

Homework 4

1. Leif consumes only three goods: pickled herring (x_1), Swedish meatballs (x_2), and coffee (x_3). Suppose we know that the demand functions for pickled herring and Swedish meatballs are:

$$\begin{aligned} x_1(p_1, p_2, p_3, m) &= \frac{3m}{8p_1} \\ x_2(p_1, p_2, p_3, m) &= \frac{2m}{8p_2}. \end{aligned}$$

what is the demand function for coffee?

2. A demand change due to a change in the price of good x_1 can be decomposed into:

$$\text{Income effect: } \Delta x_1^N = x_1(p'_1, p_2, m) - x_1(p'_1, p_2, m')$$

$$\text{Substitution effect: } \Delta x_1^S = x_1(p'_1, p_2, m') - x_1(p_1, p_1, m)$$

where (p_1, p_1, m) are the original prices and income p'_1 is the new price and m' is the income that makes the demanded bundle given the original prices and income exactly affordable after the change in the price.

Suppose the utility function is given by $u(x_1, x_2) = x_1^a x_2^{1-a}$.

- (a) Calculate the income and substitution effects for good x_1 given a price change from (3, 3) to (3, 4) if the income is 100.
- (b) Redo the same exercise with $u(x_1, x_2) = \min\{x_1, x_2\}$
3. Consider a consumer/worker with no non-labor income. Assume that the endowment of time is $\bar{L} = 24$, and denote labor supply by L (say in hours). Let the wage be 5 dollars/hour and the dollar price of the consumption good be 1 dollar/unit. Suppose that the optimal labor supply (given the $(p, w) = (1, 5)$) is $L^* = 8$.
- (a) Suppose that consumption and leisure are both normal goods and that the wage increases to 10 dollars/hour. Show carefully in a graph how the change in labor supply may be decomposed in a substitution and an income effect. Can you say for sure whether labor supply will increase or decrease?
- (b) Suppose instead that the worker is offered the contract:

$$w(L) = \begin{cases} 5L & \text{if } L \leq 8 \\ 5L^* + 10(L - L^*) & \text{if } L > 8 \end{cases},$$

In a new graph, depict the budget set corresponding to this wage contract (still assuming $p = 1$). Can you say whether labor supply will increase or decrease relative the case with the wage being 5 dollars/hour?

4. Tweedeldum and Tweedeldee consume two goods only: weekday wireless minutes and weekend wireless minutes. Tweedeldum has signed on with a service that gives him 1000 weekday minutes, but no weekend minutes and Tweedeldee has a service that gives him 1000 weekend minutes, but no weekday minutes. If you want to make life easy, assume that weekday minutes cannot be used in the weekend and the other way around.
- In a carefully labeled graph, put in the endowment described above together with some preferences for Tweedeldee and Tweedeldum. Draw the preferences in such a way that both Tweedeldee and Tweedeldum prefers 500 weekday minutes and 500 weekend minutes to their initial endowment.
 - Explain carefully what a competitive equilibrium is in this environment and illustrate how a competitive equilibrium would look like graphically in a NEW GRAPH. *Don't change the preferences or the endowment!*
 - Consider the following alternative trading institution. Tweedeldum first suggests an allocation. Then Tweedeldee either agrees, in which case they consume the allocation suggested by Tweedeldum. If Tweedeldee doesn't agree, they both consume their respective endowments. In a new graph, show how Tweedeldum will choose the allocation. Will this trading institution result in a Pareto Optimal equilibrium?
5. Suppose that Mrs A has preferences given by $u_A(x, y) = \pi x + (1 - \pi)y$ and Mr B has preferences given by $u_B(x, y) = \pi\sqrt{x} + (1 - \pi)\sqrt{y}$, where $\pi \in (0, 1)$. Also suppose that A has an endowment of $(10, 10)$ and B has an endowment of $(5, 15)$. If it helps you, you may consider the case with $\pi = 1/2$. Notice that you may think of π as the probability that B suffers a loss of 10.
- Suppose that $\frac{p_x}{p_y} > \frac{\pi}{1-\pi}$. Can this be an equilibrium? **Hint:** Use the edgeworth box. You need to be explicit about the slope of the indifference curve for A . For B you only need to figure out the general shape of the indifference curve. Alternatively, you may use the indifference curve for A to figure out what the equilibrium allocation would have to be and then check how the utility of this allocation compares to the endowment point for B .
 - Suppose that $\frac{p_x}{p_y} < \frac{\pi}{1-\pi}$. Can this be an equilibrium? **Hint:** Same as above.
 - Suppose that $\frac{p_x}{p_y} = \frac{\pi}{1-\pi}$. Can this be an equilibrium?
 - In the equilibrium, how does x^B relate to y^B ?
 - Interpret this in terms of how a competitive market allocates risk. Who takes on the risk and why? Is the outcome efficient?