Summary to <u>Hw2</u>

1 (a) The correct transformation of Y is given by:

$$Y = \pm\sqrt{(1-W^2)Z}$$

However, some students only considered either $Y = +\sqrt{(1-W^2)Z}$ or $Y = -\sqrt{(1-W^2)Z}$ and didn't consider both cases.

(4)
- Some students got $\mathrm{corr}(Y, Z) = 1$ which is not correct.
- Some students got $\mathrm{Cov}(Y, Z) = 0$ which is not correct.

(6)
- To solve this problem, we need to use

$$n! = n^{n+\frac{1}{2}}e^{-n}\sqrt{2\pi}e^{\alpha_n} \qquad \text{where } \frac{1}{12n+1} < \alpha_n < \frac{1}{12n}$$

However, some students only used the following form of Stirling's formula

$$n! \approx n^{n+\frac{1}{2}}e^{-n}\sqrt{2\pi}$$

which can not derive the inequality.

- The correct inequality used in the proof is:

$$e^{\alpha_n} - e^{\alpha_m} - e^{\alpha_{n-m}} < e^{\frac{1}{12n}} - e^{\frac{1}{12m+1}} - e^{\frac{1}{12(n-m)+1}}$$

Some students got

$$e^{\alpha_n} - e^{\alpha_m} - e^{\alpha_{n-m}} < e^{\frac{1}{12n}} - e^{\frac{1}{12m}} - e^{\frac{1}{12(n-m)}}$$

which is not correct

(9) The assumptions are Y is independent of X, Y and X have same distributions. Some students only assumed independence and forgot the identical distribution part.

(10) We need to define random variable X as:

$$X = \begin{cases} a & \text{w.p. } 1/2 \\ b & \text{w.p. } 1/2 \end{cases}$$

Some students didn't explicitly define this random variable but implicitly used this r.v. in the proof.

(11) When they used Holder's inequality, some students didn't define random variables X and Y appropriately. They define X and Y as following:

$$\begin{aligned} X &= (t^{x-1}e^{-t})^{\alpha} \\ Y &= (t^{y-1}e^{-t})^{(1-\alpha)} \end{aligned}$$

and got

$$\mathbb{E}XY = \int_0^{+\infty} (t^{x-1}e^{-t})^{\alpha}(t^{y-1}e^{-t})^{1-\alpha}dt$$

which is not correct since there is no density in the integrand.

(13)
- Some students assumed Λ to be finite which is incorrect.
- Some students claimed that $\exists\lambda_0$, s.t. $f_{\lambda_0} = f$ which is incorrect.