Assignment 7

1. Redo everything you didn’t or couldn’t do from assignment 6

2. Consider the following signalling game due to Cho and Kreps. There are two players, Arnie and Barnie. First, nature draws Arnie’s type. With probability \( p \) Arnie is “tough” and with probability \( (1 - p) \) Arnie is a wimp. Knowing his type, Arnie decides whether to have beer or quiche for breakfast. If Arnie is tough, then beer gives an incremental unit of utility and if arnie is a wimp quiche gives him an incremental unit of utility. Barnie observes Arnie having breakfast, and then has a decide between dueling Arnie or not. Notice that Barnie doesn’t observe the type.

If there is no duel, Arnie gains another two incremental utils, whereas no utils are gained or lost if there is a util. Type is irrelevant for Arnies preferences over dueling. In contrast, Barnie gains a util from dueling Arnie if he is a wimp and gains one util if he does not duels Arnie if he is tough.

1. Draw the extensive form.
2. For \( p = \frac{9}{10} \), solve for all sequential equilibria of the game.
3. For \( p = \frac{9}{10} \), derive the normal form payoff matrix and solve for all Nash equilibria of the game.
4. In a general signalling game, let \( S \) be an arbitrary subset of \( \Theta \) and let

\[
B (m, S) = \left\{ a \in A | \exists \mu \in \Delta (S) \mbox{ s.t. } \sum_{\theta \in S} u_2 (a, m, \theta) \mu (\theta) \geq \sum_{\theta \in S} u_2 (a', m, \theta) \mu (\theta) \ \forall a' \in A \right\} .
\]

In words, \( a \in B (m) \) if there is some belief that the receiver can hold which makes \( a \) a best response after seeing message \( m \). Moreover, given an equilibrium and an out of equilibrium message \( m \), let \( u^*_1 (\theta) \) denote the equilibrium payoff for a sender of type \( \theta \) and

\[
S (m) = \left\{ \theta \in \Theta | u^*_1 (\theta) > \max_{a \in B(m, \theta)} u_1 (a, m; \theta) \right\} .
\]

That is, \( S (m) \) is the set of types for which message \( m \) is equilibrium dominated (factoring in that receiver will not play dominated strategy). Then, we say that the equilibrium fails the Intuitive criterion if there exists some type \( \theta' \) such that

\[
u^*_1 (\theta') < \min_{a' \in B(m, \theta')} u_1 (a', m; \theta').\]

Apply the intuitive criterion to the equilibria in the example.

5. Redo everything for \( p = \frac{1}{2} \). Does this change anything?

3. Consider yet another signalling game, now with type space \( \Theta = \{ \theta_1, \theta_2, \theta_3 \} \) and prior \( p = \left( \frac{1}{3}, \frac{1}{3}, \frac{1}{3} \right) \). The sender can play LEFT or RIGHT. If the sender picks LEFT, then the game is over with payoffs \( (1, 1) \) regardless of type. If the sender picks RIGHT, then the receiver chooses between \{TOP, MIDDLE, BOTTOM\}.

Assume that:

- Sender of type \( \theta_1 \) always get utility 0 if going RIGHT
- Sender of type \( \theta_2 \) gets 3 if receiver plays TOP and 0 otherwise.
- Sender of type \( \theta_3 \) gets 3 if receiver plays BOTTOM and 0 otherwise
- If sender is \( \theta_1 \) the receiver gets 1 from MIDDLE and 0 otherwise
- If sender is \( \theta_2 \) the receiver gets 1 from TOP and 0 otherwise.
• If sender is $\theta_3$ the receiver gets 1 from BOTTOM and 0 otherwise.

1. Derive the full set of sequential equilibria (including beliefs).
2. Which equilibria pass equilibrium dominance?
3. Which equilibria pass the intuitive criterion?

4. In the simple version of the job market signalling model discussed in class, construct a “hybrid equilibrium” where there is some length of education that is picked with positive probability by both types, but where some other level of education is chosen by one type only.