Moving Mirrors

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An accelerating observer in the traditional Minkowski vacuum state will observe a thermal spectrum of particles which implies that there is thermal radiation in the prsence of a black hole event horizon. The Unruh effect implies the Hawking effect. The motion of a single reflecting boundary can also create particles. Through a process of irreversible production of entropy, new quanta become present.

A Planck spectrum with temperature

$$T = \frac{a}{2\pi k}$$

occurs for this thermal radiation.

This reveals the essentially thermal nature of the vacuum in quantum field theory. If there is no energy -momentum, how can Rindler observers detect particles? When you accelerate a detector, energy is not conserved, work is pumped in to keep it accelerating constantly. The detector is not being excited by the background energy-momentum tensor, but from the energy used to keep the detector moving.

Consider the moving mirror in two-demensional Minkowski space moving along trajectory:

$$x = z(t)$$

The massless scalar field ϕ with field equation

$$\Box\phi = \frac{\partial^2\phi}{\partial u \partial v} = 0$$

is constrained to vanish at the mirror.

Allowing the mirror to be static in some inertial frame, define positive frequency 'in' modes of the form:

$$u_k^{in} = \frac{i}{\sqrt{4\pi\omega}}(e^{-i\omega v} - e^{-i\omega u}) = \frac{\sin\omega x}{\sqrt{\pi\omega}} e^{-i\omega t}$$

This is a positive frequency mode with respect to Minkowski time t and as $t \leq 0$. The field, ϕ may be expanded in terms of these modes and we can also define a vacuum state:

$$\phi = \sum_{k>0} [a_k u_k^{in} + a_k^\dagger (u_k^{in})^*]$$

$$a_k|0, in\rangle = 0$$

This vacuum state is our very friendly no particle state of standard quantum field theory. If we had some constant velocity particle detector, it would detect nothing. This can be proved by using the Wightman function for the 'in' region

$$\langle in, 0|\phi(x)\phi(x')|0, in\rangle = -\frac{1}{4\pi} ln[\frac{(u-u'-i\epsilon)(v-v'-i\epsilon)}{(v-u'-i\epsilon)(u-v'-i\epsilon)}]$$

and substituting into the detector response function,

$$F(E) = \int_{-\infty}^{\infty} d\tau \int_{-\infty}^{\infty} d\tau' e^{-eE(\tau-\tau')} G^{+}(x(\tau), x(\tau')).$$

But what we are most interested in, is when the mirror starts moving. Once the mirror accelerates we have something akin to gravity. When the mirror accelerates as $t > 0$ the field modes will suffer distortion for $u > 0$ from

$$u_k^{in} = \frac{i}{\sqrt{4\pi\omega}}(e^{-i\omega v} - e^{-i\omega u})$$

to the more general form

$$u_k^{in} = \frac{i}{\sqrt{4\pi\omega}}(e^{-i\omega v} - e^{-i\omega(2\tau_u - u)})$$

Here τ_u is determined implicity by the trajectory $x = z(t)$ through $\tau_u - z(\tau_u) = u$. There is an asymmetry between u and v chosen to correspond to the usual retarded boundary condition. Notice that the right moving waves, the ones that are reflected from the mirror are complicated because of the Doppler shift suffered.

This moving mirror plays the same role as a time-dependent background geometry, that is, a gravitational field. What makes it different is that the $2\tau_u - u$ term is unchanged all along the null ray as the light ray goes out forever, and the distortion of the modes occurs suddenly after reflection rather than gradually over a long time as is the case in a gravitational geometrical disruption.

So lets look at this for a moment. Look at the right moving piece:

$$e^{-i\omega(2\tau_u - u)}$$

This reduces in the region $u < 0$ to the standard form for a right-moving wave, $e^{-i\omega u}$. For $t > 0$ we associate the friendly vacuum state, defined by

$$a_k|0, in\rangle = 0$$

with it, as long as $u < 0$. But in the region $u > 0$, it does not reduce to the standard right moving waves. The vacuum state becomes a state with particles! The Doppler complication ends up exciting field modes and particles seemingly come into existence. It is said that moving mirrors create particles, which travel away to the right along null rays, $u = constant$.

To verify that these quanta exist, take an inertial particle detector in the region of interest, $u > 0$, and look at its response. The Wightman function in the 'in' vacuum is

$$D^{+}(u, v; u', v') = -\frac{1}{4\pi} ln[\frac{(p(u) - p(u') - i\epsilon)(v - v' - i\epsilon)}{(v - p(u') - i\epsilon)(p(u) - v' - i\epsilon)}]$$

where $p(u) = 2\tau_u - u$. Here, for a general $x(t)$, and hence $p(u)$, the Fourier transform of D^{+} will be nonzero. Substituting D^{+} into the response funtion, $F(E)$, will yield a nonzero response from the detector. This can be explicitly illustrated by looking at the trajectory for the mirror with asymptotic form

$$z(t \to \infty) \to -t - Ae^{-2\kappa t} + B$$

Where A, B and κ are constants greater than zero. Consequently, this trajectory yields a response :

$$\frac{F(E)}{unit\ time} = \frac{1}{E(e^{E/kT} - 1)}$$

with

$$kT = \frac{\kappa}{2\pi}\sqrt{\frac{1-w}{1+w}}$$

The square root factor is the Doppler shift due to the motion of the detector at velocity w relative to the radiation, where the inertial detector follows a path

$$x = x_0 + wt$$

If the detector is at rest in the special frame defined by the mirror when it was intially static, that is, when $w = 0$, then we have

$$T = \frac{\kappa}{2\pi k}$$

By evaluating the Bogolubov transformation between the in and out modes we can derive the same spectrum. Evaluating the transformation at $t = 0$ is convenient and consists of the simple u_k^{in} modes,

$$u_k^{in} = \frac{i}{\sqrt{4\pi\omega}}(e^{-i\omega v} - e^{-i\omega u})$$

but complicated u_k^{out} modes. Using only the region $v < B$ for the out modes, the Bogolubov coefficents are solved using

$$\alpha_{ij} = (\bar{u}_i, u_j) \qquad \beta_{ij} = -(\bar{u}_i, u_j^*)$$

and we have

$$|\beta_{\omega',\omega}|^2 = \frac{1}{2\pi\kappa\omega'}\left(\frac{1}{e^{\omega/kT} - 1}\right)$$

with

$$T = \frac{\kappa}{2\pi k}$$