Strategy set for player 1:
\((s_1', s_2^2, s_3^3)\)

\[
\begin{align*}
S_1' &: H_1 \rightarrow X_1' \\
S_1^2 &: H_2 \rightarrow \epsilon A_1 R_2 \\
S_1^3 &: H_3 \rightarrow X_1^3
\end{align*}
\]

Strategy set for player 2:
\((s_2', s_2^2, s_3^3)\)

\[
\begin{align*}
S_2' &: H_1 \rightarrow \epsilon A_1 R_2 \\
S_2^2 &: H_2 \rightarrow X_2^2 \\
S_2^3 &: H_3 \rightarrow \epsilon A_2 R_2
\end{align*}
\]
Pure strategies

Player 2 has:

- 0 actions off 101 info sets
- 101 actions off 1 info set
- 2 actions off 101 info sets

=> $2^{101} \cdot 101 \cdot 2^{101}$

Player 1 has:

- 101 actions off 1 info set
- 2 actions off 101 info sets
- 101 actions off 1 info set

=> $101^2 \cdot 2^{101}$
1.2 Non-Backwards induction Nash Equilibrium,

- Player 1 always demands 1, and refuses all smaller offers.
- Player 2 always offers \( x = 1 \), and accepts any offer.

Check if Nash: Can either player profitably deviate?

Player 1: Knowing player 2's strategy, he can not do any better than demanding 1, and refusing all smaller offers.

Player 2: Knowing Player 1's strategy, he can do no better than zero, so his Best Response is to offer \( x = 1 \) and accept any offer.

\[ \Rightarrow \text{Neither player can profitably deviate.} \]
Stage 3
Player 2 will accept any offer $>0$ (Not $=0$ because of the discrete nature of the game)

$\therefore$ Player 1 offers $\frac{1}{100}$, Player 2 accepts.

Stage 2

Player 1 will accept any offer $> 99\frac{1}{100}$

$\Rightarrow$ Player 2 will get $0$, so he prefers to have Player 1 reject

$\therefore$ Player 2 offers any amount below $99\frac{1}{100}$, Player 2 rejects.

Stage 1

Player 2 will accept any offer $> \frac{1}{100}$

$\Rightarrow$ Player 1 will get less than $99\frac{1}{100}$, he prefers to have Player 2 reject

$\therefore$ Player 1 offers $\frac{1}{100}$ or $0$, Player 2 rejects.
1.3 Continued

Backwards induction equilibria

Stage 1:

Player 1 offers $x_1 = \frac{1}{100}$ or 0

Player 2 accepts iff $x_1 > \frac{1}{100}$

Stage 2:

Player 2 offers $x_2 \leq \frac{99}{100}$

Player 1 accepts iff $x_2 > \frac{99}{100}$

Stage 3:

Player 1 offers $x_3 = \frac{1}{100}$

Player 2 accepts iff $x_1 > 0$
Question 2

1 strategy set for player 1:
\((S_1^1, S_1^2, \ldots, \ldots)\):
where \(S_i^i : \mathbb{R} \rightarrow [0,1] \) for \(i\) even
\(S_i^i : \mathbb{R} \rightarrow \mathbb{A} iR^3\) for \(i\) odd

Strategy set for player 2
\((S_2^1, S_2^2, \ldots, \ldots)\):
where \(S_2^i : \mathbb{R} \rightarrow \mathbb{A} iR^3\) for \(i\) even
\(S_2^i : \mathbb{R} \rightarrow [0,1] \) for \(i\) odd

2 Assuming we are in a stationary equilibrium \(\Rightarrow S_i^i = S_i^{i+2}\)
if firm 1 demands \(X_1\) in period \(j\) and it is accepted
Then, in \(j-1\) firm 2 can demand \(1-\delta X_1\) and it will be accepted.
Furthermore in \(j-2\) firm 1 can demand \(1-\delta_2 (1-\delta X_1)\)

Stationarity \(\Rightarrow X_1 = 1-\delta_2 (1-\delta X_1)\)
\(\Rightarrow X_1 = \frac{1-\delta_2}{1-\delta_2 \delta_1}\)
Thus we can conclude the stationary backward induction equilibrium is Player i always demands a share $\frac{1-s_j}{1-s_i s_j}$ when it is his turn to make an offer. He accepts any share equal to or greater than or equal to $\frac{s_i (1-s_j)}{1-s_i s_j}$ and refuses any smaller share.

Note: $\frac{1-s_j}{1-s_i s_j} = 1 - \frac{s_j (1-s_i)}{1-s_i s_j}$

so a demand of $\frac{1-s_j}{1-s_i s_j}$ to the other player is receiving $\frac{s_j (1-s_i)}{1-s_i s_j}$

Now we need to make sure neither player can profitably deviate:

- Player i's demand of $\frac{1-s_j}{1-s_i s_j}$ is the highest share for player i that is accepted by player j (see note above)
continued

- Player i cannot gain by making a lower demand (it will be accepted)

- Making a higher, and rejected demand and waiting to accept player j's offer next period hurts player i:

\[
P_i \left(1 - \frac{1 - P_i}{1 - P_i \cdot P_j}\right) = P_i^2 \frac{1 - P_j}{1 - P_i \cdot P_j} < \frac{1 - P_j}{1 - P_i \cdot P_j}
\]

player j's demand

\[
\text{player i's payoff from waiting}
\]

It is optimal for player i to accept any offer of \(\frac{P_i}{1 - P_j}\) and reject lower shares since if he rejects he receives the share \(\frac{1 - P_j}{1 - P_i \cdot P_j}\) in the next period (same payoff).
Question 2

3) Proof that this is the unique backward induction equilibrium of the game.

First a relevant definition: The continuation payoffs of a strategy profile in a subgame starting at $t$ is the utility in time units of the outcome induced by that profile.

Proof
Define $H_i$ and $L_i$ to be player $i$'s highest and lowest continuation payoffs for player $i$ when player $i$ makes an offer.

Define $h_i$ and $l_i$ to be player $i$'s highest and lowest continuation payoffs for player $i$ when player $j \neq i$ makes an offer.

(i) When player 1 makes an offer, player 2 will accept any share $> S_2 H_2$.

$$\Rightarrow l_1 \geq 1 - S_2 H_2$$

Similarly, player 1 will accept $S_i H_1$.

$$\Rightarrow l_2 \geq 1 - S_i H_i$$

(ii) Since player 2 will never offer player 1 a share greater than $S_i H_i$,

$h_1 = S_i H_i$
2.3 continued

(iii) Since player 2 can obtain at least $L_2$ by rejecting player 1's offer, player 2 will reject any $X$ such that:

$$1 - X \leq S_2 L_2$$

$$\Rightarrow H_1 \leq \max \left( 1 - S_2 L_2, S_1 H_1 \right)$$

$$\leq \max \left( 1 - S_2 L_2, S_1^9 H_1 \right) \text{ (by ii)}$$

(iv) Claim: $\max \left( 1 - S_2 L_2, S_1^2 H_1 \right) = 1 - S_2 L_2$

if not $H_1 \leq S_1^2 H_1 \Rightarrow H_1 = 0$

$$\Rightarrow 1 - S_2 L_2 > S_1^2 H_1 \text{ (since } S_2, L_2 \leq 1)$$

(v) $H_1 \leq 1 - S_2 L_2$

by symmetry

$H_2 \leq 1 - S_1 L_1$

(vi) Combining inequalities yields

$$L_1 \geq 1 - S_2 H_2 \geq 1 - S_2 (1 - S_1 L_1)$$

or

$$L_1 \geq \frac{1 - S_2}{1 - S_1 S_2}$$
2.3 continued

and \( H_1 \leq 1 - \delta \alpha (1 - \delta, H_1) \)

or \( H_1 \leq \frac{1 - \delta \alpha}{1 - \delta_1 \delta \alpha} \)

(vii) \( H_1 \geq L_1 \Rightarrow H_1 = L_1 \)

* can do the same exercise for player 2

(viii) As \( h_1 \leq S_1, H_1 \) and \( l_1 \leq S_1, L_1 \)

we can also conclude that \( h_1 = l_1 \) and \( h_2 = l_2 \)

"* The perfect equ. continuation payoffs are unique

(see Game theory: Fundenberg + Tirole, pp 115-116 for a more thorough treatment)"
For simplicity I will solve this game assuming \( u_i > 0 \), it can also be done assuming \( u_i < 0 \).

1. \[ \begin{array}{c|ccc}
   & X & Y & 3 \text{ votes X} \\
   \hline
   X & 0,0,0 & 0,0,0 & X \\
   Y & 0,0,0 & u_i,u_i,u_i & Y \\
\end{array} \]

2. \[ \text{Nash} = \{ (x,x,x) \text{, } (y,y,x) \text{, } (x,y,y) \text{, } (y,x,y) \text{, } (y,y,y) \} \]

Since both outcomes are a Nash, this has no implications on whether a policy can be eliminated.

3. When no agent plays a weakly dominant strategy, the only result is \((y,y,y)\). Thus the outcome of \(x\) being implemented is no longer an option. \(y\) is the equilibrium outcome.
**Normal Form**

Player I plays $x$.

<table>
<thead>
<tr>
<th></th>
<th>$xx$</th>
<th>$xy$</th>
<th>$yx$</th>
<th>$yy$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$xx$</td>
<td>$U$</td>
<td>$U$</td>
<td>$U$</td>
<td>$U$</td>
</tr>
<tr>
<td>$xy$</td>
<td>$U$</td>
<td>$U$</td>
<td>$U$</td>
<td>$U$</td>
</tr>
<tr>
<td>$yx$</td>
<td>$U$</td>
<td>$U$</td>
<td>$U$</td>
<td>$U$</td>
</tr>
<tr>
<td>$yy$</td>
<td>$U$</td>
<td>$U$</td>
<td>$U$</td>
<td>$U$</td>
</tr>
</tbody>
</table>

Nash = All strategy sets that provide a payoff of $U$ as well as:

$(x, xx, xxxx)$

$(x, xx, yxxx)$

where $xx$ means $x$ if $x$ was played

$x$ if $y$ was played.

$xxxx$ means $x$ if $(xx)$ was played

$x$ if $(x,y)$

$x$ if $(y,x)$

$x$ if $(yy)$
$BI_{eqv} = x(y, y), (y, x, y), (y, y, x), (y, y, y)$
The 1st step in solving for SPE is to find the Nash EG of A, B, and C. Then update the extensive form with the resulting payoffs. C yields a, B yields b, and A yields c. Continued.
\[ \text{Even outcome } \Rightarrow \text{x wins, y wins } \Rightarrow \text{ x vs y, } \]

\[ \Rightarrow \text{ Even is technically an ill-defined pair. } \]

Now we solve each of these games.

\[ \text{Backwards induction } \]

[Diagram of a game tree with various outcomes marked with x and y, showing the decision paths and outcomes.]