

Homework 5

1. Consider the production function $f(x_1, x_2) = x_1^\alpha x_2^{1-\alpha}$
 - (a) Formulate the cost minimization problem. That is, the problem that gives you the cheapest way to produce a given output y when factor prices are w_1 and w_2 .
 - (b) Derive the optimality conditions.
 - (c) Derive the minimal cost function?
 - (d) Suppose instead that factor 1 is fixed at $\bar{x}_1 = 1$. What is the solution to this restricted cost minimization problem. What is the associated cost function? For $y = 1, \alpha = \frac{1}{3}, p = 3$ and $w_1 = 1$, what does this solution simplify to? Compare the “short-term” minimized costs with the solution to the problem with both factors variable for the same parameter values. How do the costs compare if $w_1 \neq 1$, but all other parameter values are held fixed.
2. Derive the cost functions for the following technologies:
 - (a) $f(x_1, x_2) = \min\{ax_1, bx_2\}$
 - (b) $f(x_1, x_2, \dots, x_n) = \min\{x_1, x_2, \dots, x_n\}$
 - (c) $f(x_1, x_2) = ax_1 + bx_2$
 - (d) $f(x_1, x_2) = \min\{ax_1, bx_2\} - k$, where $k > 0$. (There is a little problem with this formulation. Think about that what that could be and how to fix it)
3. Suppose a competitive firm has cost function $c(y) = 1 + y + y^2$.
 - a. Derive and sketch the average variable cost, the average cost curve and the marginal cost curve.
 - b. Draw (the inverse of) the firm supply function. Give an explicit expression for the firm supply function $S(p)$.
4. Radiant Power Co. is a firm specializing in producing electricity by Nuclear power. For simplicity, we assume that the only factors of production are R (reactor capacity) and L (labor) and that the output and factor prices are known to be w_R, w_L and p respectively. The technology is as follows: with probability π the operation is successful and the output is $L^\alpha R^{1-\alpha}$. However, with the complementary probability $1 - \pi$ there is a meltdown, in which case no electric power is produced and Radiant Power Co. will need to spend $KL^\alpha R^{1-\alpha}$ dollars to clean up and compensate neighbors for the lost value of the land (i.e., the costs from a disaster are proportional to how much waste is emitted).
 - (a) Formulate the profit maximization problem for Radiant Power.
 - (b) Derive the first order (necessary) conditions for an interior solution to the problem.
 - (c) Let $L(w_R, w_L, p)$ and $R(w_R, w_L, p)$ be some optimal choices of labor and reactor capacity. From the first order conditions in part b. Solve out for $\frac{L(w_R, w_L, p)}{R(w_R, w_L, p)}$.

- (d) Instead, suppose that the firm has decided to produce 10 units of electricity in the case there is no accident. Formulate the problem from which you can solve for the cheapest way to do this (you don't need to solve it).
- (e) Finally, suppose that the industry is regulated so that the reactor capacity is fixed at \bar{R} . Formulate the relevant profit maximization problem and check how the firm responds to some improved safety procedures that leads to a decrease in π . **Hint:** the last part of the question is easiest to approach in a graph where you first derive the lines in y, L -space that gives constant profits and the "short-run" production possibilities set.
5. Carrboro Mad Cows is a professional hockey team. The ticket price is \$5 and they have an audience of 3000 for each game in a building with 5000 seats.
- (a) Assume that the costs of admitting an extra spectator is zero. Can this pricing be consistent with profit maximization? Explain, using a graph.
- (b) This week they play Wake Forest Lampreys who offers to buy an unlimited amount of tickets at \$4 a piece. The offer is non-negotiable, but the Mad Cows owner may decide how many tickets to sell. Should the Mad Cows sell any tickets to the Lampreys? Should the price they charge to their own fans be below \$4, between \$4 and \$5, \$5 exactly or above \$5? (You should take as given that the greedy owner of the Mad Cows wants as high profits as possible).
6. Consider a monopolist that produces meals with cost function $C(y) = 2c$. 50% of the customers are students who are relatively poor and each has an inverse demand function given by $p^S(y) = 4 - y$. The other 50% of the customers are rich professors, where each professor has inverse demand $p^P(y) = 12 - y$.
- (a) Draw the demand function for each type customer.
- (b) Suppose that the monopolist must charge the same price to both types of customers. What is the monopoly price? Does it matter how *many* of each type of customers the monopolist has?
- (c) Suppose instead that the monopolist can charge different prices (for example by asking customer for ID to get student price). Which price will students be charged and which price will professors be charged?