

Homework 1

Here is the first homework installment. Try to provide organized answers, where you are careful to explain what you are doing! In order to be able to allocate partial credit on exams, it is crucial that the logic is clear. It is therefore important that you try to get into the good habit of explaining your steps when solving the homework problems.

1. Given prices (p_1, p_2) and income m we have that the set of affordable consumption bundles consists of all (x_1, x_2) such that $p_1x_1 + p_2x_2 \leq m$ and $x_1 \geq 0$ and $x_2 \geq 0$.
 1. For $p_1 = 3, p_2 = 6$ and $m = 12$, draw this set carefully. Be explicit about intercepts with the x_1 and x_2 -axis.
 2. Show what happens to the budget set if the price on good x_1 increases to $p_1 = 4$. Also (for $(p_1, p_2) = (4, 6)$) show what happens if m changes.
 3. Now compare the budget set in part a. with the one for $(p_1, p_2, m) = (6, 12, 24)$. Interpret!
 4. Argue that, in general, the budget given prices and income (p_1, p_2, m) is the same for that given prices and income (tp_{x_1}, tp_{x_2}, tm) for any positive t .
2. A professor is allocated a research budget of \$2000, which can be used only for copying and postage. However, the chair of the department doesn't think that it is worth the effort to charge for small copying expenses and has therefore decided that the first 4000 pages are free, but that if copying exceeds 4000 pages then 10 cent/copy is charged (also for the first 4000 copies). For the postage, assume that each letter is 33 cents and that a charge will be imposed from the very first letter.
3. Eric is a college student at a big state university. He has no cash, but he is in possession of 10 CD records. He also has 15 cans of coke. Assume that Eric only consumes CDs and coke, where we for simplicity imagine that "CDs" is a homogenous good. Eric can buy and sell CDs for 10 dollars a piece and can buy and sell coke for 50 cents a can.
 1. Carefully graph Eric's budget set.
 2. Now suppose that the greedy owner of the CD store only gives Eric 5 dollars for CDs when Eric is selling (while he still charges 10 dollars/CD if Eric buys. Depict the new Budget set carefully.
 3. Finally, suppose that Eric simply cannot sell cans of coke, but that he can go to the store and buy additional cans in addition to the 15 he already has. Maintain the assumption that he buys CDs for 10 dollars and sells for 5 and carefully draw the relevant Budget set.
4. Now suppose that the prices on the market are $(p_1, p_2) = (2, 1)$ and the income is $m = 20$. However, the government thinks that good x_1 is very very important and gives away 4 units for free.
 1. Draw the budget set.

2. The (per capita) cost for the policy to give away 4 units for free is 8 dollars. Hence, for the same cost the government could simply increase the disposable income of the consumer to 28. Which is better for the consumer? **Hint:** use graph!
 3. Suppose instead that the government pays a subsidy $s = 1$ for each unit of good x_1 the consumer buys (total cost will thus be the number of units the consumer buys). Draw the new budget set for the consumer. Sketch some indifference curves such that the best consumption bundle for the consumer is $(10, 10)$ with this subsidy. What is the total cost for the subsidy paid by the government? Suppose the government would give this to the consumer instead of subsidizing consumption. Would the consumer be better or worse off? Interpret!
5. Let \succeq be a rational preference ordering (transitive and complete) over X . Define the new object \succ (for pedantic people—a binary relation on $X \times X$) as follows

$$x' \succ x'' \text{ if } x' \succeq x'' \text{ and } [\text{not } x'' \succeq x']$$

1. Is \succ transitive? If yes provide an argument that is as formal as possible. If no, provide a counter example.
 2. Is \succ complete? **Hint:** Use an example!
6. Instead of using \succeq as the primitive and defining \succ from \succeq as in the previous question it is possible to go the other way and use \succ as a primitive and define \succeq from \succ .

1. Show how \succeq can be defined from \succ .
2. Rather than putting the basic assumptions on the weak preference relation one can make assumptions directly on the strict preference order to ensure that the preferences are rational. It turns out that 2 requirements that do the job are *asymmetry* and *negative transitivity*, which are defined as follows:

Definition 1. The preference relation is **asymmetric** if there exists no pair x' and x'' such that $x' \succ x''$ and $x'' \succ x'$

Definition 2. The preference relation is **negatively transitive** if for any $x' \succ x''$ and any third choice x''' , either $x' \succ x'''$ or $x''' \succ x''$ (or both).

Show that asymmetry and negative transitivity implies that the preference relation is transitive.

3. **(Very optional)** Show that if \succ is asymmetric, then \succeq is complete. Also show that if \succeq is complete, then \succ is asymmetric.
4. **(Hyper-optional).** Show that if \succ is negatively transitive, then \succeq is transitive. Also show that if \succeq is transitive, then \succ is negatively transitive.