Modeling of Debris Deposition in an Extrusion Filter Medium

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Abstract

The goal of this work is to predict reasonable lifetime of a filter used to remove debris (e.g. foreign particles and gels) from the melt stream of an extrusion process. We are developing models which incorporate non-Newtonian porous media flow through a medium whose porosity changes as debris accumulates. Boundary conditions are based on the assumption of constant flow rate and coupling with other process stages. Governing equations consist of a mass balance equation for flow of the suspension coupled with a Darcy velocity, the non-Newtonian constitutive equation, and equations for modeling particle transport and deposition. The model is being developed in a manner which allows for generalization to various domains in higher dimensions and more complex constitutive models. One-dimensional Newtonian and non-Newtonian flow models will be presented and compared to one another. Plans for continuing work will also be discussed.

1 Introduction

At the Center for Advanced Engineering Fibers and Films (CAEFF), we are interested in simulating a fiber spinning process from the polymer melt to the finished product. One of the components of the process is the extrusion filter, which separates debris particles from the polymer. Properties of the fiber may be severely compromised if these particles remain in the polymer during spinning.

The filter is often composed of a sintered metal, compressed with sufficient force to produce a cake material, or layers of wired mesh, with mesh spacings small enough to trap particles a few microns in diameter. As debris accumulates in the filter, the pressure drop across the filter must be increased in order to maintain a constant mass flow rate. The primary goal in filter management is to predict when the pressure drop across the filter becomes so large that it could damage the pumping mechanism.

Replacing the filter requires that the entire spinline be taken out of service. While in-line filter replacement processes are being introduced, many industrial facilities do not yet use this technology. A predictive model will allow companies to better manage filter replacement. In addition, an accurate filter simulation tool will enable companies that design and fabricate filters to optimize their products.

Flow of non-Newtonian fluids in porous media has been considered for the one-dimensional case in [1, 2, 3]. A three-dimensional simulation of non-Newtonian flow through porous media is presented in [4]. Analytical results for non-Newtonian flows through a porous medium, where the domain consists of periodic repetitions of an elementary cell of unit size, are developed in [5, 6, 7]. We refer the reader to Pearson and Tardy [8] for a summary of continuum models that have been developed for non-Newtonian flows in porous media.

Deposition of particles inside a filter medium has been studied previously in the literature, and much of the early work assumed a Newtonian transport fluid. In [9, 10, 11], the authors analyze the flow of particles suspended in a Newtonian fluid, and develop models for the mechanisms of capture and retention of particles. Herzig, Leclerc, and Le Goff [9] were among the first to derive differential equations for the filtration process, and they analyzed a variety of capture and retention mechanisms for the particles. Tien, Turian, and Pendse in [10] considered the amount and morphology of the particle deposition, but again for Newtonian fluids.

Debris transport and deposition in higher-dimensional Newtonian porous media is presented in [12]. The 1D non-Newtonian filtration problem was the subject of [13, 14]. This work focused on cake filtration under constant pressure. One of the few efforts of which we are aware to model debris deposition through a non-Newtonian porous medium in three dimensions is the work in

[15], which was based on a finite difference solution of the conservation equations in integral form.

In this paper, we consider the debris filtration stage of a polymer process line, focusing on non-Newtonian flow through the filter along with debris deposition. Our goal is to use the finite element method to solve the governing equations which characterize fluid flow and debris deposition in a filter medium. Prototype 1D Newtonian and non-Newtonian models are developed, in order to provide intuitive insight and serve as benchmarks for more complex simulations. Governing equations are developed with the intent of generalizing to higher dimensions with flexibility in characterization of the medium, the debris, and the fluid.

The remainder of this paper is outlined as follows. In §2, we briefly describe debris filtration as a stage in the fiber spinning process. In §3, we write the model equations, and we present numerical results in §4. We provide concluding remarks and directions for future work in §5.

2 Filtration in the Fiber Melt-Spinning Process

A schematic of the fiber melt-spinning process is given in Figure 1. The metering pump is used to maintain a constant inflow rate; large pressure drops due to debris loading can adversely affect the pump. In addition, debris particles must be removed before the polymer reaches the spinneret. After that point, any remaining debris particles will be spun into the fiber, leading to degradation in fiber properties.

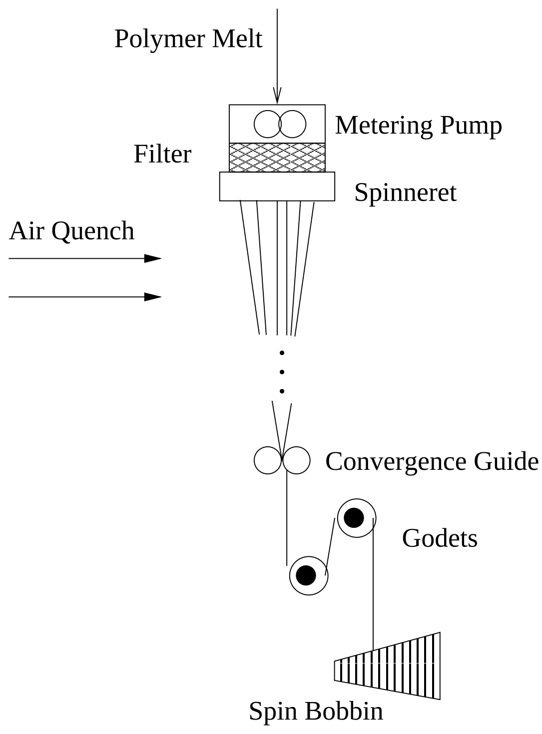

Figure 1: Fiber melt-spinning process

A complete filtration model includes characterizations of the debris in the melt and the filter bed. Example descriptions of debris and filter beds are given in [16] and [17].

3 Model Equations

The model for the extrusion filter is a coupled system of ordinary and partial differential equations. We use a Darcy velocity, corrected for non-Newtonian effects, for the flow of the slurry, and we use a standard transport equation to model the particle deposition. The system is completed by using a standard mass balance and a permeability relationship that is a function of the changing porosity.

3.1 Equations for suspension flow

The Darcy velocity $\boldsymbol{u}$ for a Newtonian fluid is given in [8] as

$$\boldsymbol{u} = -\frac{k}{\mu}(\nabla p - \rho \mathbf{g}).$$

Here, k is the intrinsic permeability, μ is the fluid viscosity, p is the fluid pressure, and $\mathbf{g}$ is gravitational force.

Following the characterization of generalized Newtonian fluids given in [18], the viscosity μ is a function of the strain rate $|\mathbf{E}| = \sqrt{\frac{1}{2}\sum_{i,j} E_{ij}E_{ij}}$, where $\mathbf{E}$ is the rate-of-strain tensor

$$\mathbf{E} = \left(\nabla \mathbf{v} + \nabla \mathbf{v}^T\right).$$

The velocity $\mathbf{v}$ above is the pore-scale velocity, so that $|\mathbf{E}|$ must be adjusted when using a Darcy velocity. In particular, Pearson and Tardy [8] suggest that $|\mathbf{E}|$ (denoted E subsequently) is proportional to $\frac{1}{\sqrt{k}}|\boldsymbol{u}|$ using a dimensionality argument.

Therefore, for a generalized, non-Newtonian, power-law fluid, with characteristic viscosity μ_0 and characteristic strain rate E_0, we have

$$\mu_{GN} = \mu_0 \left(\frac{E}{E_0}\right)^{n-1},$$

where the power-law index n is normally taken between 0 and 1. Incorporating this into the Darcy relationship gives

$$\begin{aligned} \boldsymbol{u} &= -\frac{k}{\mu_{GN}}\nabla p \\ &= -\frac{k}{\mu_0}\left(\frac{E}{E_0}\right)^{1-n}\nabla p, \end{aligned}$$

assuming that the vertical dimension of the filter is small enough to allow us to ignore any gravitational effects.

The governing equations will be developed in a manner which allows other constitutive models to be adopted, such as the Carreau-law constitutive model:

$$\mu_C = \mu_0 \left(1 + \delta\,(E)^2\right)^{\frac{n-1}{2}},$$

where μ_0 is the zero-shear-rate viscosity, δ is a time constant, and n is a power-law exponent, between 0 and 1. This model is considered more realistic than the power-law model because it exhibits Newtonian-like constant viscosity behavior at low shear rates, [18].

The permeability parameter k is currently modeled using a Blaze-Kozeny relationship that depends on the average diameter of the filter pore size, d_p, and the current filter porosity η. The filter porosity is a dimensionless quantity that measures the volume of the void space to the total volume of the medium. The relationship for k, whose derivation is provided in [19], is

$$k = \frac{d_p^2 \eta^3}{150(1-\eta)^2}.$$

More recent work suggests that the Blaze-Kozeny relationship can be improved [20]. Specifically, more accurate macroscale representations for permeability are required for Newtonian fluid flow, especially under critical values for the Reynolds number. We do not address the validation of permeability representations in this work, but we realize that this issue should be considered. In addition, Pearson and Tardy in [8] suggest that the geometry of the porous medium plays a significant role in the simulation of non-Newtonian fluids, but these considerations are beyond the scope of the present work.

3.2 Particle transport and deposition

The model equation for the transport of the debris particles in the fluid, given in [12], is:

$$\frac{\partial}{\partial t}(\eta C(s) + \sigma) = \nabla \cdot (\boldsymbol{D}\nabla C(s)) - \nabla \cdot (\boldsymbol{u}C(s)) \tag{1}$$

where $\boldsymbol{D}$ is the hydrodynamic dispersion tensor and σ is the volume fraction of particles retained by the filter. The concentration of debris particles in the fluid is denoted by $C(s)$, where s is the particle diameter. Thus $C(s)$ is the volume of particles in the slurry with respect to the volume of the slurry. The particles are transported via advection and diffusion mechanisms acting within the filter volume.

Expanding the left side of (1) gives

$$\eta\frac{\partial}{\partial t}C(s) + C(s)\frac{\partial}{\partial t}\eta + \frac{\partial}{\partial t}\sigma = \nabla \cdot (\boldsymbol{D}\nabla C(s)) - \nabla \cdot (\boldsymbol{u}C(s)). \tag{2}$$

We account for changes in porosity η by noting that, relative to the initial porosity η_0, $\eta(\mathbf{x}, t)$ can be defined as in [12] by

$$\eta(\mathbf{x}, t) = \eta_0(\mathbf{x}) - \beta\sigma(\mathbf{x}, t), \tag{3}$$

where β is the reciprocal of the compaction factor, and $1 \leq \beta \leq 1 + \epsilon$. The value of β accounts for any liquid that is trapped due to particle deposition and thus acts as an additional clogging mechanism. Therefore, from (3) we have

$$\frac{\partial}{\partial t}\eta = -\beta\frac{\partial}{\partial t}\sigma. \tag{4}$$

The volume fraction of retained particles changes based on filter parameters and suspension velocities as

$$\frac{\partial}{\partial t}\sigma = \lambda|\boldsymbol{u}|C(s)F(\sigma), \tag{5}$$

where λ is the filter coefficient and $F(\sigma)$ is the formation factor [12]. In deep bed filtration, which is defined in [10] as an "engineering operation in which the removal of suspended particulate ... is effected by passing the stream through porous media composed of granular substances," $F(\sigma) = 1$, [12]. Thus from (2), (4), and (5) we have

$$\begin{aligned}
\eta\frac{\partial}{\partial t}C(s) &= -C(s)\frac{\partial\eta}{\partial t} - \frac{\partial}{\partial t}\sigma + \nabla \cdot (\boldsymbol{D}\nabla C(s)) - \nabla \cdot (\boldsymbol{u}C(s)) \\
&= (\beta C(s) - 1)\frac{\partial}{\partial t}\sigma + \nabla \cdot (\boldsymbol{D}\nabla C(s)) - \nabla \cdot (\boldsymbol{u}C(s)) \\
&= (\beta C(s) - 1)(\lambda|\boldsymbol{u}|C(s)) + \nabla \cdot (\boldsymbol{D}\nabla C(s)) - \nabla \cdot (\boldsymbol{u}C(s))
\end{aligned}$$

We use the example of the filter coefficient λ given in [10]

$$\lambda = \frac{1}{l}\ln\frac{1}{1-f},$$

where l is the filter length and f is the fraction of particles that enter the filter and are removed.

3.3 Divergence equation

We use as the mass balance equation for porous media a variation on that found in [12]

$$\frac{\partial}{\partial t}(\eta\rho S) + \nabla \cdot \rho\boldsymbol{u} = q, \tag{6}$$

where ρ is the density of the suspension fluid, S is the level of saturation, and q (the variation from the equation in [12]) is a generic source/sink term. The saturation S measures the amount of the void space occupied by the fluid, or the ratio of the fluid volume to the volume of the voids. We assume that our medium is always fully saturated, and thus $S = 1$.

Equation (6) leads to the standard divergence-free condition on $\mathbf{u}$. Assuming an incompressible fluid (so that $\rho = 1$) and full saturation of the medium, the relationship simplifies to

$$\frac{\partial}{\partial t}\eta + \nabla \cdot \boldsymbol{u} = q. \tag{7}$$

If $\boldsymbol{u}$ is the Darcy velocity of the slurry, which includes the debris particles along with the suspension fluid, then q must account for the volume of slurry lost due to filtration. In fact, using the definition of η along with equations (4) and (5), it follows that

$$q = -\beta\lambda|\boldsymbol{u}|C(s),$$

which is consistent with (7), (4) and (5) if

$$\nabla \cdot \boldsymbol{u} = 0.$$

3.4 Equation summary

In summary, the model equations are given as

$$\begin{aligned}
\boldsymbol{u} &= -\frac{k}{\tilde{\mu}}\nabla p && (8)\\
\nabla \cdot \boldsymbol{u} &= 0 && (9)\\
\frac{\partial \eta}{\partial t} &= -\beta\lambda\,|\boldsymbol{u}|\,C(s) && (10)\\
\frac{\partial}{\partial t}\left(\eta C(s) + \sigma\right) &= \nabla \cdot \left(\boldsymbol{D}\nabla C(s)\right) - \nabla\left(\boldsymbol{u}C(s)\right), && (11)
\end{aligned}$$

where $\tilde{\mu}(\tilde{E})$ is the effective viscosity, and the reduced shear rate $\tilde{E}$, by definition, is proportional to $|\mathbf{u}|\, k^{-1/2}$, [8].

4 Numerical Results

Numerical results were obtained for both Newtonian and non-Newtonian one-dimensional, prototype models. The motivation for this work was to establish benchmark results for higher dimensional models, as well as to match physical intuition with the assumed model equations.

In obtaining the numerical results, we make the following assumptions on the particles and fluid:

1. particles are captured only through direct interception with the filter pore spaces, and they are retained due to the fluid pressure [9];
2. only constrictive retention sites are considered [9];
3. once particles are trapped in the filter, they are not released back into the slurry;
4. particles do not partially block any filter pore space; i.e, the pore space is either completely open or completely blocked;
5. the amount of dead liquid in the filter, i.e., liquid that is trapped by particle deposition and effectively acts as a blocking mechanism in the filter, is negligible;
6. the density of the particles is negligible when compared with the density of the slurry;
7. the particles do not react chemically with the filter;
8. Darcy's law is a valid approximation for the velocity of the slurry;
9. dispersion of the fluid is assumed to be negligible

In addition, since we assume a constant mass flow rate, it follows from (9) that the 1-D velocity, U is constant. The simulation was terminated when clogging occured at the inlet, i.e. once $\eta = 0$.

4.1 Newtonian flow

For the one-dimensional, Newtonian flow, we used the governing equations

$$\begin{aligned} \eta \frac{\partial C}{\partial t} &= (\beta C - 1)\,\lambda U C - U \frac{\partial C}{\partial z} \\ \frac{\partial \eta}{\partial t} &= -\beta \lambda U C, \end{aligned}$$

with boundary and initial conditions

$$\begin{aligned} C(0,t) &= C_0, \quad t > 0 \\ C(z,0) &= 0, \quad \eta(z,0) = \eta_0, \quad 0 < z \leq L. \end{aligned}$$

The equations were nondimensionalized using

$$t = \lambda U t_{physical}, \quad z = \lambda z_{physical},$$

which results in modified equations

$$\begin{aligned} \eta \frac{\partial C}{\partial t} &= (\beta C - 1)\,C - \frac{\partial C}{\partial z} \\ \frac{\partial \eta}{\partial t} &= -\beta C. \end{aligned}$$

Since the filter clogs at the inflow region when $\eta(0,t) = 0$, we can find t by resolving the ordinary differential equation

$$\frac{d\eta(0,t)}{dt} = -\beta C(0,t),$$

which gives $\eta(0,t) = -\beta C(0,t)t + \eta(0,0)$. Thus, $\eta(0,t) = 0$ when $t = \dfrac{\eta_0}{C_0}$.

We linearize the coupled partial differential equations using the iteration scheme

$$\begin{aligned} \eta^{(i-1)} \frac{\partial C^{(i)}}{\partial t} &= \left(\beta C^{(i-1)} - 1\right) C^{(i)} - \frac{\partial C^{(i)}}{\partial z} \\ \frac{\partial \eta^{(i)}}{\partial t} &= -\beta C^{(i)}, \end{aligned}$$

and discretize using continuous piecewise linear finite elements. The initial parameters for the simulation were $C_0 = 0.1$, $\eta_0 = 0.5$, $\beta = 1$, and $0 \leq z \leq 7$. The results are given in Figure 2. The plots show the volume fraction of particles in the filter versus filter depth, the porosity of the filter versus filter depth, and the pressure as a function of filter depth, all at different time slices. The last plot in the figure shows the (absolute) pressure drop as a function of time. The simulation results indicate that the model equations agree with physical intuition. For our simple, one-dimensional case, we see that the porosity is lowest at $z = 0$, where primary clogging occurs, and that as we progress through the filter, the volume fraction of retained particles decreases. The pressure drop increases monotonically as a function of time, and increases dramatically as $\eta(0,t)$ approaches 0.

4.2 Non-Newtonian flow

The Newtonian one-dimensional model was adjusted to account for a power-law fluid. As mentioned earlier, the viscosity $\tilde{\mu}$ is a function of the reduced shear rate $\tilde{E}$, which is proportional to $|\mathbf{u}|\,k^{-1/2}$. Thus, we have

$$u^n \sim \left[-\left(k\right)^{\frac{n+1}{2}} \frac{\partial p}{\partial z}\right],$$

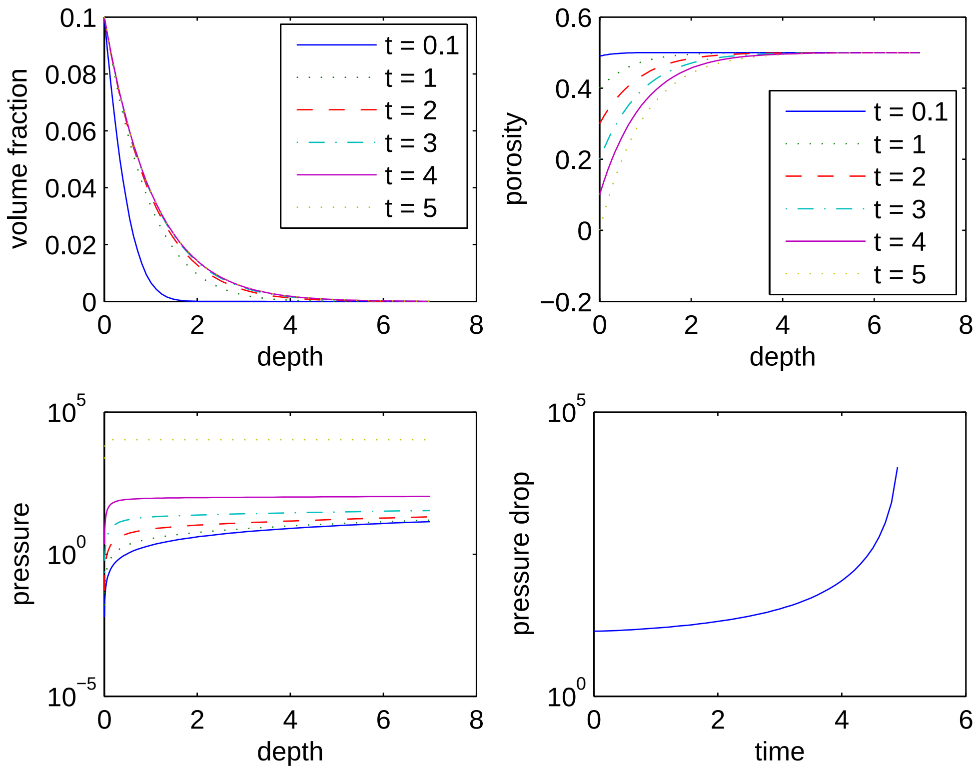

Figure 2: Results from one-dimensional prototype, Newtonian flow

which, when using the Darcy relationship, results in a normalized one-dimensional equation for the pressure p

$$\frac{\partial p}{\partial z} \sim \left[- \left(\frac{(1-\eta)^2}{\eta^3} \right)^{\frac{n+1}{2}} \right].$$

Plots of the pressure drop as a function of time, and varying choices of the power-law exponent n, are given in Figure 3.

5 Conclusions

We have presented a set of model equations that described the mechanisms of debris deposition in a filter medium. We have adjusted the equation for Newtonian fluids to account for non-Newtonian effects, and we have validated these equations using one-dimensional prototype models.

Further work on this problem will include the incorporation of statistical models for the sizes of the debris particles, coupled with experimental retention information for a variety of filters. This information will give a more dynamical model for λ, the filter coefficient, and allow for use of more physical realistic parameters.

Full, three-dimensional simulations will be performed, so that the model may be included in the viscoelatic flow code being developed at CAEFF. The incorporation of the filtration model will require that we couple the Darcy flow equation with non-Newtonian, Navier-Stokes equations, as in [5, 7], or fully viscoelastic flow equations. Work on coupling Darcy and Stokes flows is an active area of research for Newtonian fluids, which we believe will give insight into our problem.

We are encouraged by the numerical simulations completed thus far, and we plan to use efficient solution algorithms in the context of filter optimization. Thus, the model equations will not

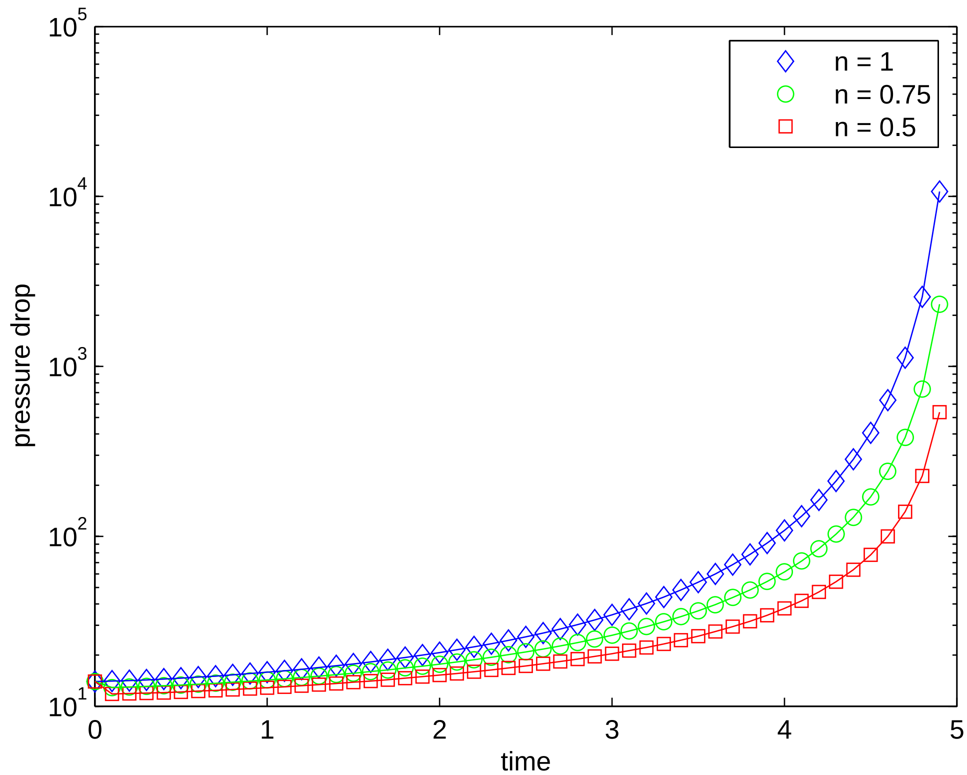

Figure 3: Result from one-dimensional prototype, power-law fluid

only be a useful component in a simulation of an entire fiber-spinning process, but they may also be used to help industrial partners select or design appropriate filters for their use.

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