12 Adverse Selection and Insurance; The Case with a Monopoly

- Suppose that there are two "types" of consumers. Call them $\{H, L\}$
- Type H has a probability of an accident given by π_H
- Type L has a probability of an accident gives by π_L
- For both types, the endowment is $\{m_G, m_B\}$, where $m_G > m_B$ (i.e., "state B" is when loss occurs).
- Risk neutral monopolist selling insurance;

12.1 Benchmark; Observable Types

If the monopolist knows which types consumer he/she deals with one may be inclined to proceed as follows. Let p be the per unit price of insurance. Let $D_J(p)$ be type Js demand for insurance as a function of the price, that is

$$D_J(p) = \max_z \pi_J u(m_B + z(1-p)) + (1-\pi_J) u(m_G - z)$$

Then, the monopolist should solve

$$\max_p D_J(p)(p - \pi_J).$$

One **could** analyze this problem, but in general the monopolist could do better! The reason is that allowing the consumer to buy any number of units at the same unit price necessarily leaves some consumer surplus to the consumer. That is, we know (assuming risk aversion) that $p = \pi_J$ in order for the consumer to fully insure. But that would give no profit to the monopolist. Hence, whatever the profit maximizing unit price would be, it must involve under insurance.

Suppose instead that the monopolist proceeds as follows. The consumer may get either full insurance and consume x_J units (regardless of whether an accident occurs or not) or get no insurance at all, where

$$u(x_J) = \pi_J u(m_B) + (1 - \pi_J) u(m_G)$$

The expected profit of this arrangement is

$$\pi_J (m_B - x_J) + (1 - \pi_J)(m_G - x_J)$$

Proposition 1 *There is no insurance contract that both gives the monopolist a higher profit and makes the consumer willing to buy insurance. I.e., a profit maximizing monopolist fully insures the consumer and extracts all the consumer surplus.*

To see this, suppose that x_B, x_G are the consumptions for the consumer in a better contract. If u is concave/consumer is risk averse we have that

$$u(\pi_J x_B + (1 - \pi_J) x_G) \geq \pi_J u(x_B) + (1 - \pi_J) u(x_G)$$

If the expected profit is higher from (x_B, x_G) than from (x_J, x_J) then

$$\begin{aligned} \pi_J (m_B - x_J) + (1 - \pi_J)(m_G - x_J) &< \pi_J (m_B - x_B) + (1 - \pi_J)(m_G - x_G) \\ &\Longleftrightarrow \\ x_J &> \pi_J x_B + (1 - \pi_J) x_G \end{aligned}$$

But u is strictly increasing, so

$$\begin{aligned} \pi_J u(m_B) + (1 - \pi_J) u(m_G) &= u(x_J) > u(\pi_J x_B + (1 - \pi_J) x_G) \\ &\geq \pi_J u(x_B) + (1 - \pi_J) u(x_G), \end{aligned}$$

meaning that the consumer is better off buying no insurance at all.

Remark 1 *Contract specifying consumption in each state is without loss of generality. This is called a "revelation principle". The idea is that if the monopolist designs any sort of* *contract, say, where the price is a highly non-linear function of how much insurance is bought, when the optimal choice is eventually made the agent ends up with some CONSUMPTION in each state. We can always replicate this by removing from the choice set all levels of insurance that are not purchased (except* 0 *since we take the view that the consumer must be willing to buy).*

12.2 Non-Observable Types (Private Information)

Again, for the same reasons as above, an insurance contract can be viewed as two numbers (x_B, x_G). From these numbers we may define concepts that may be more familiar in real world insurance.

$$\begin{aligned}\text{Premium} &= P = m_G - x_G \\ \text{Benefit} &= B = x_B + P - m_B = x_B + m_G - x_G - m_B\end{aligned}$$

Notationally it is simpler to perform analysis in terms of (x_B, x_G), but it is equivalent with maximizing over (P, B).

The crucial aspect when the monopolist cannot see who is who is that L must be willing to prick contract designed for L and H must be willing to pick contract designed for H. This yields the following problem. The monopolist designs two contracts, $\left(x_B^H, x_G^H\right)$ and $\left(x_B^L, x_G^L\right)$ to solve

$$\max_{x_B^H, x_G^H, x_B^L, x_G^L} \alpha \underbrace{\left[\pi_L \left(m_B - x_B^L\right)(1-\pi_L)\left(m_G - x_G^L\right)\right]}_{\text{expected profit if type is } L} \tag{1}$$

$$+(1-\alpha)\underbrace{\left[\pi_H \left(m_B - x_B^H\right) + (1-\pi_H)\left(m_G - x_G^H\right)\right]}_{\text{expected profit if type is } H}$$

$$\pi_J u\left(x_B^J\right) + (1-\pi_J) u\left(x_G^J\right) \ \geq \pi_J u(m_B) + (1-\pi_J) u(m_G) \tag{2}$$

$$\pi_L u\left(x_B^L\right) + (1-\pi_L) u\left(x_G^L\right) \ \geq \pi_L u\left(x_B^H\right) + (1-\pi_L) u\left(x_G^H\right) \tag{3}$$

$$\pi_H u\left(x_B^H\right) + (1-\pi_H) u\left(x_G^H\right) \ \geq \pi_H u\left(x_B^L\right) + (1-\pi_H) u\left(x_G^L\right) \tag{4}$$

We will be able to use graphs for most of the analysis. But, to get to this point we need to be able to compare slopes of the indifference curves for type L and H.

Letting the indifference curve be described by a function $f_L(x_G)$ that solves

$$\pi_L u(f_L(x_G)) + (1-\pi_L) u(x_G) = \pi_L u(x_B^*) + (1-\pi_L) u(x_G^*)$$

for every x_G (in some interval around x_G^*). Taking derivatives we get

$$\begin{aligned}
\frac{d}{dx_G}\left[\pi_L u(f_L(x_G)) + (1-\pi_L) u(x_G)\right] &= \pi_L u'(f(x_G)) \frac{df_L(x_G)}{dx_G} + (1-\pi_L) u'(x_G) \\
&= \frac{d}{dx_G}\left[\pi_L u(x_B^*) + (1-\pi_L) u(x_G^*)\right] = 0 \\
&\Leftrightarrow \\
\text{Slope of indifference curve for type } L &= \frac{df_L(x_G)}{dx_G} = -\frac{(1-\pi_L) u'(x_G)}{\pi_L u'(f(x_G))}
\end{aligned}$$

Finally, evaluate at $x_G^* = x_B^* \Rightarrow f(x_G^*) = x_B^*$;

$$\text{Slope of indifference curve for } L \text{ at } (x_G^*, x_B^*) = \frac{df_L(x_G^*)}{dx_G} = -\frac{(1-\pi_L) u'(x_G^*)}{\pi_L u'(x_B^*)}$$

Obviously, we can do same thing for type

$$\text{Slope of indifference curve for } H \text{ at } (x_G^*, x_B^*) = \frac{df_H(x_G^*)}{dx_G} = -\frac{(1-\pi_H) u'(x_G^*)}{\pi_H u'(f(x_G^*))}$$

12.3 The Low Risk Type Has Steeper Indifference Curves Everywhere

Now, just comparing the slopes at any point (x_B^*, x_G^*) we have that

$$\begin{aligned}
\frac{\text{Slope of indifference curve for } L \text{ at } (x_B^*, x_G^*)}{\text{Slope of indifference curve for } H \text{ at } (x_B^*, x_G^*)} &= \frac{\frac{df_L(x_G^*)}{dx_G}}{\frac{df_H(x_G^*)}{dx_G}} = \frac{\frac{(1-\pi_L)u'(x_G^*)}{\pi_L u'(x_B^*)}}{\frac{(1-\pi_H)u'(x_G^*)}{\pi_H u'(x_B^*)}} \\
&= \frac{\frac{(1-\pi_L)}{\pi_L}}{\frac{(1-\pi_H)}{\pi_H}} = \frac{(1-\pi_L)\pi_H}{\pi_L(1-\pi_H)} > 1
\end{aligned}$$

13 Monopolist "Indifference Curves" (Isoprofits)

Suppose that the monopolist sells contract (x_B, x_G) to a consumer with low risk. Then, the expected profit is

$$\pi_L(x_B - m_B) + (1-\pi_L)(x_G - m_G),$$

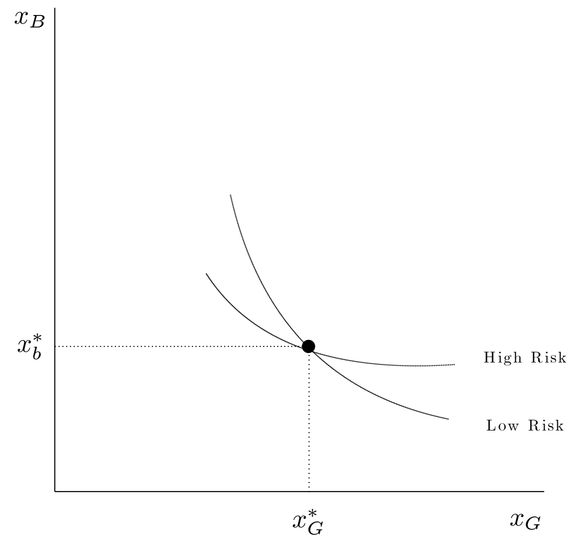

Figure 1: Relative Slopes of Indifference Curves

where we usually would have that $x_B - m_B < 0$ and $x_G - m_G > 0$. An "isoprofit" is then simply a line with constant profits, that is, solutions to

$$\begin{aligned}\pi_L (x_B - m_B) + (1 - \pi_L)(x_G - m_G) &= k \\ x_B &= -\frac{1-\pi_L}{\pi_L} x_G + \frac{k + \pi_L m_B + (1-\pi_L) m_G}{\pi_L}\end{aligned}$$

I.e., straight lines with slope $-\frac{1-\pi_L}{\pi_L}$. Similarly, the relevant isoprofit lines for a high risk individual are

$$x_B = -\frac{1-\pi_H}{\pi_H} x_G + \frac{k + \pi_H m_B + (1-\pi_H) m_G}{\pi_H}$$

14 The Profit Maximizing Contract

Step 1 *If the low risk type isn't insured (eg., if $\left(x_B^L, x_G^L\right) = (m_B, m_G)$), then optimal contract fully insures the high risk type at a premium that extracts all the consumer surplus from the high risk type.*

Proof. See Picture. The straight line is the isoprofit when selling to the low risk type only that goes through the full insurance point at indifference curve through endowment. Any

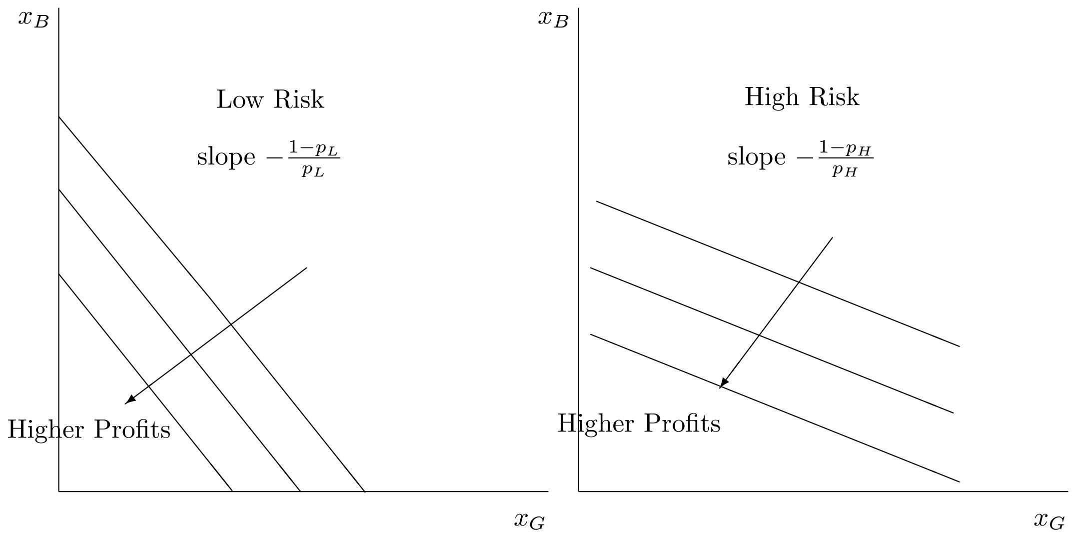

Figure 2: Constant Profit Loci (Isoprofits) for Low and High Risk Type

Plan that gives a higher profit therefore violates the Individual rationality constraint for the high risk type. ■

Step 2 $x_G^L \geq x_G^H$

Proof.

See the Figure. Fix $\left(x_B^L, x_G^L\right)$ arbitrarily. For IC-L to be satisfied (i.e., for L to be better off with $\left(x_B^L, x_G^L\right)$ than with $\left(x_B^H, x_G^H\right)$) it must be that $\left(x_B^H, x_G^H\right)$ is below the indifference curve for the low risk type. For IC-H to hold (i.e., for H to be better off with $\left(x_B^H, x_G^H\right)$ than with $\left(x_B^L, x_G^L\right)$) it must be that $\left(x_B^H, x_G^H\right)$ is above the indifference curve for the high risk type. Hence, only the shaded area in the Figure remains, which proves the claim. ■

Step 3 $x_G^L \leq m_G$ *(no "anti-insurance").*

Proof. Suppose that $x_G^L > m_G$. Consider Fig 5, where l is the hypothetical optimal constract for type L and point A is the point where the indifference curve through the endowment for the high risk type intersects the indifference curve for the low risk type though point l. To satisfy IR-H and IC-L it is therefore necessary that the contract for type H is in the wedge beginning at point A. Note then that;

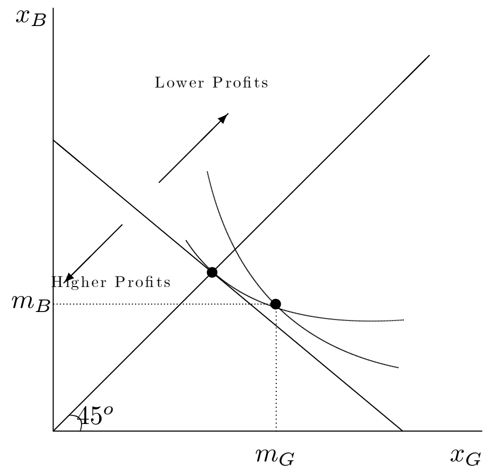

Figure 3: Why High Risk Type is Fully Insured if No Insurance Sold to Other Type

1) If (as in Figure) type L is at a higher indifference curve than the one through the endowment, then it is possible to reduce the consumption for L in (say) the bad state and keep everything the same. This increases the expected profit for monopolist. If instead type L is at the same indifference curve as the endowments (redraw the picture) point A coincides with the endowment. Moving type L to the endowment will keep IC-H satisfied. Since this is a movement in the direction of increased insurance along a given indifference curve the monopolist will increase its profit.

Step 4 *IC-H binds*

Proof. See Figure. If the incentive constraint for the high risk type is not binding the monopolist may reduce the consumption in one state of the world for the high risk type and keep everything else the same. Because of Step 3, point l in the graph is at least as good as the endowment for the high risk type, so the movement from h to h' (which corresponds to reducing the consumption for H in case of accident) will satisfy both IR-H and IC-H. Obviously this increases the profits for the monopolist.

Notice that this argument uses the result that the low risk type doesn't get "anti-insurance" in Step 3 to rule out the possibility that point l is worse for H than the endowment, in which case h' would not satisfy IR-L. ■

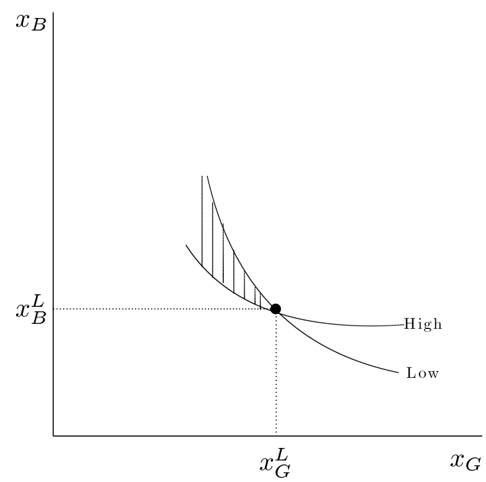

Figure 4: Why x_G^L is at least as large as x_G^H in Optimal Solution to Contracting Problem

Then, we realize that moving the high risk contract to the endowment increases profits on the high risk type (profits increasing in direction of full insurance). For the low risk type, moving the contract down to the point on indifference curve that goes through the endowment increases profits (you give less in case of a loss an keep consumption constant when there is no accident). Finally, moving the low risk to the endowment increases the profits further (profits increasing in direction of full insurance). ■

Step 5 *IR-L binds*

Proof. See Picture. Fixing the contract for type L (point l in graph) we know that the contract for H must be in the "wedge". Call that contract h. Now, offer l' to type L, where the only difference is that the consumption in the case of an accident is reduced to make IR-L binding (consumption when no accident is unchanged. This is obviously better for monopolist, but could possibly upset IC-H. However, by simultaneously reducing the consumption in case of an accident for type H by moving from h to h' we see that both incentive constraints will hold, and, again, reducing the consumption in case of an accident and keeping it constant when there is no accident is better for the monopoly provider.

Step 6 *L is not over insured.*

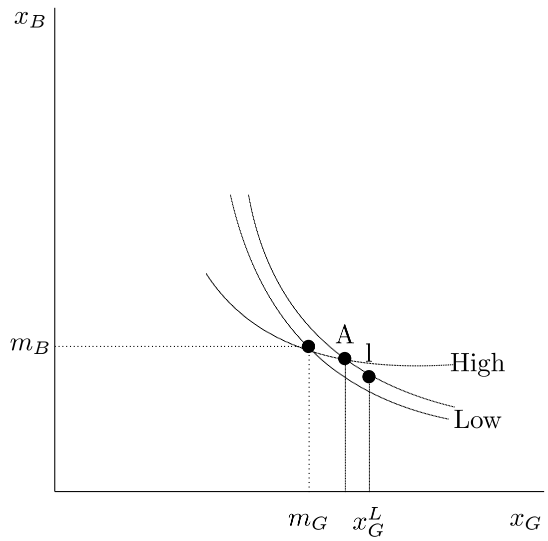

Figure 5: No Anti-Insurance at the Optimum

Proof. See Figure. If L is over insured it must be that he gets a contract like the point l. Since IC-H binds, H gets a contract on the indifference curve through l and to the left of l. Hence, moving from l to l' doesn't change the utility for L and the incentive constraint for H remains satisfied. The picture is a bit bad, but the straight line is supposed to be the isoprofit for the firm (when selling to L), which is tangent to the indifference curve at point l' where L gets full insurance. Hence, l' gives a higher profit than l (since it is a movement on an indifference curve in the direction of full insurance). ■

Step 7 *Full insurance for High Risk Type*

Proof. Draw a Picture! Fix $\left(x_B^L, x_G^L\right)$ anywhere between full insurance point and endowment point on indifference curve going through the endowment. Highest profit along indifference curve for high risk type is full insurance (just like the reasoning in previous picture). ■

Hence we have;

Proposition 2 *The optimal contract has the following features.*

1. *High risk type fully insured*

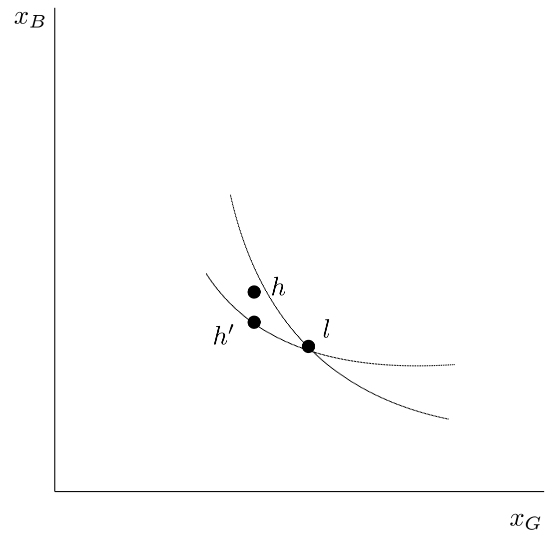

Figure 6: IC-H Binds

2. High risk type indifferent between his and low risk type contract

3. Low risk type at reservation utility level.

Remarks;

1. High risk type earns informational rents. Can get some of gains from trade due to informational advantage.

2. Trade-off for monopolist. Efficiency gains of full insurance versus how much surplus can be extracted from High risk type.

3. Example of price discrimination/non-linear pricing

4. Also note; any observable variable that would be correlated with risk or willingness to pay should be used by monopolist. In example, no such observable variables exist.

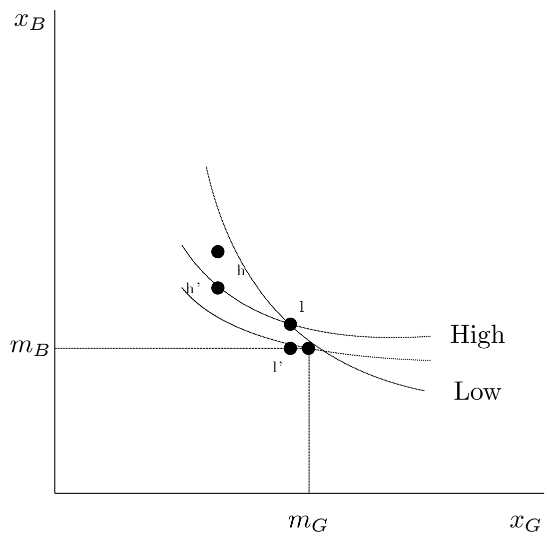

Figure 7: IR-L holds with Equality

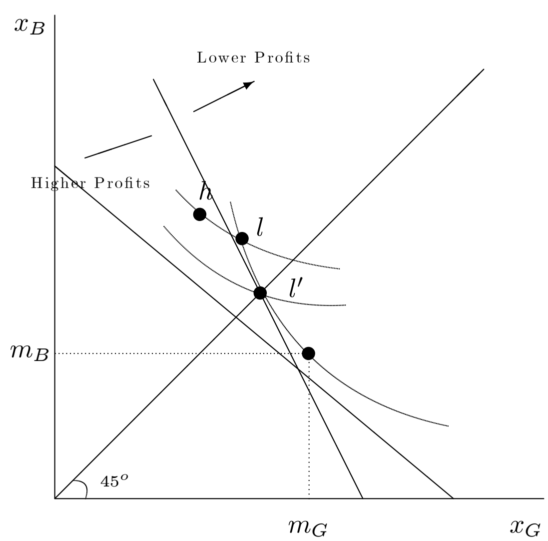

Figure 8: Monopolist can improve on contract L is overinsured