Econ 711
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Problem Set 4

1. Consider an infinite repetition of a static Cournot duopoly game with inverse demand $p(q) = 1 - q$ and constant unit cost $c = 0$.Suppose that both firms discount the future at rate δ.

    1. Derive the static Nash equilibrium as well as the symmetric "cartel outcome" that maximizes industry profit.
    2. Derive a condition on δ under which the symmetric "cartel outcome in every period" can be supported as a subgame perfect equilibrium in the infinitely repeated game by use of Nash reversion strategies.
    3. Suppose the condition on δ above is not satisfied. Is it possible to use Nash reversion to support any other outcomes than the symmetric cartel outcome?
    4. Again, suppose that the condition on δ above is not satisfied. Can you construct a subgame perfect equilibrium that supports the symmetric cartel outcome for some additional range of values for δ by making the punishment more severe than Nash reversion?

2. Let G be an arbitrary stage game and consider the T-fold repetition G^T. Say that a strategy profile is history independent if for every $t \geq 1$ the continuation strategies satisfy $s|h_t = s|h'_t$ for any two histories h_t, h'_t of length t.

    1. Show that any subgame perfect strategy profile that is history independent must induce a stage game Nash equilibrium outcome in every stage of G^T
    2. Show that any history independent strategy profile that induces Nash equilibrium play in every stage of G^T is subgame perfect.
    3. Construct an example of a strategy profile that induces stage game Nash equilibrium outcomes in every stage but is not subgame perfect.
    4. Construct an example of a strategy profile which is subgame perfect but does not induce Nash equilibrium play in every stage of G^T.

3. Axel, Birgitta and Clint are engaged in a *simultaneous* three way duel (truel?). Each player is given one shot and they all shoot at one of the two opponents simultaneously. The preferences of the players are simple: the higher the probability of survival, the happier is the agent. Suppose that Axel is the worst shot, he hits his target with probability $\frac{1}{2}$. Birgitta hits her target with probability $\frac{3}{4}$, whereas Clint is a perfect shot who hits his target for sure.

    1. Carefully depict the extensive form of this simultaneous move game. Use the probabilities of survival as the payoffs (Clint survives with probability $\frac{1}{8}$ if both Axel and Birgitta aims at him). Also note that since all players shoot at the same time, the probability that Axel hits Clint is unaffected by whether Clint aims at Axel or Birgitta.
    2. Find all pure strategy Nash equilibria of the game.

3. Add a second round to the “truel”, where any survivors again has one bullet and shoot simultaneously. Can there be a subgame perfect Nash equilibrium where Axel aims at Birgitta and Birgitta aims at Axel in the first round ?
4. Construct a subgame perfect equilibrium of the 2-round version of the model.