1. (10 points)
a. (5 points) Solve the linear system of equations
\[
\begin{align*}
    x + y + z &= 2 \\
    2x - 2y - z &= 1 \\
    x + y - z &= 0
\end{align*}
\]
if possible.
b. (3 points) Write the system in the matrix form \( A\vec{x} = \vec{b} \). 
c. (2 points) Are the vectors \((1, 1, 1), (2, -2, -1)\) and \((1, 1, -1)\) linearly independent or dependent?
Problem 1 Cont.
2. (10 points)
Find the general solution of the equation
\[ y^{(4)} + y = 0. \]
Problem 2 Cont.
3. (10 points)
   a. (5 points) Find the kernel of the linear transformation $A : \mathbb{R}^3 \to \mathbb{R}^3$
      
      $A(x, y, z) = (x + y, y + z, x - z)$.

   b. (4 points) Find a basis for the kernel.
   c. (1 point) What is the dimension of the kernel?
Problem 3 Cont.
4. (10 points)
Find a particular solution to the equation
\[ y^{(2)} + y = \sin(t). \]
Problem 4 Cont.
5. (10 points)
Does the equation

\[ y^{(4)}(t) + y^{(2)}(t) = 0 \]

have a solution that exists near \( t = 0 \)?
(Hint: Compute the determinant of the Wronskian matrix at \( t = 0 \))
Problem 5 Cont.