Bels, decibels, and intensity

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Based on Moore (1989, Ch. 1–2).

1 Bels and decibels

Bels are a unit for *comparing* two quantities. Whenever you use them, you are (at least implicitly) saying that "Thing 1 is r Bels **as compared to** Thing 0". This means Thing 1 is 10^r times as big as Thing 0, or, equivalently, that (Thing 1)/(Thing 0) = 10^r, or, equivalently,

$$\log_{10}\left(\frac{Thing1}{Thing0}\right) = r$$

Example 1: Pretend there are 10,000 people in Carrboro, and 1,000,000 people in Boston. Then the population of Boston is 100 times that of Carrboro; i.e., 2 Bels above that of Carrboro.

Example 2: Suppose the population is growing at the rate of 5% per year. How fast is that in Bels per year?

Example 3: The Earth is one Astronomical Unit (AU) from the Sun. Mercury is 0.4 AU from the Sun, and Pluto is (usually about) 40 AU from the sun. What are the distances of Mercury, Earth, and Pluto from the Sun, expressed in Bels relative to the distance of the Earth?

A decibel (dB) is just a tenth of a Bel, so a 2-Bel difference is a 20-dB difference. If you double a quantity, you increase it by $\log_{10}(2/1) \approx 0.3$ Bels = 3 dB. If you halve it, how many dB does it change by?

2 Pressure and intensity

The Praat waveform display shows *pressure* as a function of time. The spectrogram display shows *intensity* as a function of frequency *and* time (where intensity is the rate at which the sound carries energy through an imaginary surface). For a sinusoid, the intensity is related to the peak pressure[1] P by the formula

[1] You may see people using "RMS" (root-mean-square) pressure instead of peak pressure. For sinusoids, RMS pressure = $P/\sqrt{2}$.

$$I = kP^2$$

The constant k doesn't matter to us, because for everything we're doing, it will be cancelled out by another k.

Say the intensity of Sound 1 is I_1, and that of Sound 0 is I_0. Then you would say that the intensity of Sound 1 is $\log(I_1/I_0)$ Bels relative to that of Sound 0. The peak *pressure amplitudes* of the two sounds would be P_1 and P_0, and so the intensity of Sound 1 relative to that of Sound 2 would be

$$\begin{aligned}
\log_{10}\left(\frac{kP_1^2}{kP_0^2}\right) \text{ Bels} &= \log_{10}\left(\left(\frac{P_1}{P_0}\right)^2\right) \text{ Bels} \\
&= 2\log_{10}\left(\frac{P_1}{P_0}\right) \text{ Bels} \\
&= 20\log_{10}\left(\frac{P_1}{P_0}\right) \text{ dB}
\end{aligned}$$

Example 4: If Sound 1 is $1/4$ as intense as Sound 0, what is the difference in intensities, expressed in dB? If both are sinusoids, what is their difference in peak pressures, expressed in dB?

Loudness, the psychological percept corresponding to physical intensity, does not scale directly with intensity or with amplitude. If Sound 1 is twice as loud as sound 0, the difference in intensities is about 10 dB. I.e., loudness goes up slower than intensity.

When people say that Sound 1 has an intensity of "60 dB", without specifying what Sound 0 is, they usually mean that Sound 0 is the standard reference intensity of $10^{-12}\text{W}/\text{m}^2$, which corresponds to a sinusoid with peak amplitude of $20\sqrt{2}\mu\text{Pa}$ (micropascals)—a sound barely audible at 1000 Hz. Intensities specified with respect to this standard are called "dB SPL", for "sound pressure level."

On the vertical scale of the Praat waveform display, 1 corresponds to a nominal pressure of 1 Pa. This is a fiction; how intense the sound will actually be when you play it depends on what kinds of speakers you have, how your volume control is set, how far away from the speaker you are when you measure the intensity, etc. A piece of software has no way to figure this out. All Praat can really do is show you the differences between the intensities of two sounds (which is usually what you want anyway).

Example 5: Assuming that the Praat waveform display is correct, what would be the peak amplitude of a sinusoid with an intensity of 0 dB SPL? If you ask Praat to show you the "intensity contour" of your sound, you can see its intensity expressed in dB relative to the intensity of a sound with that peak amplitude.

References

Moore, B. C. J. (1989). *An introduction to the psychology of hearing* (3rd ed.). San Diego: Academic Press.