

Problem Set 2

1. Consider the normal form game  $G^1$  which can be represented in matrix form as

		Birgitta	
		$L$	$R$
Axel	$U$	4, 1	1, 4
	$D$	0, 0	2, 2

Create a new game  $K$  by playing  $G^1$  twice. That is, first the simultaneous move game is played once. Then, both players observe the outcome. Finally, they play the simultaneous move game again, after which payoffs equal to the sum of the payoffs in the two rounds are realized.

- (a) Sketch the extensive form.
  - (b) How many nodes does  $K$  have? How many endnodes? How many information sets?
  - (c) How many pure strategies are available for each player?
2. Consider the Cournot duopoly model given continuous inverse demand  $p(q)$  and (common) cost function  $c(q)$ . Assume that  $c(0) = 0, c(q') < c(q'')$  for every  $q' < q''$  and that there exists  $\bar{q} > 0$  such that  $p(q) > 0$  for every  $q < \bar{q}$ .
- (a) Provide a set of sufficient conditions for profits to be strictly concave in  $q_i$ .
  - (b) Given strict concavity, prove that the best reply correspondence is singleton valued (and hence a function).
  - (c) Let  $\beta_i(q_j)$  be the best response. Prove that there exists function  $\beta(q)$  such that  $\beta_1(q) = \beta_2(q) = \beta(q)$ .
  - (d) Using the Theorem of the maximum, prove that  $\beta(\cdot)$  is continuous.
  - (e) Using the intermediate value theorem, prove that a symmetric equilibrium exists.
  - (f) Show that there may exist multiple symmetric equilibria as well as asymmetric equilibria.
3. Consider a twice repeated Prisoner's dilemma with payoffs

	Cooperate	Defect
Cooperate	1, 1	-1, 2
Defect	2, -1	0, 0

The first stage outcome is observed before the second stage and payoffs at the terminal nodes are the sum of the payoffs in the two rounds of play.

- (a) How many histories and strategies is there?
- (b) Derive the reduced normal form.

- (c) Derive all Nash equilibria.
  - (d) Which strategies survive iterated elimination of strictly dominated strategies?
  - (e) Which strategies survive iterated elimination of weakly dominated strategies?
  - (f) Generalize everything to a  $T$ -fold repetition.
4. Consider the Stackelberg duopoly model with inverse demand  $p(q) = \max\{1 - q, 0\}$  and constant unit cost equal to zero. The difference between the Cournot and the Stackelberg model is that in the Stackelberg model the leader (firm 1) moves first. The follower (firm 2) observes the quantity chosen by firm 1 before setting its quantity.
- (a) What are the strategy spaces for the two firms?
  - (b) Given strategy profile  $(s_1, s_2)$ , what is the equilibrium outcome?
  - (c) Derive the unique backwards induction equilibrium.
  - (d) Characterize some necessary conditions for there to exist a strategy profile supporting quantities  $(q_1, q_2)$  as a Nash equilibrium.
  - (e) Let  $Q^2 \subseteq R_+^2$  denote the set of quantities such that there exists a Nash equilibrium with  $(q_1, q_2) \in Q^2$  being the equilibrium outcome. Find  $Q^2$ .
5. Prove that the order of deletion does not matter for the set of strategies surviving iterated deletion of strictly dominated strategies.
6. Prove that if iterated deletion of strictly dominated strategies results in a single strategy profile, then that strategy profile is a Nash equilibrium.