

Problem Set 1

1. Consider the second price auction with  $n$  bidders considered in class:
  - (a) Carefully show that bidding the valuation is the unique weakly dominant strategy for each player.
  - (b) Are there any Nash equilibria in the game that are different from the dominant strategy equilibrium?
2. Suppose that Axel and Birgitta runs a partnership. Output depends on whether they put in high or low effort. Let effort  $e^J$  take on values  $H$  or  $L$  and assume that output is given by

$$\begin{aligned}y(H, H) &= 2 \\y(H, L) &= y(L, H) = 1 \\y(L, L) &= 0\end{aligned}$$

Also assume that there is utility cost  $c \in (0, 1)$  from exerting effort level  $H$  and that they split profits equally.

- (a) Find the parameter range of  $c$  such that  $(H, H)$  is Pareto optimal.
  - (b) Find the parameter range for  $c$  such that  $L$  is a strictly dominant strategy.
3. Axel and Birgitta suffer identical losses and must submit claims to an insurance company. The insurance company is afraid they'll lie and therefore devises the following procedure. Axel and Birgitta simultaneously announces a loss, which is an integer value between 1 and 100. If the claims agree they will be paid that amount. If not, the company pays them the value of the smaller claim, plus an extra 2 for the person who announces the smaller claim (to reward honesty!).
    - (a) Which strategies (if any) are strictly dominated?
    - (b) Which strategies (if any) are weakly dominated?
    - (c) Which strategies (if any) can be eliminated by iteratively elimination of weakly dominant strategies?
    - (d) What are the pure strategy Nash equilibria of the game?
  4. Consider a duopoly model where firms compete in prices. Let the demands be

$$\begin{aligned}D_1(p_1, p_2) &= \left(\frac{p_2}{p_1}\right)^\alpha \\D_2(p_1, p_2) &= \left(\frac{p_1}{p_2}\right)^\beta\end{aligned}$$

and assume that firms have constant unit costs  $c$ .

- (a) Given that firms seek to maximize profits and prices are set simultaneously, what is the normal form?

- (b) Is it possible to set  $\alpha$  and  $\beta$  so that there is a dominant strategy equilibrium? If so, calculate it and explain what is special with the demands.
5. Consider the Bertrand duopoly model where firms simultaneously set prices, but where the good is homogenous with demand  $D(p) = 1 - p$ . Again firms set prices simultaneously, but now consumers only purchase from the firm that offers the lowest price. If prices are equal, the firms each get half the market demand. Let the constant unit cost be equal to  $c$ .
- (a) Write down the payoff functions (assuming that firms want to maximize profits).
- (b) Find all weakly dominated strategies.
- (c) Find all Nash equilibria.
6. Suppose that three players, 1, 2 and 3, are voting in a committee. There are three options,  $x, y$  and  $z$  and payers have the following utilities over these outcomes

	1	2	3
$x$	2	0	1
$y$	1	2	0
$z$	0	1	2

First, each player submits a secret vote. If there is a majority for any outcome, then that outcome is implemented. If instead there is a three-way tie, then player 1 acts as a tie breaker and decides which outcome to implement.

- (a) What are the strategy sets (does it matter whether the votes are secret or not?).
- (b) Solve the game by iterative deletion of weakly dominated strategies.
7. MWG 8.B.2
8. MWG 8.D.4