

# Thermalizing the Vacuum

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## 1 Unification

The importance of Hawking's unification of relativity, quantum theory and thermodynamics must not be overlooked. The ramifications of Hawking's discoveries has provided physicists with many of the deepest insights into the nature of quantum gravity. The Unruh effect is the flat space analog of Hawking's effect. An observer who is accelerating with respect to the traditional zero-temperature Minkowski vacuum state will observe a thermal spectrum of particles with temperature

$$T = \frac{\hbar}{2\pi ck} a \quad (1)$$

This result was pioneered by Davis [1], Fulling [2], and Unruh [3]. It beautifully parallels Hawking's temperature of  $T_H = \frac{\hbar}{2\pi ck} \kappa$ , where  $\kappa$  is the surface gravity, for the Schwarzschild black hole. The Unruh effect has been used to imply that there is indeed thermal radiation in the presence of a black hole event horizon. The Unruh effect is more fundamental than the Hawking effect and has more potential for empirical verification. [4] [5] [6]. To many physicists the Unruh effect is radically changing the notion of the vacuum and the debunking the idea that 'particles' are fundamental entities in quantum field theory.

## 2 A Mysterious Radiation

This simple relationship, though not nearly as famous but is at least of the same rank as  $E = mc^2$ , has a great deal of controversy surrounding it. Most physicists agree that the Unruh effect is real, i.e. that a vacuum thermalizes at a temperature (the Unruh temperature) for a system undergoing uniform acceleration. The confusion arises when physicists confront the issue of whether the system actually radiates or not.

So there is a distinct difference between 'Unruh radiation' and the 'Unruh effect'. The Unruh effect is the accelerated observer/detector/oscillator finding itself in a thermal bath despite acceleration through the vacuum of Minkowski flat space. That is, you get hot when you accelerate. But Unruh radiation, to many, is the result of energy flowing from a hot body to a cold body. It is the proposed radiation given off by the system due to the immersion in this thermal bath. The existence of Unruh radiation is tentative.

## 3 A History of Opposition

It wasn't until about 10 years after the discovery of the Unruh effect that the question of whether the system actually radiates or not was questioned. Grove [7] was the first to go against the prevailing

opinion and argue that radiation does not occur. Supported later by Raine et al [8], the full-fledged controversy began.

Those in the opposition such as Barut et al [9] claim the thermal photons have no independent existence outside the accelerating detector. Hu et al [10] at Maryland, very confidently joined in the debate, claiming ‘there is absolutely no emitted radiation from a uniformly accelerated oscillator in equilibrium conditions’. On the far end of the spectrum, the Russians, Narozhny et al [11] have at least seven papers which they claim show the absence of a theoretical basis for the existence of ‘acceleration radiation’ [12]. Most recently Ford et al [13], in an effort to subdue the controversy, have produced a pedagogical paper with solid, easy-to-understand arguments that support the notion of no radiation.

Still, these treatments seem to be the minority as numerous others assume that Unruh radiation exists [14], [15] [16] [17] [18] [19] [20] [21] [22] Others also believe it may soon be detectable using lasers such as Chen[4] and Schutzhold[5], as well as through microwave cavity conditions [23]. Perhaps the most vocal believer is Unruh, as he is reported to have said “It is real enough to roast a steak.”.[24]

## 4 Empirical Surprises

Lasers may be used to discover whether Unruh radiation exists. [4] [5] Ultra-high strength electric field lasers such as the petawatt class lasers currently in development (which we may have access to) may shed light on three very interesting effects: Unruh Radiation, Schwinger’s Limit and Vacuum Birefringence.

It is generally considered that using high strength lasers are the most promising way to go, so far, to detect Unruh radiation. This is mostly due to overcoming the loud background noise found in numerous other experimental setups. [25] It is claimed that the Unruh radiation will have distinct polarizations and angular distributions in these set ups.

Habs [5] points out that the Schwinger limit must be taken into account with high strength electric fields. If our lasers produce too strong of fields then the creation of electron-positron pairs will appear in the vacuum. This can be appreciated intuitively because we know that fields carry energy and with enough energy we can create particles. That is, the energy density is proportional to the electric field squared. He reminds us that the field strength can’t be too far below the Schwinger limit or the Unruh radiation will be too hard to measure.

The Schwinger limit is the maximum electric field possible in the vacuum in order to avoid the birth of copious electron-positron pairs. The destruction of the vacuum occurs around:

$$eE_{max} \approx \frac{mc^2}{\lambda} = \frac{mc^2}{\hbar/(mc)} = \frac{m^2c^3}{\hbar}$$

But the Schwinger limit has not yet been reached in the laboratory. So simply reaching this would be a great step in understanding the quantum vacuum.

The quantum vacuum holds more mystery than one may imagine. There is interest in performing this type of experiment with strong lasers for yet another reason: The QED vacuum is predicted to exhibit birefringent [26] and dichroic effects when strong electric fields are applied. [27] That is, light passed through the vacuum that is subjected to these fields will decompose depending on polarization just like the double image seen from using a calcite crystal. This type of experiment would offer us

the opportunity to test the nature of the quantum vacuum.

## 5 A Couple of Simple Physical Derivations

Here I want to offer two very simple derivations of the Unruh effect in order for us to feel more comfortable and at home.

### 5.1 A Game of Units

First let me start with a basic game of playing with units. Imagine I take an elevator dubbed an ‘Einstein lift’. At the bottom of this elevator I will place several electrons and I begin to accelerate the Einstein lift. As I accelerate the lift, the amount of energy absorbed by one of the electrons is equal to

$$\Delta E = ma\Delta x$$

I am going to require that this energy be equal to the amount of energy it takes to make an  $e^-e^+$  pair. That is, I am forced to set this equal to

$$\Delta E = 2mc^2$$

Solving for the change in distance, i.e. the space-slice  $\Delta x$ , with  $2mc^2 = ma\Delta x$ , we have

$$\Delta x = \frac{2c^2}{a}$$

The electron-positron pair is confined to an amplitude of  $\Delta x$ . There is an inherent uncertainty associated with the energy of a single electron. Using  $\Delta E\Delta t = \hbar/2$  with  $\Delta t = \Delta x/c$ , the uncertainty in energy is:

$$\Delta E = \frac{\hbar c}{2\Delta x} = \frac{\hbar a}{4c}$$

Now this energy may be thought of classically. If we assume the thermal agitation energy is due to this uncertainty, then the single electron has energy  $E = \frac{3}{2}kT$  so:

$$\frac{3}{2}kT = \frac{\hbar a}{4c} \rightarrow T = \frac{\hbar a}{6ck}$$

Here we see that

$$T = \frac{\hbar a}{6ck} \approx \frac{\hbar a}{2\pi ck}$$

### 5.2 Doppler Shift Derivation

The next derivation is not quite so simple but is suitable for an advanced undergraduate. It is likely the second easiest derivation of Unruh’s effect to grasp. So let’s see if there is any indication of a thermal effect for acceleration. Allow me to pull forth the well-known hyperbolic trajectory resulting from uniform acceleration:

$$t(\tau) = \frac{c}{a} \sinh\left(\frac{a\tau}{c}\right) \quad z(\tau) = \frac{c^2}{a} \cosh\left(\frac{a\tau}{c}\right)$$

Now allow me to directly consider the standard nonaccelerated Minkowski plane wave

$$e^{i\phi_{\pm}} \equiv e^{i(Kz \pm \omega_K t)}$$

Understand that the phase,  $\phi_{\pm}(\tau)$ , is time-dependent. The accelerated observer will see waves that involve time-dependent Doppler shifts. Now just solving for the time-dependent phase, by plugging in the trajectory in:

$$\phi_{\pm}(\tau) = Kz(\tau) \pm \omega_K t(\tau) = \frac{\omega_K c}{a} e^{\pm \frac{a\tau}{c}}$$

with  $K = \omega_K/c$ . Let's allow the wave to propagate in the  $-z$  direction so our time-dependent phase is

$$\phi(\tau) = \frac{\omega_K c}{a} e^{-\frac{a\tau}{c}}$$

The observer will see a frequency spectrum,  $S(\Omega)$ , proportional to:

$$\left| \int_{-\infty}^{\infty} d\tau e^{i(\Omega\tau + \phi)} \right|^2$$

So let me pause for a moment and remind you what I'm looking for. The rest of this derivation will be plugging in the  $\phi$  into our frequency spectrum and seeing if there is a standard Planck factor  $\frac{1}{e^{\hbar\Omega/kT} - 1}$  that indicates a Bose-Einstein distribution for bosons with our desired temperature  $T = \frac{\hbar a}{2\pi ck}$ .

Moving forward, our spectrum is proportional to:

$$\left| \int_{-\infty}^{\infty} d\tau e^{i\Omega\tau} e^{i(\omega_K c/a) e^{-a\tau/c}} \right|^2$$

Changing variables to  $y = e^{a\tau/c}$  we obtain

$$A \equiv \int_{-\infty}^{\infty} d\tau e^{i\Omega\tau} e^{i(\omega_K c/a) e^{-a\tau/c}} = \frac{c}{a} \int_0^{\infty} dy y^{(i\Omega c/a - 1)} e^{i(\omega_K c/a)y}$$

The integral on the right hand side is:

$$A = \frac{c}{a} \Gamma\left(\frac{i\Omega c}{a}\right) \left(\frac{\omega_K c}{a}\right)^{-i\Omega c/a} e^{-\pi\Omega c/2a}$$

Obtaining this may be considered kind of tricky, but you can look up the solution in any standard integral book, by breaking the exponent up via Euler's relation,  $e^{iax} = \cos(ax) + i\sin(ax)$ . Use  $\int_0^{\infty} x^{u-1} \sin(ax) dx = [\Gamma(u)/a^u] \sin(u\pi/2)$  and  $\int_0^{\infty} x^{u-1} \cos(ax) dx = [\Gamma(u)/a^u] \cos(u\pi/2)$ . Then, using  $|\Gamma(i\Omega c/a)|^2 = \pi/[(\Omega c/a) \sinh(\pi\Omega c/a)]$  we have,

$$|A|^2 = \frac{2\pi c}{\Omega a} \frac{1}{e^{2\pi\Omega c/a} - 1}$$

Where the desired Planck factor shows up because of the time-dependent Doppler shift detected by the accelerated observer if

$$T \equiv \frac{\hbar a}{2\pi ck}$$

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