Econ 301
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Homework 2

1. Carefully depict some of the indifference curves for the following utility functions. In each case, check whether the preferences are **monotonic** and whether preferences are **convex.** Be very precise! Once you have depicted the indifference curves it is OK to use the graphs to check monotonicity and convexity, but then you need a short explanation and show how you perform the graphical test. It is of course also possible to use some algebra and the definitions of convexity and monotonicity.

   (a) $u(x_1, x_2) = x_1 + \sqrt{x_2}$

   (b) $u(x_1, x_2) = x_1^{\frac{1}{3}} x_2^{\frac{2}{3}}$

   (c) $u(x_1, x_2) = 25 + \left(x_1 + \sqrt{x_2}\right)^2$

   (d) $u(x_1, x_2) = k \min\{2x_1, x_2\} + 3$ where $k > 0$.

   (e) $u(x_1, x_2) = \sqrt{x_1} + x_2$

   (f) $u(x_1, x_2) = -\sqrt{(x_1 - 2)^2 + (x_2 - 4)^2}$

   (g) $u(x_1, x_2) = \text{integer}(x_1) + \text{integer}(x_2)$, where integer (x) means the "integer part of x". That is

   $$\begin{aligned} \text{integer}(x) &= 0 \text{ if } 0 \leq x < 1 \\ \text{integer}(x) &= 1 \text{ if } 1 \leq x < 2 \\ \text{integer}(x) &= 2 \text{ if } 2 \leq x < 3 \end{aligned}$$

   and so on.

   (h) $u(x_1, x_2) = \frac{1}{2}(x_1)^2 + \frac{1}{2}(x_2)^2$

2. Let preferences be represented by $u(x_1, x_2) = x_1^a x_2^b$ and suppose that the income is m and that prices are given by p_1 and p_2

   (a) Set the consumer choice problem up as a problem in a single variable (skip if you are comfortable using Lagrangian methods).

   (b) Can you assume that $b = 1 - a$? Why/why not? Why would you want to make such an assumption?

   (c) Write down the necessary conditions for an interior solution (first order conditions) for the consumer choice problem.

   (d) Solve out explicitly for a bundle (x_1^*, x_2^*) that satisfies the first order conditions. Which two bundles except for the one you just derived may solve the problem? Evaluate the utility function at these two bundles and at (x_1^*, x_2^*). What is the solution to the utility maximization problem?

   (e) **(Optional)** Use the Lagrangian to solve the problem and convince yourself that you get the same solution.

3. Now suppose that $u(x_1, x_2) = \ln(x_1 + x_2)$

   (a) Formulate the *consumer choice problem.*

   (b) Suppose that the solution is *interior.* What must be true about p_1 and p_2? If the solution is interior, how many solutions are there? Illustrate with a picture.

   (c) Under what condition does the consumer only consume x_1 in the solution? Under what condition does the consumer only consume x_2 in the solution? What is the most likely type of solution with this type of preferences, corner solutions or interior solutions?

4. Now suppose that preferences are represented by $u(x_1, x_2) = x_1 + \ln x_2$, the income is m and that prices are given by p_1 and p_2

   (a) Carefully depict the indifference curves for these preferences. In particular, be very careful about the slopes near the x_1-axis and the x_2-axis.

   (b) Formulate the consumer choice problem and write down the necessary conditions for an interior solution (first order conditions). Will the solution always be interior? If $p_1 = p_2 = 1$, what level of m is required for the solution to be interior? Can you come up with more general conditions?

5. Consider a farmer from the Eau Claire region in Wisconsin who lives a two period life and is the proud owner of an Airshyre diary cow. The cow produces 10000 pounds of milk in each period and we imagine that the farmer/consumer only consumes milk and that the milk goes bad unless it is consumed within the period. The farmer has preferences $u(c_1) + \delta u(c_2)$ where c_1 and c_2 denotes the milk consumption in each period. Assume that $u(c)$ is strictly increasing.

   (a) Suppose that there is a bank where the farmer can borrow and lend milk. Let r be the interest rate, so that if the farmer deposits s pounds of milk in the bank he gets $(1+r)s$ pounds back in the second period. Carefully derive and depict the budget set.

   (b) Formulate the relevant choice problem for the farmer.

   (c) Derive the first order condition (the necessary condition for optimality under the assumption that the solution is not at a corner)

   (d) Assume that $u(c) = \ln c, r = 0$ and $\delta = 1$. Solve for the optimal consumption plan.

   (e) Now, assume that there is no credit market. However, the farmer can give the cow antibiotics in period 1, which increases the milk production in period 1, but decreases it in period 2. Let x be the quantity antibiotics given to the animal and assume that $0 \leq x \leq 100$. Suppose that the milk production as a function of x is $m_1(x) = 10000 + x$ in the first period and $m_2(x) = 10000 - x^2$ in the second period. Carefully depict in a graph the set of feasible consumption plans (c_1, c_2) given that the farmer has this technology available, but has no access to any kind of market.