Statistics 654 Homework 3

1. Let $\text{Bin}(n, p)$ denote the binomial distribution with parameters $n \geq 1$ and $p \in [0, 1]$. Show that for each $1 \leq k \leq n$ and each $p \in [0, 1]$ that the following identity holds:

$$P(\text{Bin}(n, p) \geq k) \;=\; \frac{n!}{(k-1)!(n-k)!} \int_0^p u^{k-1}(1-u)^{n-k} du$$

Hint: Fix $1 \leq k \leq n$. Let $f(p)$ and $g(p)$ be, respectively, the left- and right-hand sides of the equation. Show that f, g are equal when $p = 0$. Then show that $f'(p) = g'(p)$ for each p. To do this, write $f(p)$ as a sum, differentiate each summand, and then note that terms in successive summands cancel.

2. Following the arguments in class, show that for a canonical exponential family generated by T, h with normalizing function $A(\eta)$, we have $\nabla^2 A(\eta) = \text{Var}_\eta(T(X))$.