Statistics 654: Midterm Practice Problems

1. Let $X \geq$ be a random variable. Provided that all the expectations exist, show that

$$(EX^{1/3})^2\,(EX^{2/3})^2 \;\leq\; (EX)^2 \;\leq\; EX^{2/3}\,EX^{4/3}.$$

2. Let $X \sim \chi^2_5$ be a chi-squared random variable with 5 degrees of freedom. Find the numerical value of EX^3.

3. Let (X, Y) be jointly distributed discrete random variables.

a. Write the formula for the joint entropy $H(X, Y)$ in terms of $p(x, y)$.

Now suppose that X, Y are independent.

b. Show that $H(X, Y) = H(X) + H(Y)$.

c. Show that $H(X|Y) = H(X)$.

4. Suppose that $f, g : \mathbb{R} \to \mathbb{R}$ are convex, and that g is also increasing. Show that the composition $h(x) = g(f(x))$ is also convex.

5. Let X be a random variable with an invertible CDF F. Find the distribution of $F(X)$.

6. Show carefully that $1 + x \leq e^x$ for all $x \in \mathbb{R}$.

7. State Hoeffding's inequality for $X_1, \ldots, X_n$ i.i.d. with $EX_i = 0$ and $a \leq X_i \leq b$. (You don't need to prove anything.)

8. Let X, Y be jointly distributed discrete random variables.

a. Define $E(Y \mid X = x)$ and $E(Y \mid X)$

b. Show that $E[E(Y \mid X)] = EY$

9. Let X, Y be independent random variables with $Y > 0$. What can you say about the relation between $E(X/Y)$ and EX/EY?