Assignment 6

1. Consider the following simplification of poker that we used as an example in class. There are two players, a professor and a student.

First, each player first puts a dollar down (the ante). Notice however that there is no choice here, so you don’t need to make this part of the extensive form of the game (the reason for describing the setup this way is that it is a natural way of generating the payoffs).

Then the professor draws a single card from a deck consisting of an equal number of queens and kings. Only the professor draws a card.

After observing the card, the professor may either “bet” or “fold”. If the professor folds, the student wins the pot containing the ante (implying that the student wins 1 and the professor loses 1).

If the professor “bets”, he places another dollar in the pot. The student then has the option between “fold” and “call”. If the student folds, then the professor takes the pot (implying a net transfer of 1 from the student to the professor). If the student “calls” he adds another dollar to the pot. In this case the professor wins if he has a king (in which case there is a net transfer of 2 from student to professor) and the student wins if the professor has a queen (in which case there is a net transfer of 2 from professor to student);

1. Draw the extensive form of the game.
2. Calculate all Nash equilibria in pure as well as mixed strategies.
3. Is the game “fair” in the sense that both players can expect to break even on average?

2. Consider a sequential game between McMartBucks (called the incumbent for short) and a sequence of entrants (in geographically separated markets);

- There is one potential entrant for every period \( n = 1, \ldots, N \)
- The entrant can “enter” or “stay out”. If there is entry the incumbent can choose between “fight” or “accomodate”. If the entrant stays out the incumbent enjoys a monopoly profit of \( a > 0 \) and the entrant gets a payoff of zero.
- The twist in the model is that there is a probability \( p_0 \) that McMartBucks is a “crazy/tough” firm, which simply enjoys fighting. With the complementary probability \( 1 - p_0 \) McMartBucks is a normal firm, which gets payoff 0 of accomodating entry and \(-1\) if fighting entry. The entrant gets a payoff of \( b > 0 \) if the incumbent accommodates and \(-1\) if the incumbent fights.

1. Suppose \( p_0 = 0 \). Derive the unique backwards induction equilibrium.
2. Consider the case with \( p_0 > 0 \). Assume that \( N = 1 \) (that is, there is only a single period). Under what condition will the entrant enter? Under what condition will the entrant stay out?
3. Now consider the case with \( N = 2 \) and that \( p_0 \) is such that entry would be deterred in the one period model. Show that the normal firm must fight entry and the entrant must stay out in both periods in this case.
4. Suppose instead that \( p_0 \) is such that the entrant would enter in the 1 period model. Show that;
   - It cannot be an equilibrium for the normal firm to fight for sure in first period of the two period model (Hint: think about what would happen in the second period in this case)
   - It cannot be an equilibrium for the normal firm to accomodate for sure in the first period of the two period model (Hint: think about what fighting in the first period would imply in this case)
5. (Again in the case where \( p_0 \) is such that the entrant would enter in the 1 period model). Write down the condition on entry probabilities in the second period that makes the incumbent willing to randomize upon entry in the first period.

6. (Again in the case where \( p_0 \) is such that the entrant would enter in the 1 period model). Write down the condition on the posterior probability that the incumbent is crazy that rationalizes randomization by the entrant in the second period.

7. Use your conclusions above to derive a condition for when the entrant wants to enter. Compare with the 1 period model.

8. Show that if \( N \to \infty \), then, for any \( p_0 \), almost all entrants will stay out, the normal incumbent would almost always fight entry, and the incumbent average payoff approaches \( a \) [Hint: use induction to show that the prior \( p_0 \) needed to stop entry shrinks geometrically with the remaining time horizon]

3. Consider the Cournot duopoly with inverse demand \( P(Q) = \max \{1 - Q, 0\} \) and constant unit costs \( c_1 = c_2 = 0 \).

1. Derive (or recall) the Nash equilibrium in the simultaneous move Cournot game and the backwards induction equilibrium in the Stackelberg game with firm 1 moving first.

2. Let \( (q_1^C, q_2^C) \) and \( (q_1^S, \beta_2(q_1^S)) \) be the equilibrium outcomes in the Cornot and Stackelberg games respectively where \( \beta_2(q_1) \) is the best response function for firm 2. Consider the following extensive form. In stage 1, firm 1 can pick \( q_1 \in \{q_1^C, q_1^S\} \). Then, nature moves and a noisy signal of \( q_1 \) called \( s \) is drawn. Let \( s = q_1 \) with probability \( p > \frac{1}{2} \) and \( s \neq q_1 \) with probability \( (1 - p) \). Finally, firm 2 observes \( s \) and picks \( q_2 \in \{q_2^C, \beta_2(q_1^S)\} \). Derive an equilibrium. What happens as \( p \to \frac{1}{2} \)? What happens as \( p \to 1 \)?

3. Now, let \( q_1 \in [0, 1] \). Then, nature draws \( s \) in accordance with conditional cumulative probability \( F(s; q_1) \) given by

\[
F(s; q_1) = \begin{cases} 
(1 - p)s & \text{if } s < q_1 \\
(1 - p)s + p & \text{if } s \geq q_1 
\end{cases}
\]

Then, firm 2 observes \( s \) and selects some \( q_2 \in [0, 1] \). Again, derive an equilibrium. What happens as \( p \to \frac{1}{2} \) and \( p \to 1 \).

4. MGW 9.C.4

5. MGW 9.C.5