1 Quantative Skills Refresher

If you have problems with basic algebra skills, you will find it difficult to work through quantative economic problems and to manipulate the many lines and curves used to illustrate economic concepts. A good grounding in mathematics will allow you to concentrate on the economics rather than on the algebra. This note will help you refresh your quantative skills.

1.1 Function Notation

If consumption is related to price in a linear fashion, one can write:

$$\text{Sales} = 20 - 2 * \text{Price}$$

Other times one may not have the explicit equation and can only write the general functional form: Sales = f(Price), where $f()$ denotes that there is a functional relationship. A *function* is denoted:$y = f(x)$, and is read 'y equals f of x.' A function is a rule or method for assigning a unique value of y for each x. We say that y is the dependent variable, which is determined by the independent variable x.

1.2 Linear Functions

The *slope* of the a function is the rate of change in the dependent variable as the independent variable changes. One can express the slope as $\frac{dy}{dx}$, where $dy = y2 - y1$ and $dx = x2 - x1$. It can be read as 'the change in y caused by a change in x.'

For example, the price for good X increases from 60 to 70. Over the same interval, sales falls from 100 to 95. Thus,

$$\frac{dy}{dx} = \frac{\text{Sales}}{\text{Price}} = \frac{95 - 100}{70 - 60} = \frac{-5}{10} = -0.5$$

-0.5 indicates on the rate of change in the sales if the price goes down from 60 and 70.

Be aware that the slope of the linear function is constant.

In general, any linear function can be expressed as $y = a + bx$, where a and b are fixed numbers. The a is the intercept (the point where the line crosses the y-axis). The value of a can be found if one sets $x = 0$ and solve for a. The term b is defined as the slope. One can solve for b if sets $y = 0$ and solve for b.

Example 1.

$$y = 30 + b * x$$

1. Find the intercept of the given function.

2. Find the slope of the given function at $x = 10$ and $y = 20$.

Solution 1.

1. $y = 30 + b * 0 = 30$. The intercept is equal to 30.
2. $20 = 30 + b * 10$
   $b = -1$. The slope of the given function is -1.

Example 2.

Suppose one is given two points on a line and asked to find that line's equation. The points are $(x_1, y_1) = (2, 46)$ and $(x_2, y_2) = (15, 20)$

Solution 2.
You know that any line's equation can be given as follows:

$$y = a + bx$$

1. Find the slope of the line.
   $\frac{dy}{dx} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{20-46}{15-2} = \frac{-26}{13} = -2$
2. Find the intercept of the line.
   $y = a - 2x$, use any of the given points. Lets use the first point
   $46 = a - 2 * 2$
   $a = 50$
   Check the obtained result using the second point
   $20 = a - 2 * 15$
   $a = 50$

The line's equation is $y = 50 - 2x$.

1.3 Solving system of equations

In order to have a unique solution the number of equation must be equal to the number of unknowns.

Case 1. One equation and one unknown.

$$38 = 50 + 3x$$

Solution
$38 - 50 = 3x$
$-12 = 3x$
$-12 * \frac{1}{3} = 3 * \frac{1}{3}x$
$-4 = x$
The answer is $x = -4$

Case 2. Two equations and two unknowns.

$$\begin{cases} y = 50 + 3x \\ y = 30 - 2x \end{cases}$$

Solution

Solve for x:
$50 + 3x = 30 - 2x$
$20 = -5x$
$-4 = x$

Substitute x in any of the equations and solve for y.

$y = 50 + 3 * (-4)$
$y = 50 - 12$
$y = 38$
.

The answer is $(x, y) = (-4, 38)$

Case 3. Three equations and three unknowns.

$$\begin{cases} y = 50 + 3x + 2z \\ y = 30 - 2x + z \\ z = -3x + 2y \end{cases}$$

Substitute z in the third equation into the z in the first and the second equations.

$$\begin{cases} y = 50 + 3x - 6x + 4y \\ y = 30 - 2x - 3x + 2y \end{cases}$$

After simplification we get:

$$\begin{cases} y = \frac{50-3x}{-3} \\ y = 5x - 30 \end{cases}$$

Solve for x
$\frac{50-3x}{-3} = 5x - 30$
$50 - 3x = -15x + 90$
$15x - 3x = 90 - 50$
$12x = 40$
$x = \frac{40}{12} = \frac{10}{3}$

Substitute x in any of the equations and solve for y.
$y = 5 * \frac{10}{3} - 30 = \frac{50}{3} - 30 = \frac{-40}{3}$

Substitute x and y into the third equation and solve for z
$z = -3 * \frac{10}{3} + 2\frac{-40}{3} = -\frac{30}{3} - \frac{80}{3} = -\frac{110}{3}$

2 Exponent rules

If $a > 0$, $b > 0$, x, y, and n Real Numbers

1. $a^x = a^y \longleftrightarrow x = y$
2. $a^x a^y = a^{x+y}$
3. $\frac{a^x}{a^y} = a^{x-y}$
4. $a^{-x} = \frac{1}{a^x}$
5. $(a^x)^y = a^{xy}$
6. $a^x b^x = (ab)^x$
7. $a^{\frac{x}{y}} = \sqrt[y]{a^x}$
8. $a^0 = 1$

3 Approaching economic problems

Most economic problems can be approached in three different ways:

1. *Intuitive approach.* As you progress, your economic intuition will develop. However, intuition does not provide the exact answer. For example, intuitively you know that P and Q have the negative relationship in the demand schedule, but the intuitive approach does not allow to find the magnitude of the slope. Use this approach in case of solving essay or multiple choice problems.
2. *Graphical approach.* This approach captures the essence of the problem. I recommend to use this approach for all problems. Especially in case of solving problems where you need to find the exact answer, the graphical approach helps determine the set of conditions.
3. *Mathematical approach.* Use this approach for any problem, which requires any computation.

In order to be successfull in this course you have to master all these three approaches. The right solutions for the majority of questions from exams and homework can be easier found if you use all these approaches simulteneously.