

### Homework 3

1. Lisa has decided to invest all her wealth in stocks. She is only considering 2 stocks suggested by her financial advisor: Fanny and Freddy. Stocks in both Fanny and Freddy cost 1 dollar each and Lisa has 40 dollars to invest. Each stock in Fanny will be worth 10 cents with probability  $\frac{1}{2}$  and 10 dollars with probability  $\frac{1}{2}$ . Each stock in Freddy will be worth 10 cents with probability  $\frac{1}{2}$  and 10 dollars with probability  $\frac{1}{2}$ . Moreover, the value of the Fanny stock and the value of the Freddy stock are independent outcomes, so to sum up we have that:

probability	dollar value of 1 Fanny stock	dollar value of 1 Freddy stock
$\frac{1}{4}$	0.1	0.1
$\frac{1}{4}$	10	0.1
$\frac{1}{4}$	0.1	10
$\frac{1}{4}$	10	10

Label consumption when both stocks are worth 10 cents  $c_{LL}$ . and consumption when both stocks are worth 10 dollars  $c_{HH}$ . Next, let the consumption when Fanny is worth 10 dollars and Freddy is worth 10 cents be  $c_{HL}$ , and the consumption when Fanny is worth 10 cents and Freddy is worth 10 dollars be  $c_{LH}$ . Finally, assume that Lisas' expected utility function is

$$U(c_{HH}, c_{HL}, c_{LH}, c_{LL}) = \frac{1}{4}\sqrt{c_{HH}} + \frac{1}{4}\sqrt{c_{HL}} + \frac{1}{4}\sqrt{c_{LH}} + \frac{1}{4}\sqrt{c_{LL}}$$

- (a) Suppose Lisa puts all her wealth in Fanny. Calculate  $c_{HH}, c_{HL}, c_{LH}, c_{LL}$  and Lisas expected utility.
  - (b) Suppose Lisa puts all her wealth in Freddy. Calculate  $c_{HH}, c_{HL}, c_{LH}, c_{LL}$  and Lisas expected utility.
  - (c) Suppose Lisa puts 20 dollars each in Fanny and Freddy respectively. Calculate  $c_{HH}, c_{HL}, c_{LH}, c_{LL}$  and Lisas expected utility. **Demonstrate** that it cannot be optimal for Lisa to invest all her wealth in a single asset. This can be done by constructing an alternative portfolio that Lisa think is better. Explain as well as you can what is going on.
  - (d) Formulate the relevant maximization problem for Lisa from which the optimal investment strategy can be derived.
  - (e) **(Optional)** Solve the problem.
2. Consider an investor choosing between a safe bond and a risky investment in normandotcom (a company specializing in annoying pop-up windows). The safe bond earns an interest of 10%, so for each dollar invested, the investor gets 1, 1 dollar back. However, each in normandotcom costs \$1 and with probability 0.01 they are worth \$120 each when the uncertainty is resolved, while with the complementary probability of 0.99 the company goes bankrupt and the shares are worthless.
    - (a) Let  $c_g$  denote the consumption of the investor when normandotcom succeeds and  $c_b$  the consumption when the company fails and suppose the investor has wealth  $w = \$1000$  to invest. Write down the budget equation in a form where  $c_g$  and  $c_b$  appears (but not number of dollars invested in the company). To construct this budget set you need to assume that what is not invested in the company is invested in the safe bond. Explain why this is a natural assumption (that is, why it would be stupid to put dollars under the mattress).

- (b) Suppose the investor is a risk neutral expected utility maximizer. What would the investor do?
- (c) Suppose the investor has utility function  $0.1 \ln c_g + 0.99 \ln c_b$ . Will the investor buy *some* shares in normandotcom or will the investor put all the money in the safe bond? **Hint:** to answer this, suppose that all money is put in the safe bond and investigate the slope of the indifference curve through the corresponding point. A picture may help.
3. Gunnar is a farmer who owns 5 pigs. His neighbor Leif is willing to buy them all for 1000 Kronas each. If he doesn't sell, the pigs will multiply and he will have 50 pigs to sell next year. However, pigs are costly to raise, so he'll have to pay 800 Kronas per pig for food, sty-rental etc. Moreover, the pork price is uncertain, so with probability  $\frac{1}{2}$  he will get 800 Kronas per pig when selling in the next period, whereas with probability  $\frac{1}{2}$  he can sell them for 1000 Kronas each also in the next period. Assume that Gunnar only care about the consumption in the second year when making this decision.
- (a) Suppose that Gunnars' ex post utility function is  $u(c) = \sqrt{c}$ . Will Gunnar accept Leifs' offer?
- (b) Graph an ex post utility function that would be consistent with rejecting the offer is the only rational choice for Gunnar and explain why you have drawn it the way you do.
4. Consider a tycoon who owns a really good race horse who would generate earnings of  $H$  if there are no injuries, but only  $L$  if there is an injury, where  $H > L$ .
- (a) Suppose the tycoon can purchase insurance at a cost  $p$  per unit of coverage (each unit paying 1 dollar if there is an injury) and that the tycoon has assets worth  $W$  (for sure) apart from the horse. Derive and graph the budget set for the tycoon carefully, assuming that the coverage can't go negative.
- (b) Suppose that the tycoon has preferences given by  $u(c_H, c_L) = \pi u(c_H) + (1 - \pi) u(c_L)$ , where  $\pi$  is the probability that there are no injuries and  $c_H$  and  $c_L$  is the consumption in the case of no injury and an injury respectively. Assume that  $\frac{du(c)}{dc}$  is strictly decreasing. Calculate the slope of the indifference curve at a point  $(c'_H, c'_L)$  where  $c_H = c_L$ .
- (c) Let  $(c''_H, c''_L)$  be a point at the same indifference curve as  $(c'_H, c'_L)$ , but where  $c''_H > c'_H$ . Compare the slope of the indifference curve at  $(c'_H, c'_L)$  and  $(c''_H, c''_L)$ . If you can't do it in general pick some function  $u(c)$  for which you can compute the derivative ( $\frac{du(c)}{dc}$  must be strictly decreasing).
- (d) Let  $(c'''_H, c'''_L)$  be a point at the same indifference curve as  $(c'_H, c'_L)$ , but where  $c'''_H < c'_H$ . Compare the slope of the indifference curve at  $(c'_H, c'_L)$  and  $(c'''_H, c'''_L)$ .
- (e) In a very carefully constructed graph, illustrate the optional solution to the insurance problem for the tycoon under the assumption that  $p = \pi$ . You may use solve this from the first order conditions if you wish, but if you do so you should still construct the graph. You can also proceed with a graph by using facts from above.
- (f) Suppose instead that  $p > \pi$ . Illustrate the solution in a graph (again very carefully drawn) and determine whether the consumption will be higher when there is an injury to the horse or when there is no injury to the horse.