Econ 890-005
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Problem Set 3

1. In an arbitrary metric space, prove that if $\langle x_n \rangle$ is a Cauchy and contains a subsequence $\langle x_{n_k} \rangle$ converging to x, then $\langle x_n \rangle$ converges to x.

2. Prove that if set in a metric space is totally bounded, then it is bounded. Give an example showing that the converse is not true.

3. Show that if $M_1 = \{X_1, \rho_1\}$ and $M_2 = \{X_2, \rho_2\}$ are complete, then $M = \{X_1 \times X_2, \rho\}$ where

$$\rho(x, y) = \max\{\rho_1(x_1, y_y), \rho_2(x_2, y_2)\}$$

   is a complete metric space.

4. Give a direct proof that a closed and bounded subspace of R^n is sequentially compact.

5. Prove that any space with a finite number of points is compact.

6. If A and B are subsets of a metric space B is contained in $\overline{A}$ (the closure of A) we say that A is **dense** in B.Show that the rationals are dense in the real numbers.

7. Metric space $M = \{X, \rho\}$ is called **separable** if there exists a countable set A that is dense in X. Show that R^n is a separable metric space.