

Problem Set 5

1. Consider the system

$$\begin{aligned}\dot{x}(t) &= ax(t) - 3y(t) \\ \dot{y}(t) &= x(t) + y(t).\end{aligned}$$

1. Under what conditions is the system stable?
2. Solve the system for $a = 3$.

2. Consider the system

$$\dot{x}(t) = \begin{pmatrix} 1 & 4 \\ -1 & 1 \end{pmatrix} x(t) + \begin{pmatrix} 4 \\ 2 \end{pmatrix}$$

1. Find a stationary solution to the system
2. Write down the general solution to the system.
3. Under what conditions will the system be near the stationary solution in the long run? Illustrate with a graph.

3. Consider the system

$$\begin{aligned}\dot{x}(t) &= e^x - 2y - y^2 - 1 \\ \dot{y}(t) &= 2e^{2x} \left(1 - \frac{3}{2}y\right) - y^2 - 1\end{aligned}$$

1. Find a stationary solution (staring and guess/verify is the best method)
2. Check whether the stationary solution is locally stable and illustrate the dynamics near the stationary point with a phase diagram.

4. Consider the second order differential equation $x''(t) + ax'(t) + bx(t) = 0$ when $\frac{1}{4}a^2 - b = 0$.

1. Show that the characteristic equation has a double root.
2. Show that the general solution is on form $x(t) = (A + Bt)e^{\lambda t}$. It may be useful to use an intermediate step where you show that

$$x(t) = f(t)e^{\lambda t}$$

solves the problem provided that $\frac{d^2 f(t)}{dt^2} = 0$.

5. Solve the system

$$x_{t+1} = \begin{pmatrix} -1 & 2 \\ -2 & -1 \end{pmatrix} x_t + \begin{pmatrix} -2 \\ 14 \end{pmatrix}.$$

and illustrate in a phase diagram.