*(\not L)\, \partial \theta} = 0, \forall \theta \leq 0 \quad (58)$$

This proves

$$\frac{\partial^2 e_1^*\left(\theta \mid \sigma_2^*(L), \sigma_2^*(\not L)\right)}{\partial \sigma_2^*(\not L)\, \partial \theta} \leq 0, \forall \theta > 0 \quad (59)$$

Lemma 3. As $\sigma_2^*(L)$ increases, the change in equilibrium first period effort is a weakly increasing function of type θ.

$$\frac{\partial^2 e_1^*(\theta \mid \sigma_2^*(L), \sigma_2^*(\not{L}))}{\partial \sigma_2^*(L)\,\partial\theta} \geq 0,\ \forall\theta \tag{60}$$

Proof: From Lemma 1, we know that $\frac{\partial e_1^*\left(\theta \mid \sigma_2^*(L), \sigma_2^*\left(\not{L}\right)\right)}{\partial \sigma_2^*(L)} > 0$ if and only if $\frac{\partial \psi(\theta)}{\partial \sigma_2^*(L)} > 0$. Furthermore, from Lemma 1,

$$\frac{\partial \psi(\theta)}{\partial \sigma_2^*(L)} = \theta\gamma + F \tag{61}$$

Therefore,

$$\frac{\partial^2 \psi(\theta)}{\partial \sigma_2^*(L)\,\partial\theta} = \frac{\partial[\theta\gamma + F]}{\partial\theta} \tag{62}$$

$$= \gamma \tag{63}$$

Therefore,

$$\frac{\partial^2 \psi(\theta)}{\partial \sigma_2^*(L)\,\partial\theta} > 0,\ \forall\psi(\theta) > 0 \tag{64}$$

By Lemma 1, this proves

$$\frac{\partial^2 e_1^*(\theta \mid \sigma_2^*(L), \sigma_2^*(\not{L}))}{\partial \sigma_2^*(L)\,\partial\theta} > 0,\ \forall\psi(\theta) > 0 \tag{65}$$

Therefore,

$$\frac{\partial^2 e_1^*(\theta \mid \sigma_2^*(L), \sigma_2^*(\not{L}))}{\partial \sigma_2^*(L)\,\partial\theta} > 0,\ \forall\psi(\theta) > 0 \tag{66}$$

$$\frac{\partial^2 e_1^*(\theta \mid \sigma_2^*(L), \sigma_2^*(\not{L}))}{\partial \sigma_2^*(\not{L})\,\partial\theta} = 0,\ \forall\psi(\theta) < 0 \tag{67}$$

■

Lemma 4: The infringer's expected profits from entry, after observing the existence of litigation in period 1 $(h_1 = L)$, is

(4.1) monotonically increasing in $\sigma_2^*(\not{L})$

(4.2) monotonically decreasing in $\sigma_2^*(L)$

Proof of (4.1): Consider the change in expected profits from entry upon observing the existence of litigation in period 1 $(h_1 = L)$ as we change $\sigma_2^*(\not{L})$.

$$E_{net}^{IN}\left[\Pi^{IN} \mid L\right] \tag{68}$$

Taking the derivative of net expected profit with respect to the infringer's equilibrium entry probability, conditional on observing the absence of litigation we get,

$$\frac{\partial\left[E_{net}^{IN}\left[\Pi^{IN} \mid L\right]\right]}{\partial\sigma_2\left(\not{L}\right)} \tag{69}$$

$$= \frac{\partial\int\left[F-\theta\gamma\right]\phi(\theta \mid L)d\theta}{\partial\sigma_2^*\left(\not{L}\right)} \tag{70}$$

$$= \int\left[F-\theta\gamma\right]\frac{\partial\phi(\theta \mid L)}{\partial\sigma_2^*\left(\not{L}\right)}d\theta \tag{71}$$

by Lemma 2 we have

$$\frac{\partial^2 e_1^*\left(\theta \mid \sigma_2^*\left(L\right),\sigma_2^*\left(\not{L}\right)\right)}{\partial\sigma_2^*\left(\not{L}\right)\partial\theta} < 0 \tag{72}$$

In other words, as we increase $\sigma_2^*\left(\not{L}\right)$, any type $\widehat{\theta}$ is reducing its first period effort at a faster rate than any type $\theta < \widehat{\theta}$. Therefore, since

$$\phi\left(\theta \mid L\right) = \frac{p\left(e_1^*\left(\theta\right)\right)\phi_0\left(\theta\right)}{\int\limits_{\theta\in\Theta} p\left(e_1^*\left(\theta\right)\right)\phi_0\left(\theta\right)d\theta} \tag{73}$$

then

$$\frac{\partial\phi(\theta \mid L)}{\partial\sigma_2^*\left(\not{L}\right)\partial\theta} < 0 \tag{74}$$

Therefore,

$$\frac{\partial E_{net}^{IN}\left[\Pi^{IN} \mid L\right]}{\partial\sigma_2\left(\not{L}\right)} > 0 \tag{75}$$

In short, as $\sigma_2^*\left(\not{L}\right)$ increases, the existence of litigation in period 1 will become less threatening, because in relative terms, lower types will increase the probability that they create litigation faster than higher types as $\sigma_2^*\left(\not{L}\right)$ increases.

Proof of (4.2): Consider the change in expected profits from entry upon observing the existence of litigation in period 1 ($h_1 = L$) as we change $\sigma_2^*\left(L\right)$.

.

Taking the derivative of net expected profit with respect to the infringer's equilibrium entry probability, conditional on observing the existence of litigation we get,

$$\frac{\partial E_{net}^{IN}\left[\Pi^{IN} \mid L\right]}{\partial \sigma_2(L)} \tag{76}$$

$$= \frac{\partial \int \left[F - \theta\gamma\right] \phi(\theta \mid L) d\theta}{\partial \sigma_2^*(L)} \tag{77}$$

$$= \int \left[F - \theta\gamma\right] \frac{\partial \phi(\theta \mid L)}{\partial \sigma_2^*(L)} d\theta \tag{78}$$

Recall that by Lemma 3 we know

$$\frac{\partial^2 e_1^*\left(\theta \mid \sigma_2^*(L), \sigma_2^*(\not L)\right)}{\partial \sigma_2^*(L)\, \partial\theta} > 0 \tag{79}$$

and therefore

$$\frac{\partial^2 \phi(\theta \mid L)}{\partial \sigma_2^*(L)\, \partial\theta} > 0 \tag{80}$$

$$\frac{\partial E_{net}^{IN}\left[\Pi^{IN} \mid L\right]}{\partial \sigma_2(L)} < 0 \tag{81}$$

■

Lemma 5: The infringer's expected profits from entry after observing the absence of litigation ($h_1 = \not L$) in period 1, is

(5.1) monotonically decreasing in $\sigma_2^*(\not L)$

(5.2) monotonically increasing in $\sigma_2^*(L)$.

Proof of (5.1): Consider the change in the infringer's expected profits from entry upon observing the absence of litigation in period 1 ($h_1 = \not L$) as we change $\sigma_2^*(\not L)$

Taking the derivative of expected profit from entry with respect to the infringer's equilibrium entry probability, conditional on observing the absence of litigation we get,

$$\frac{\partial E_{net}^{IN}\left[\Pi^{IN} \mid \not L\right]}{\partial \sigma_2(\not L)} \tag{82}$$

$$= \frac{\partial \int \left[F + \theta p\left(e_2^*(\theta \mid Out)\right) - \theta\gamma p\left(e_2^*(\theta \mid In)\right)\right] \phi(\theta \mid \not L) d\theta}{\partial \sigma_2^*(\not L)} \tag{83}$$

$$= \int \left[F + \theta p\left(e_2^*(\theta \mid Out)\right) - \theta\gamma p\left(e_2^*(\theta \mid In)\right)\right] \frac{\partial \phi(\theta \mid \not L)}{\partial \sigma_2^*(\not L)} d\theta \tag{84}$$

where

$$\text{where } e_2^*(\theta \mid In) = \underset{e_2 \in R_+}{\operatorname{argmax}} \left\{\theta(\gamma+1)\, p(e_2) - e_2\right\}$$

$$\text{where } e_2^*(\theta \mid Out) = \underset{e_2 \in R_+}{\operatorname{argmax}} \left\{\theta p(e_2) - e_2\right\}$$

By Lemma 2 we have

$$\frac{\partial e_1^*\left(\theta \mid \sigma_2^*\left(L\right), \sigma_2^*\left(\not{L}\right)\right)}{\partial \sigma_2^*\left(\not{L}\right) \partial \theta} < 0 \tag{85}$$

In other words, as we increase $\sigma_2^*\left(\not{L}\right)$, any type $\widehat{\theta}$ is reducing its first period effort at a faster rate than any type $\theta < \widehat{\theta}$. Therefore, since

$$\phi\left(\theta \mid \not{L}\right) = \frac{\left(1 - p\left(e_1^*\left(\theta\right)\right)\right) \phi_0\left(\theta\right)}{\int\limits_{\theta \in \Theta} \left(1 - p\left(e_1^*\left(\theta\right)\right)\right) \phi_0\left(\theta\right) d\theta} \tag{86}$$

Lemma 2 proves that

$$\frac{\partial^2 \phi(\theta \mid \not{L})}{\partial \sigma_2^*\left(\not{L}\right) \partial \theta} > 0 \tag{87}$$

Therefore,

$$\frac{\partial E_{net}^{IN}\left[\Pi^{IN} \mid \not{L}\right]}{\partial \sigma_2\left(\not{L}\right)} < 0 \tag{88}$$

In short, as $\sigma_2^*\left(\not{L}\right)$ increases, the absence of litigation in period 1 will become more threatening, because in relative terms, lower types will increase the probability that they create litigation faster than higher types as $\sigma_2^*\left(\not{L}\right)$ increases

Proof of (5.2): Consider the change in the infringer's expected profits from entry upon observing the existence of litigation in period 1 ($h_1 = L$) as we change $\sigma_2^*\left(L\right)$.

$$\frac{\partial E_{net}^{IN}\left[\Pi^{IN} \mid \not{L}\right]}{\partial \sigma_2\left(L\right)} \tag{89}$$

$$= \frac{\partial \int \left[F + \theta p\left(e_2^*\left(\theta \mid Out\right)\right) - \theta \gamma p\left(e_2^*\left(\theta \mid In\right)\right)\right] \phi(\theta \mid \not{L}) d\theta}{\partial \sigma_2^*\left(L\right)} \tag{90}$$

$$= \int \left[F + \theta p\left(e_2^*\left(\theta \mid Out\right)\right) - \theta \gamma p\left(e_2^*\left(\theta \mid In\right)\right)\right] \frac{\partial \phi(\theta \mid \not{L})}{\partial \sigma_2^*\left(L\right)} d\theta \tag{91}$$

where

$$\text{where } e_2^*\left(\theta \mid In\right) = \underset{e_2 \in R_+}{\operatorname{argmax}} \left\{\theta\left(\gamma + 1\right) p\left(e_2\right) - e_2\right\}$$
$$\text{where } e_2^*\left(\theta \mid Out\right) = \underset{e_2 \in R_+}{\operatorname{argmax}} \left\{\theta p\left(e_2\right) - e_2\right\}$$

Recall that by Lemma 3 we know

$$\frac{\partial^2 e_1^*\left(\theta \mid \sigma_2^*\left(L\right), \sigma_2^*\left(\not{L}\right)\right)}{\partial \sigma_2^*\left(L\right) \partial \theta} > 0 \tag{92}$$

and therefore

$$\frac{\partial^2 \phi(\theta \mid \not{L})}{\partial \sigma_2^*(L)\, \partial\theta} < 0 \tag{93}$$

$$\frac{\partial E_{net}^{IN}\left[\Pi^{IN} \mid \not{L}\right]}{\partial \sigma_2(L)} > 0 \tag{94}$$

■

Lemma 6: In any Bayesian equilibrium $\sigma_2^*(L) > 0$ only if $\sigma_2^*(\not{L}) = 1$.

Proof: We know,

$$\left[E_{SG3}^{IN}(\Pi^{IN} \mid \theta) - E_{SG4}^{IN}(\Pi^{IN} \mid \theta)\right] > E_{SG2}^{IN}(\Pi^{IN} \mid \theta) - E_{SG1}^{IN}(\Pi^{IN} \mid \theta) \tag{95}$$

Therefore using (23) and (24) we see that if $\phi(\theta \mid \not{L}) \geq \phi(\theta \mid L)$ then

$$E_{net}^{IN}\left[\Pi^{IN} \mid \not{L}\right] > E_{net}^{IN}\left[\Pi^{IN} \mid L\right]. \tag{96}$$

Furthermore, if $\sigma_2^*(\not{L}) < \sigma_2^*(L)$ then from (28) and (29) we know $\frac{\partial e_1^*\left(\theta \mid \sigma_2^*(L), \sigma_2^*\left(\not{L}\right)\right)}{\partial\theta} \geq 0\ \forall\theta$, which implies $\phi(\theta \mid \not{L}) \geq \phi(\theta \mid L)$ and thus $E_{net}^{IN}\left[\Pi^{IN} \mid \not{L}\right] > E_{net}^{IN}\left[\Pi^{IN} \mid L\right]$ which contradicts $\sigma_2^*(\not{L}) < \sigma_2^*(L)$. Therefore we conclude that in any equilibrium $\sigma_2^*(\not{L}) \geq \sigma_2^*(L)$.

Furthermore, since the inequality in (98) is strict, we cannot have

$$E_{net}^{IN}\left[\Pi^{IN} \mid \not{L}\right] = E_{net}^{IN}\left[\Pi^{IN} \mid L\right] = 0. \tag{97}$$

Therefore, the infringer cannot be indifferent to entry under both histories $\{\not{L}\}$ and $\{L\}$. Thus $\sigma_2^*(L) > 0$ only if $\sigma_2^*(\not{L}) = 1$.■

Proposition1: For any setting of the parameters and any continuous function $p(\cdot)$ under our assumptions, and for any possible prior beliefs $\phi_0(\cdot)$ and any type space $\Theta \subset R$, there exists a unique Bayesian equilibrium.

Proof of Existence: We can check to see if there is an equilibrium when the infringer best responses are in pure strategies using the following procedure:

1. Consider a hypothetical best response $\{\hat{\sigma}_2^*(L), \hat{\sigma}_2^*(\not{L})\}$ from the set

$$\{\hat{\sigma}_2^*(L), \hat{\sigma}_2^*(\not{L})\} \in \{\{Out, Out\}, \{Out, In\}, \{In, In\}\},$$

excluding $\{\hat{\sigma}_2^*(L), \hat{\sigma}_2^*(\not{L})\} = \{In, Out\}$ because of Lemma 6.

2. Take $\{\hat{\sigma}_2^*(L), \hat{\sigma}_2^*(\not{L})\}$ and plug it into each patent holder's profit maximization problem to find the optimal choice of effort for each patent holder type:

$$\{e_1^*(\theta \mid \hat{\sigma}_2^*(L), \hat{\sigma}_2^*(\not{L})) \ \forall\theta\}$$

3. Using $\{e_1^*(\theta \mid \hat{\sigma}_2^*(L), \hat{\sigma}_2^*(\not{L})) \ \forall\theta\}$, calculate $\phi(\theta \mid L)$ and $\phi(\theta \mid \not{L})$.

4. Using $\phi(\theta \mid L)$ and $\phi(\theta \mid \not{L})$, calculate $E_{net}^{IN}[\Pi^{IN} \mid L]$ and $E_{net}^{IN}[\Pi^{IN} \mid \not{L}]$.

5. Using $E_{net}^{IN}[\Pi^{IN} \mid L]$ and $E_{net}^{IN}[\Pi^{IN} \mid \not{L}]$ we know

$$\{\hat{\sigma}_2^*(L), \hat{\sigma}_2^*(\not{L}), \{e_1^*(\theta \mid \hat{\sigma}_2^*(L), \hat{\sigma}_2^*(\not{L})) \ \forall\theta\}\}$$

is a Bayesian equilibrium only if, for all $h_1 \in \{L, \not{L}\}$

$$\begin{aligned} \hat{\sigma}_2^*(h_1) &= \{In\}, \quad \text{only if } E_{net}^{IN}[\Pi^{IN} \mid h_1] > 0 \\ &\quad \text{and} \\ \hat{\sigma}_2^*(h_1) &= \{Out\}, \ \text{only if } E_{net}^{IN}[\Pi^{IN} \mid h_1] < 0 \end{aligned}$$

because

$$\begin{aligned} \sigma_2^*(h_1) &= 1, && \text{if } E_{net}^{IN}\left[\Pi^{IN} \mid h_1\right] > 0 \\ \sigma_2^*(h_1) &\in (0,1), && \text{if } E_{net}^{IN}\left[\Pi^{IN} \mid h_1\right] = 0 \\ \sigma_2^*(h_1) &= 0, && \text{if } E_{net}^{IN}\left[\Pi^{IN} \mid h_1\right] < 0 \end{aligned}$$

When the potential infringer's best responses are not in pure strategies, we can resume the search for equilibria by asking the following question. Given the optimal first period effort for each type of patent holder when infringer best responses are $\{\sigma_2^*(L), \sigma_2^*(\not{L})\} = \{Out, In\}$, which of the best responses: $\{Out, Out\}$ or $\{In, In\}$, would the infringer wish to deviate too?

Consider the following cases of the deviations to $\{Out, Out\}$ or $\{In, In\}$ given the set of patent holder effort $(e_1^*(\theta \mid Out, In))\, \forall\theta \in \Theta$.

Case 1: Assume that given the set of patent holder effort $\{e_1^*(\theta \mid Out, In)\ \ \forall\theta \in \Theta\}$, the infringer wishes to play $\{\sigma_2^*(L), \sigma_2^*(\not{L})\} = \{Out, Out\}$. Then by

$$\{\sigma_2^*(L), \sigma_2^*(\not{L})\} \notin \{\{Out, Out\}, \{Out, In\}, \{In, In\}\},$$

we know its expected profits from entry conditional on observing $\{\not{L}\}$ are negative given $\{e_1^*(\theta \mid Out, In)\ \ \forall\theta \in \Theta\}$, but positive given $\{e_1^*(\theta \mid Out, Out)\ \ \forall\theta \in \Theta\}$. Thus by Lemma 5, the infringer's second period expected profits from entry, after observing the absence of litigation, are continuous and decreasing in $\sigma_2^*(\not{L})$. Therefore, there exists a set of efforts $\{e_1^*(\theta \mid Out, \sigma_2)\ \ \forall\theta \in \Theta\}$ for some $\sigma_2 \in (0,1)$, such that the potential infringer expects zero profit from entering upon observing $\{\not{L}\}$. Thus there exists an equilibrium

$$\{e_1^*(\theta \mid Out, \sigma_2)\ \ \forall\theta \in \Theta\} \text{ and } \{\sigma_2^*(L), \sigma_2^*(\not{L})\} = \{Out, \sigma_2\}$$

for some $\sigma_2 \in (0,1)$.

Case 2: Assume that given the set of patent holder efforts $\{e_1^*(\theta \mid Out, In)\ \ \forall\theta \in \Theta\}$, the infringer wishes to play $\{\sigma_2^*(L), \sigma_2^*(\not{L})\} = \{In, In\}$. Then by

$$\{\sigma_2^*(L), \sigma_2^*(\not{L})\} \notin \{\{Out, Out\}, \{Out, In\}, \{In, In\}\},$$

we know its expected profits from entry conditional on observing $\{L\}$ are positive given $\{e_1^*(\theta \mid Out, In)\ \ \forall\theta \in \Theta\}$, but negative given $\{e_1^*(\theta \mid In, In)\ \ \forall\theta \in \Theta\}$. Thus by Lemma 4, the infringer's second period expected profits from entry, after observing the absence of litigation, are continuous and decreasing in $\sigma_2^*(L)$. Therefore, there exists a set of efforts $\{e_1^*(\theta \mid \sigma_2, In)\ \ \forall\theta \in \Theta\}$ for some $\sigma_2 \in (0,1)$, such that the potential infringer expects zero profit from entering upon observing $\{L\}$. Thus there exists an equilibrium

$$\{e_1^*(\theta \mid \sigma_2, In)\ \ \forall\theta \in \Theta\} \text{ and } \{\sigma_2^*(L), \sigma_2^*(\not{L})\} = \{\sigma_2, In\}$$

for some $\sigma_2 \in (0,1)$.

Proof of Uniqueness:

Part 1: proving unique equilibrium best response for the infringer

Case PS: (Pure strategy equilibrium): Assume there exists an equilibrium with infringer best responses:

$$\{\hat{\sigma}_2^*(L), \hat{\sigma}_2^*(\not{L})\}$$

such that

$$\{\hat{\sigma}_2^*(L), \hat{\sigma}_2^*(\not{L})\} \in \{\{Out, Out\}, \{Out, In\}, \{In, In\}\} \tag{98}$$

Case PS.1: It cannot be the case that both $\{Out, Out\}$ $\{Out, In\}$ are in equilibrium because if $\{Out, In\}$ is an equilibrium, by Lemma 5 we know that the infringer's expected profits upon observing the $\{\not{L}\}$, are monotonically increasing as $\sigma_2^*(\not{L}) \to 0$. Therefore, it cannot be the case that case that the infringer expects positive profits from entry upon observing $\{\not{L}\}$ when playing $\{\hat{\sigma}_2^*(L), \hat{\sigma}_2^*(\not{L})\} = \{Out, In\}$, but also expect negative profits from entry upon observing $\{\not{L}\}$ when playing $\{\hat{\sigma}_2^*(L), \hat{\sigma}_2^*(\not{L})\} = \{Out, Out\}$.

Case PS.2: It cannot be the case that both $\{In, In\}$ $\{Out, In\}$ are in equilibrium because if $\{Out, In\}$ is an equilibrium, by Lemma 4 we know that the infringer's expected profits upon observing $\{L\}$, are monotonically increasing as $\sigma_2^*(L) \to 0$. Therefore, it cannot be the case that case that the infringer expects negative profits from entry upon observing $\{L\}$ when playing $\{\hat{\sigma}_2^*(L), \hat{\sigma}_2^*(\not{L})\} = \{Out, In\}$, but also expect positive profits from entry upon observing $\{L\}$ when playing $\{\hat{\sigma}_2^*(L), \hat{\sigma}_2^*(\not{L})\} = \{In, In\}$.

Case PS.3: It cannot be the case that both $\{In, In\}$ $\{Out, Out\}$ are in equilibrium because if $\{In, In\}$ is an equilibrium, by Lemma 4 we know that the infringer's expected profits upon observing $\{\not{L}\}$, are monotonically increasing as $\sigma_2^*(L) \to 0$. Therefore, it cannot be the case that case that the infringer expects positive profits from entry upon observing $\{\not{L}\}$ when playing $\{\hat{\sigma}_2^*(L), \hat{\sigma}_2^*(\not{L})\} = \{In, In\}$, but also expect positive profits from entry upon observing $\{\not{L}\}$ when playing $\{\hat{\sigma}_2^*(L), \hat{\sigma}_2^*(\not{L})\} = \{Out, Out\}$.

Case MS: Mixed Strategy Best responses:

$$\{\hat{\sigma}_2^*(L), \hat{\sigma}_2^*(\not{L})\} = \{\{\hat{\sigma}_2^*, In\}, \{Out, \check{\sigma}_2^*\}\} \tag{99}$$

for some $\hat{\sigma}_2^*$ and $\check{\sigma}_2^* \in (0,1)$

It cannot be the case that $\{\{\hat{\sigma}_2^*, In\}, \{Out, \check{\sigma}_2^*\}\}$ are both in equilibrium for any $\hat{\sigma}_2^*$ and $\check{\sigma}_2^* \in (0,1)$, because if $\{\hat{\sigma}_2^*, In\}$ is in equilibrium, it must be the case that the infringer expects positive entry profits upon observing $\{L\}$ when playing $\{\hat{\sigma}_2^*(L), \hat{\sigma}_2^*(\not{L})\} = \{Out, In\}$ and expects negative profits from entry upon observing $\{L\}$ when playing $\{\hat{\sigma}_2^*(L), \hat{\sigma}_2^*(\not{L})\} = \{In, In\}$. Therefore since by Lemma 4 the infringer's expected profits from entry, upon observing $\{\not{L}\}$, are increasing as $\sigma_2^*(\not{L}) \to 0$, it cannot be the case that the infringer is indifferent to entry, upon observing $\{\not{L}\}$, for any $\sigma_2^*(\not{L}) < 1$, when there exists another equilibrium $\{\hat{\sigma}_2^*(L), \hat{\sigma}_2^*(\not{L})\} = \{\hat{\sigma}_2^*, In\}$.

Furthermore, we know that when there is a equilibrium in mixed strategies, there exists only one entry probability $\sigma_2^*(h_1)$ that makes the infringer indifferent upon observing h_1, because of the monotonicity of expected profits shown in Lemmas 4 and 5 (See Case 1 and 2 in the proof of existence above).

Part 2: Uniqueness of Patent Holder Effort:

Each patent holder's choice of effort is uniquely determined for any given best response $\sigma_2^*(L), \sigma_2^*(\not{L})$ (from (21)-(30)). Therefore, since there is only one equilibrium set of best responses $\{\hat{\sigma}_2^*(L), \hat{\sigma}_2^*(\not{L})\}$ (Shown in Part 1 of this proof), there exists only one equilibrium $\{\hat{\sigma}_2^*(L), \hat{\sigma}_2^*(\not{L})\}, \{e_1^*(\theta \mid \hat{\sigma}_2^*(L), \hat{\sigma}_2^*(\not{L})) \ \forall\theta \in \Theta\}$ such that the infringer is profit maximizing according to their beliefs, and beliefs are consistent with the patent holder's equilibrium actions.■

Proposition 2: There exists a non-empty region of the parameter space, prior beliefs $\phi_0(\theta)$, and type spaces $\Theta \subset R$ such that the equilibrium is non-trivial.

Proof: The equilibrium is non-trivial as long as the infringer's action in period two is conditional on the first period outcome with positive probability. Therefore we must find the parameter space that excludes the two equilibrium two trivial equilibria. Recall the infringer's expected profits from (18):

$$
\begin{aligned}
E_{net}^{IN} \quad & \left[\Pi^{IN} \mid \not{L}\right] = \\
& F - \int \theta(\gamma+1)\, p\left(e_2^*(\theta \mid In)\right) \phi(\theta \mid \not{L})\, d\theta + \int \theta p\left(e_2^*(\theta \mid Out)\right) \phi(\theta \mid \not{L})\, d\theta \\
E_{net}^{IN} \quad & \left[\Pi^{IN} \mid L\right] = F - \int \theta\gamma\phi(\theta \mid L)\, d\theta
\end{aligned}
$$

Therefore, for any $E_{net}^{IN}\left[\Pi^{IN} \mid \not{L}\right] > 0,$ there exists a licensing fee F, such that

$$F < \int \theta\gamma\overline{\phi}\left(\theta \mid L\right) d\theta \tag{100}$$

$$F > \left[\begin{array}{c} \int \theta\left(\gamma+1\right) p\left(e_2^*\left(\theta \mid In\right)\right) \underline{\phi}\left(\theta \mid \not{L}\right) d\theta ... \\ -\int \theta p\left(e_2^*\left(\theta \mid Out\right)\right) \underline{\phi}\left(\theta \mid \not{L}\right) d\theta \end{array}\right] \tag{101}$$

$$\text{Where } \overline{\phi}\left(\theta \mid L\right) \equiv \frac{p\left(e_1^*(\theta \mid 1,1)\right)\phi_0\left(\theta\right)}{\int\limits_{\theta\in\Theta} p\left(e_1^*(\theta \mid 1,1)\right)\phi_0\left(\theta\right) d\theta} d\theta \tag{102}$$

$$\text{Where } \underline{\phi}\left(\theta \mid \not{L}\right) \equiv \frac{\left(1-p\left(e_1^*(\theta \mid 0,0)\right)\right)\phi_0\left(\theta\right)}{\int\limits_{\theta\in\Theta}\left(1-p\left(e_1^*(\theta \mid 0,0)\right)\right)\phi_0\left(\theta\right) d\theta} \tag{103}$$

and that will result in non-trivial equilibria. From Lemma 6, we know $E_{net}^{IN}\left[\Pi^{IN} \mid \not{L}\right] > E_{net}^{IN}\left[\Pi^{IN} \mid L\right]$, therefore, there always exists an F such that

$$F - \left[\begin{array}{c} \int \theta\left(\gamma+1\right) p\left(e_2^*\left(\theta \mid In\right)\right) \phi\left(\theta \mid \not{L}\right) d\theta + \\ \int \theta p\left(e_2^*\left(\theta \mid Out\right)\right) \phi\left(\theta \mid \not{L}\right) d\theta \end{array}\right] > 0 \tag{104}$$

$$F - \int \theta\gamma\phi\left(\theta \mid L\right) d\theta < 0 \tag{105}$$

whenever $E_{net}^{IN}\left[\Pi^{IN} \mid \not{L}\right] > 0.$■

Proposition 3: In any non-trivial equilibrium, there exists $\overline{\theta} \in R$ such that first period effort is decreasing in θ if and only if $\theta > \overline{\theta}$.

Proof: From Lemma 1 we know $\frac{\partial e_1^*\left(\theta \mid \sigma_2^*(L), \sigma_2^*\left(\not{L}\right)\right)}{\partial\theta} < 0$ if and only if $\frac{\partial\psi(\theta)}{\partial\theta} < 0$. Furthermore, from (50) we know

$$\frac{\partial\psi\left(\theta\right)}{\partial\theta} = \tag{106}$$

$$= \left[1 + \sigma_2^*\left(L\right)\gamma + \sigma_2^*\left(\not{L}\right)\left(\begin{array}{c} p\left(e_2^*\left(\theta \mid Out\right)\right) + ... \\ -\left(\gamma+1\right) p\left(e_2^*\left(\theta \mid In\right)\right) \end{array}\right) - p\left(e_2^*\left(\theta \mid Out\right)\right)\right] \tag{107}$$

$$= 1 + \sigma_2^*\left(L\right)\gamma - \sigma_2^*\left(\not{L}\right)\left(\gamma+1\right) p\left(e_2^*\left(\theta \mid In\right)\right) - \left(1 - \sigma_2^*\left(\not{L}\right)\right) p\left(e_2^*\left(\theta \mid Out\right)\right) \tag{108}$$

furthermore, as $\theta \to \infty$, $p\left(e_2^*\left(\theta \mid In\right)\right), p\left(e_2^*\left(\theta \mid Out\right)\right) \to 1$. Therefore,

$$\lim_{\theta\to\infty} \frac{\partial\psi\left(\theta\right)}{\partial\theta} = 1 + \sigma_2^*\left(L\right)\gamma - \sigma_2^*\left(\not{L}\right)\left(\gamma+1\right) - \left(1 - \sigma_2^*\left(\not{L}\right)\right) \tag{109}$$

$$= \gamma\left(\sigma_2^*\left(L\right) - \sigma_2^*\left(\not{L}\right)\right) \tag{110}$$

From Lemma 6, we know $\sigma_2^*(\not{L}) > \sigma_2^*(L)$ for all non-trivial equilibria. Therefore $\sigma_2^*(L) - \sigma_2^*(\not{L}) < 0$, and γ strictly positive implies.

$$\lim_{\theta\to\infty} \frac{\partial\psi(\theta)}{\partial\theta} = \gamma\left(\sigma_2^*(L) - \sigma_2^*(\not{L})\right) < 0 \tag{111}$$

therefore there exists $\overline{\theta} \in R$ such that for all $\theta > \overline{\theta}$, $\frac{\partial e_1^*\left(\theta|\sigma_2^*(L),\sigma_2^*\left(\not{L}\right)\right)}{\partial\theta} < 0.$■

Proposition 4: In any non-trivial equilibrium, there exists $\overline{\overline{\theta}} \in R$ and $\underline{\theta} \in R$ such that $e_1^*(\theta) = 0, \forall \left\{\theta \mid \theta > \overline{\overline{\theta}} \text{ or } \theta < \underline{\theta}\right\}$.

Proof: From Proposition 3, we know that as $\theta \to \infty$

$$\frac{\partial\psi(\theta)}{\partial\theta} \to \gamma\left(\sigma_2^*(L) - \sigma_2^*(\not{L})\right) < 0 \tag{112}$$

Therefore, there exists a $\overline{\overline{\theta}} \in R$ such that for all $\theta > \overline{\overline{\theta}}$, $\psi(\theta) = 0$, and therefore for all types $\theta > \overline{\overline{\theta}}$, $e_1^*(\theta \mid \sigma_2^*(L), \sigma_2^*(\not{L})) = 0$.

Furthermore, as $\theta \to -\infty$, then $p(e_2^*(\theta \mid In)), p(e_2^*(\theta \mid Out)) \to 0$. Therefore,

$$\lim_{\theta\to-\infty} \psi(\theta) = -\infty \tag{113}$$

so there exists $\underline{\theta} \in R$ such that for all types $\theta < \underline{\theta}$, $\psi(\theta) = 0$ and therefore for all types $\theta < \underline{\theta}$, $e_1^*(\theta \mid \sigma_2^*(L), \sigma_2^*(\not{L})) = 0$.■

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