Econ 410h
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Homework 6

1. Solve for all Nash equilibria in the following games:

   (a) "Coordination game"

		B	
		L	R
A	T	$1,1$	$0,0$
	B	$0,1$	$5,5$

   (b) "Chicken"

		B	
		Tough	Week
A	Tough	$-1,-1$	$2,1$
	Weak	$1,2$	$0,0$

2. Axel, Birgitta and Clint are engaged in a *simultaneous* three way duel (truel?). Each player is given one shot and they all shoot at one of the two opponents simultaneously. The preferences of the players are simple: the higher the probability of survival, the happier is the agent. Suppose that Axel is the worst shot, he hits his target with probability $\frac{1}{2}$. Birgitta hits her target with probability $\frac{3}{4}$, whereas Clint is a perfect shot who hits his target for sure.

   (a) Carefully depict the extensive form of this simultaneous move game. Use the probabilities of survival as the payoffs (Clint survives with probability $\frac{1}{8}$ if both Axel and Birgitta aims at him). Also note that since all players shoot at the same time, the probability that Axel hits Clint is unaffected by whether Clint aims at Axel or Birgitta.

   (b) Find all pure strategy Nash equilibria of the game.

   (c) Suppose you added a second round to the "truel", where any survivors again has one bullet and shoot simultaneously. Can there be a Nash equilibrium where Axel aims at Birgitta and Birgitta aims at Axel in the first round in this 2-round version of the truel?

   (d) Construct a backwards induction equilibrium of the 2-round version of the model.

3. Birgittafilm Inc. currently has a monopoly on producing beer-commercials for males between 21 and 30. However, Axel Productions Ltd. considers to enter this profitable market. Call the decision to enter "in" and the alternative for "out". If Axel Productions enters the market Birgittafilm Inc. has the option of either "fight" (cut prices dramatically so as to hurt the opponent, which is very costly) or "accomodate" which you may think of as playing the equilibrium in some sort of Cournot game. Suppose that Birgittafilm earns a monopoly profit of 6 if Axelfilm stays out, in which case Axelfilm earns a profit of 0. If Axel Productions enters and Birgittafilm accommodates both firms earn a profit of 2, while if Axel Productions enters and Birgittafilm fights, Birgittafilm earns a profit of -2 while Axel Productions earns a profit of -1.

   (a) Carefully draw the extensive form (game-tree and payoffs) for this game.

(b) Solve for a backwards induction (credible) Nash equilibrium.

(c) Construct a Nash equilibrium that is not a backwards induction equilibrium.

(d) Explain briefly (but exactly) why it makes sense to think that the backwards induction equilibrium is a more natural prediction than other Nash equilibria.

4. Consider the following game between the IRS and a professor. Suppose that the professor can choose between cheating and not cheating when filing the tax return and IRS can choose between audit and no audit. Assume that an audit costs the IRS 10 and that it increases the revenues by 20 if the professor is cheating (it gives no additional revenues if the professor is not cheating). Furthermore, if the professor doesn't cheat, he pays a tax of 30 (independently of whether there is an audit or not), while if he does cheat he pays 20 in case he is not caught. However, if there is an audit and the professor is caught (auditing is perfect in the sense that cheaters are detected with probability 1) then the professor has to pay a tax 30 and fine of 10 (total=40).

(a) Write down the payoff matrix of the game described.

(b) Derive all Nash equilibria of the game (mixed as well as pure).

5. Consider the Cournot duopoly model with inverse demand $p(y) = 9 - y$ and constant marginal cost equal to zero.

(a) Specify what a strategy is for each player and what the payoffs (as function of strategies) are.

(b) Derive the best responses for the two firms.

(c) Solve for the Nash equilibrium and depict it graphically.

(d) Now, suppose firm 1 moves first and firm 2 moves second, *after observing the quantity choice of firm 1.* Specify what a strategy is for each player and what the payoffs (as function of strategies) are.

(e) In the duopoly game specified in part d., solve for a "credible" Nash equilibrium (Backwards induction equilibrium) in the model. Depict it graphically and compare with the Cournot model.

(f) Now, construct a Nash equilibrium supported by non-credible threats. To do this, suppose firm 2 follows strategy $s_2(y_1) = 9 - y_1$ for all $y_1 \neq 3$ and $s_2(3) = y_2^*$. For what value(s) of y_2^* is $(3, s_2(\cdot))$ a Nash equilibrium. Why is this equilibrium (equilibria) not credible?

(g) Generalize the construction above as follows: Suppose firm 2 follows strategy

$$s_2(y_1) = \begin{cases} \frac{9-\widehat{y}_1}{2} & \text{for } y_1 = \widehat{y}_1 \\ 9 - y_1 & \text{for } y_1 \neq \widehat{y}_1 \end{cases}$$

For what (if any) values of $\widehat{y}_1$ is this a Nash equilibrium.

6. Ivar and Sven were the only inhabitants in a miserable village in northern Sweden sometime before the enclosure reform. They share a common where they let their goats graze. Let g_I be the number of goats Ivar owns, and g_S the number of goats that Sven owns. Moreover, since there is less grass for each goat as the number of goats on the field increases, the value of having a goat on the field decreases in the total number of goats, G, that are grazing on the field (due to less meat and milk production say). To be specific, assume that the per goat value is $V(G) = 10 - G$. Finally, let the cost of buying and caring for a goat be c (independent of the number of goats owned by either Ivar or Sven).

(a) Suppose that Ivar and Sven simultaneously decide how many goats to get (you may assume that the goats are perfectly divisible). Specify the payoff functions for Ivar and Sven *as a function of the strategic variables (=decision variables).*

(b) Derive the best responses. For $c = 1$, solve explicitly for the Nash equilibrium.

(c) One year Sven oversleeps when the yearly goat market opens. As a result Ivar goes first to acquire goats and Sven buys his goats after Ivar is back (and after Ivar has shown his new goats). Carefully describe what a strategy is in this game for each player (assuming that Sven knows that Ivar overslept).

(d) Derive the backwards induction equilibrium of the game in part c. and illustrate using a graph.

(e) Suppose instead that Sven and Ivar builds a fence in the middle of the field. Ivar can now only have goats on his side of the fence. What is the natural per goat value when each player have his own goats grazing his half of the field. With the fence in place, have does the equilibrium quantity of goats on the field compare with the social optimum?