

## STOR 435, Sample Questions, Midterm 1

**Problem 1.** A probability space contains two events  $A$  and  $B$  such that  $P(A) = .20$ ,  $P(B) = .40$ , and  $P(A \cap B) = .10$ . Find the following.

- a.  $P(A^c)$
- b.  $P(A \cup B)$
- c.  $P(A^c \cup B^c)$

If  $C$  is a third event such that  $P(C) = .30$  and  $C \cap A = \emptyset$ , find  $P(C \cup (A \cap B))$ .

- d. Are  $A$  and  $B$  independent?

**Problem 2.** Carefully list the three components of the probability model for a random experiment.

**Problem 3.** Consider an experiment in which successive Bernoulli trials are performed and that  $X$  is the number of trials until the first success. Assume that for each trial  $P(S) = p$  and  $P(F) = q$ .

Carefully derive the distribution of  $X$  and verify that  $\sum_x P(X = x) = 1$ .

**Problem 4.** An urn contains 10 physically identical balls, numbered  $1, 2, \dots, 10$ . Consider the following experiments, in which balls are sampled from the urn.

A. If we draw five balls at random from the urn, *replacing the ball after each draw*, what is the probability that we see 3 even numbers.

B. Suppose that we select a random subset of five balls from the urn. Find the probability of the following events.

- i. Every number in the sample is greater than 3.
- ii. The sample contains at least one number greater than or equal to 8.

**Problem 4. Problem (20 points):** A executive purchases a single golf ball on six consecutive weekends for use in her home office. Assume that each golf ball is faulty with probability .40, and that the condition of each ball is independent of the others.

a. For which value of  $k$  is  $\Pr(\text{the executive encounters } k \text{ faulty balls})$  the largest?

Answer:\_\_\_\_\_.

b. What is the probability that the executive ends up with 3 faulty golf balls in all.

Answer:\_\_\_\_\_.

c. Given that she purchases 2 faulty golf balls in the first two weeks, what is the probability that she purchases 4 faulty golf balls in all?

Answer:\_\_\_\_\_.

**Problem 5:** A toy manufacturer makes red and blue marbles, which are sold in packages of five. Suppose that the color composition of the packages is distributed as follows.

50% of packages have all blue marbles

30% of packages have 3 blue and 2 red marbles

20% of packages have 1 blue and 4 red marbles

Suppose that you choose a package of marbles at random from the manufacturer's warehouse. Unaware of the contents of the package, you select one of the five marbles at random.

a. What is the probability that you select a blue marble?

Answer:\_\_\_\_\_.

b. Given that you selected a blue marble, what is the probability that all the other marbles in the package are blue?

Answer:\_\_\_\_\_.