Acoustic resonances of a half-open cylindrical tube ([ə])

Linguistics 520, UNC-Chapel Hill
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Goal: To understand why the formants of [ə] have the values they do.

Take a length of plastic hose. Smack one end of it hard with the palm of your hand, and keep your hand there. A microphone at the other end will record something like this:

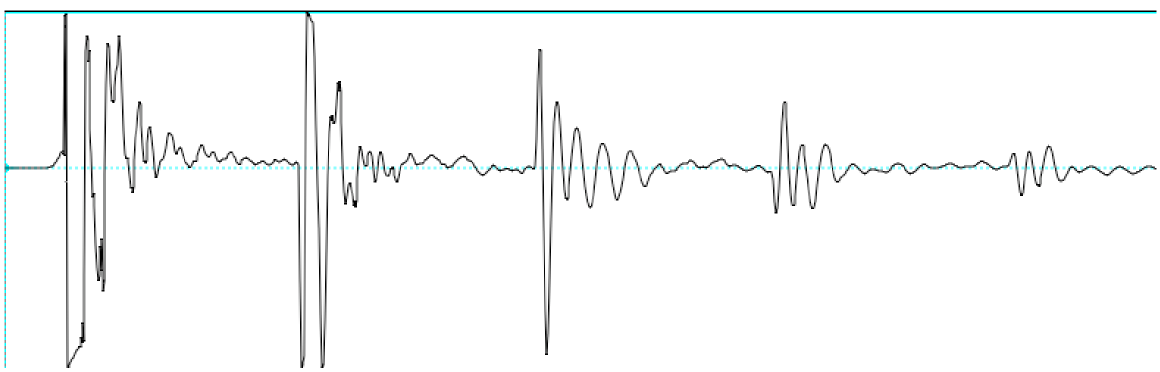

Figure 1: The smacked hose. Horizontal scale is 0.075 s.

Question 1: You can see a series of squiggles (let's call them "pulses") separated by silence. Comparing each pulse with its neighbors, how are they related?

Question 2: If the speed of sound is about $c \approx 350$m/s, how long is the hose?

What the microphone is picking up at the open end of the hose is a series of *inverted, attenuated echoes* of the original pulse as it bounces back and forth inside. The original pulse peeks out and disappears; then we see its echo, upside down and smaller; then the echo of the first echo, and so on. The echoes keep getting smaller because they lose energy travelling back and forth in the hose. Some of the energy is radiated away at the open end (which is why we can hear it), some is absorbed by the walls of the tube or the palm of the hand, etc.

It's harder to explain why the echoes get flipped. For this class, we will treat it as a fact from physics, and be content with having observed that it does happen. We should note that echo inversion is a property of half-open tubes only—it does not occur if both ends are open, or both closed. I have caricatured the process below, using an extremely simple pulse type:

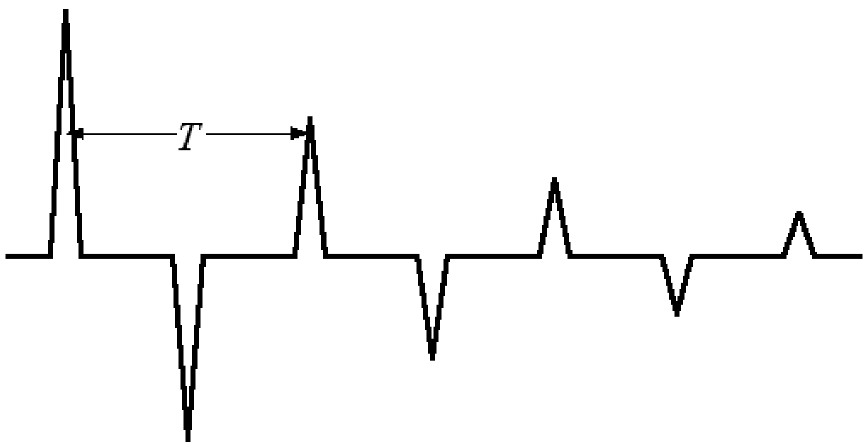

Figure 2: A single spike and its attenuated, inverted echoes in a half-open tube.

Question 3: Given the speed of sound c and the length of the tube L, what is T, the shortest time between two echoes in the same orientation?

(Note: I'm taking some liberties here. The spikes are standing in for sine waves, just because spikes are easier to keep track of in the picture. In real life, sine waves are attenuated and inverted by a half-open tube, just as we have been saying, but what happens to spikes is more complicated—notice how the pulses in Figure (1) do more than just shrink and flip. Ignore that complication for now; you'll be able to figure it out for yourself by the end of this period.)

Now suppose we have some way to put multiple spikes in at the closed end, one after another. The microphone sitting at the closed end will hear each original pulse and all of the echoes as they come along. For instance, if we put in two pulses, and wait just the right amount of time after the first before putting in the second, the microphone will pick up the *first echo* (softer and upside-down) of the first pulse at the same time as the *original* of the second pulse:

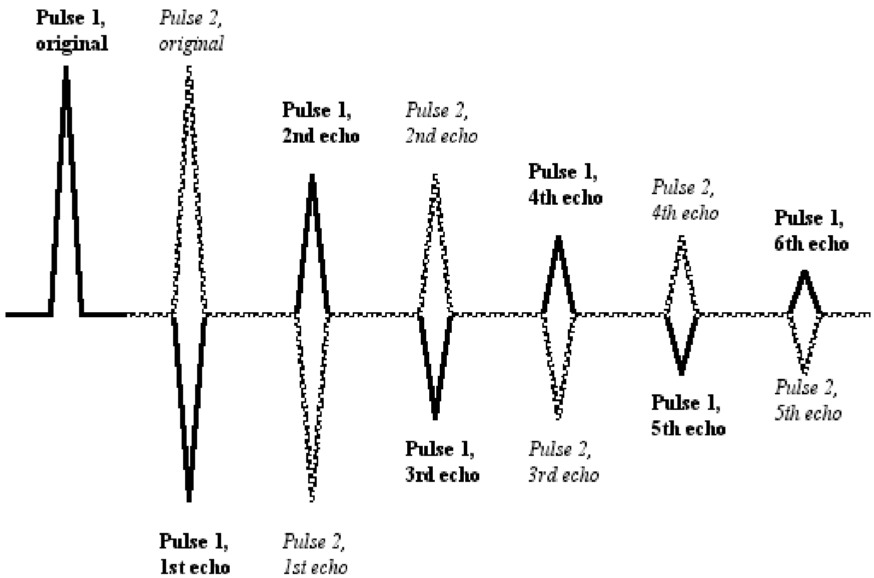

Figure 3: Two spikes timed so that their echoes cancel out.

Succeeding echoes cancel each other out: The $(n+1)$st echo of the second pulse gets clobbered by the nth echo of the first. Cancellation isn't complete, since the dueling echoes aren't perfect inverses (the echo of the second pulse is a bit fresher and less attenuated), but the microphone picks up a much quieter wave than it otherwise would. (Use your pencil to draw the combined wave onto Figure (3). This is our old high-school friend, Destructive Interference.

Question 4: To get this to happen, by how much time should the second pulse lag the first?

On the other hand, by choosing a different lag, the pulses can be aligned so that their echoes cooperate instead of conflicting:

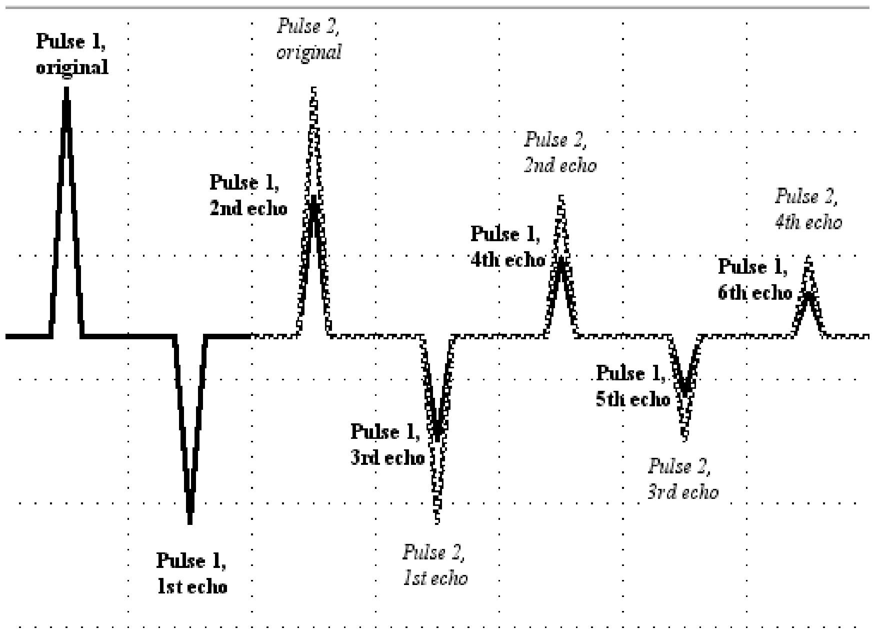

Figure 4: Two spikes timed so that their echoes reinforce each other.

Question 5: By how much time should the second pulse lag the first for the echoes to reinforce each other?

Suppose now that, instead of stopping after two pulses, we keep going, putting in pulse after pulse at regular intervals. The echoes would keep piling up on each other, and the microphone would register a very high-amplitude wave.

Question 6: What would the *frequency* of that pulse train be? I.e., how often would you have to insert a new pulse? How is it related to the tube length L and the speed of sound c? (This frequency is called the "first formant", or F_1.)

Question 7: Suppose that L, the length of the tube, were about 17.5 cm—like the simulated human vocal tract we were looking at last time. What would its F_1 be, if the speed of sound is $c = 350$ m/s? How does this relate to the experiment we did last time?

Higher-frequency pulse trains can also interfere constructively with their own echoes. This handout continues (see next page)....

Continuation of handout on resonances of schwa, LING 520 Fall 2011

		Period	Wavelength	Frequency
F3	___ ___ ___ ___ ___ ___ ___ ___ ___ _			
F2	___ ___ ___ ___ ___ ___ ___ ___ ___ _			
F1	___ ___ ___ ___ ___ ___ ___ ___ ___ _