Econ 890-005
August 18 2008
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Problem Set 1

1. Prove that for any two sets A and B the relations

$$\begin{aligned} (A \cup B) \cap C &= (A \cap C) \cup (B \cap C) \\ (A \cap B) \cup C &= (A \cup C) \cap (B \cup C) \end{aligned}$$

   hold.

2. For any three sets A, B, C, provide a careful formal proof for the relations

$$\begin{aligned} A \backslash (B \cup C) &= (A \backslash B) \cap (A \backslash C) \\ A \backslash (B \cap C) &= (A \backslash B) \cup (A \backslash C) . \end{aligned}$$

   Illustrate with Venn diagrams.

3. Generalize the claim above and prove that

$$\begin{aligned} A \backslash (\cup_i B_i) &= \cap_i (A \backslash B_i) \\ A \backslash (\cap_i B_i) &= \cup_i (A \backslash B_i) \end{aligned}$$

4. Find $\cup_{i=1}^{\infty} \left[1 + \frac{2}{n^2}, x - \frac{2}{n^2}\right]$. Under what conditions is the set non-empty?

5. Suppose that $f : X \to Y$ is bijective and let f^{-1} be its inverse. Prove that

   1. $f\left(f^{-1}(y)\right) = y$ for every $y \in Y$
   2. $f^{-1}(f(x)) = x$ for every $x \in X$

6. A binary relation may be defined as follows:

   Definition 1. *A binary relation R is defined from arbitrary sets A and B and subset C of $A \times B$ by letting xRy whenever $(x, y) \in C$.*

   We can then define an equivalence relation formally:

   Definition 2. *An equivalence relation on a set A is a binary relation (with $B = A$) such that:*

   1. *aRa for every $a \in A$*
   2. *If aRb, then bRa*
   3. *If aRb and bRc, then aRc.*

   Show that an equivalence relation defines a partition of A.