4 Public Goods

Chapter 6 in Stiglitz

4.1 Terminology

- A good is said to be *rival* if two consumer cannot consume the same unit. When we considered the Edgeworth boxes we assumed that goods are rival because the market clearing conditions were

$$\begin{aligned} x_1^{A*} + x_1^{B*} &= e_1^A + e_1^B \\ x_2^{A*} + x_2^{B*} &= e_2^A + e_2^B. \end{aligned}$$

  Obvious examples are anything that can be eaten.

- A good is *non-rival* if one consumer's consumption does not diminish the consumption possibilities for any other consumer. The ultimate example is knowledge....if one person figures out how to make a wheel, then that knowledge can be used over and over again without affecting the wheel making of the original inventor.

- For rival goods, a good is *excludable* if the owner can prevent theft.

- For non-rival goods, a good is *excludable* if there is a technology that can prevent a consumer from getting access to the good (eg., a fence around a national park or successful anti-copying software on DVDs)..

- A non-rival good is non-excludable if it is impossible to prevent any consumers to get access to the good. The best example of this is probably investments in reducing carbon dioxide emissions (or any other investment that would only global environmental gains without any particular localized benefits).

- Traditionally, non-excludable public goods are called *pure public goods.*

- Excludable rival goods (like apples) are called *private goods.*

4.2 Example of an Economy with a Pure Public Good

We will try to keep things as simple as possible:

- It is qualitatively unimportant, but notation is somewhat simplified by considering an economy with two agents. Label the agents A and B.
- Agent $J = A, B$ consumes two goods. Let x^J denote the consumption of the *private good* and y the (common) consumption of the public good.
- Agent J's preferences described by utility function $u^J(x, y)$.
- initially, there is no public good available, but each agent J has an endowment $e^J > 0$ of the private good and each agent can turn a unit private good into a unit public good on a one-to-one basis. That is, think of x as dollars and y as public radio. Obviously, it is sort of unclear what one unit of public radio is, but the assumption in the model is that there is constant returns to scale in the public radio production....which can be thought of as a choice of units.

4.3 Private Provision of Public Goods

In order to formalize the idea of citizens voluntarily providing a public good we need to write down a *strategic* model. The reason is imply that if everyone took the aggregate supply of the public good as given, then nobody would have an incentive to contribute at all.

Let y^A and y^B be the contribution towards the public good by consumers A and B respectively. Clearly, the associated consumptions of the private good are

$$\begin{aligned} x^A &= e^A - y^A \\ x^B &= e^B - y^B \end{aligned}$$

The associated happiness of agents A and B are

$$\begin{aligned} &u^A\left(e^A - y^A, y^A + y^B\right) \qquad (1) \\ &u^B\left(e^B - y^B, y^A + y^B\right). \end{aligned}$$

4.4 The Game

Well assume that the two players decide on their contributions towards the public good simultaneously. The appropriate equilirbium concept is then that of a Nash equilibrium.

Definition 1 *In the context of a game where* $\left(y^A, y^B\right)$ *are chosen simultaneously and the payoffs are given by (1) we say that* $\left(y^{A*}, y^{B*}\right)$ *is a Nash equilibrium if*

$$\begin{aligned} u^A\left(e^A - y^{A*}, y^{A*} + y^{B*}\right) &\geq u^A\left(e^A - y^A, y^A + y^{B*}\right) \text{ for every } y^A \geq 0 \\ u^B\left(e^B - y^{B*}, y^{A*} + y^{B*}\right) &\geq u^B\left(e^B - y^B, y^{A*} + y^B\right) \text{ for every } y^B \geq 0 \end{aligned} \quad (2)$$

We note that (2) is equivalent with saying that

$$\begin{aligned} y^{A*} &\text{ solves } \max_{y^A} u^A\left(e^A - y^A, y^A + y^{B*}\right) \\ y^{B*} &\text{ solves } \max_{y^B} u^B\left(e^B - y^B, y^{A*} + y^B\right) \end{aligned} \quad (3)$$

In general, it is possible that,

1. some consumer(s) don't contribute at all towards the public good, or;

2. some consumer(s) contribute all their income to the public good.

The issue that some agents may not give anything at all actually crops up in relatively reasonable examples. The other possibility, that agents may contribute all they have, cannot be ruled out, but is quite uninteresting as the ultimate reason for studying public goods is that it may lead to underprovision by a decentralized market. Such a conclusion (the possibility of underprovision) is not at a;; affected by the fact that it is possible to write down examples where the consumers have so strng preferences for the public good that they will only consume the public good (which would imply that the decentralized market is efficient).

Having warned you that we may have corner solutions, let's assume that both agents are choosing $0 < y^J < e^J$ in equilibrium. Then

$$\begin{aligned} -\frac{\partial u^A\left(e^{A*} - y^{A*}, y^{A*} + y^{B*}\right)}{\partial x^A} + \frac{\partial u^A\left(e^{A*} - y^{A*}, y^{A*} + y^{B*}\right)}{\partial y} &= 0 \\ -\frac{\partial u^B\left(e^{B*} - y^{B*}, y^{A*} + y^{B*}\right)}{\partial x^B} + \frac{\partial u^B\left(e^{A*} - y^{A*}, y^{A*} + y^{B*}\right)}{\partial y} &= 0 \end{aligned}$$

After observing that;

1. $e^{A*} - y^{A*} = x^{A*}$ and $e^{B*} - y^{B*} = x^{B*}$

2. (the consumption of the public good) $y^* = y^{A*} + y^{B*}$

We can rearrange and rewrite the condition for an interior Nash equilibrium as requiring that

$$
\begin{aligned}
MRS_A &= \frac{\frac{\partial u^A\left(x^{A*},y^*\right)}{\partial y}}{\frac{\partial u^A\left(x^{A*},y^*\right)}{\partial x^A}} = 1 \\
MRS_B &= \frac{\frac{\partial u^B\left(x^{B*},y^*\right)}{\partial y}}{\frac{\partial u^B\left(x^{B*},y^*\right)}{\partial x^B}} = 1.
\end{aligned}
$$

This equilibrium condition is almost obvious once it has been derived....the relative price between the public and the private good is one, so we can just draw an indifference curve graph (draw picture)

4.4.1 Inefficiency of the Private Market

It is *possible* that the Nash equilibrium with voluntary contributions is Pareto efficient, but only in rather trivial cases involving corner solutions for all agents. However, in the case characterized where at least two agents contribute anything in between zero and the maxmal amount, the market will always be inefficient. Indeed, this is exactly why I wanted to consider a two agent example with interior contributions from both agents.

Suppose that agents A and B would get together and agree to split the cost equally if they increase the contribution towards the public good by some $h > 0$. The utility associated with such an increase $h > 0$ can be written as

$$
\begin{aligned}
F^A(h) &\equiv u^A\left(x^{A*} - \frac{1}{2}h, y^* + h\right) \\
F^B(h) &\equiv u^B\left(x^{B*} - \frac{1}{2}h, y^* + h\right)
\end{aligned}
$$

We want to show that there are positive values for h so that both agents are made better off than in the Nash equilibrium. To see this, observe that

$$\begin{aligned}
\frac{dF^A(h)}{\partial h} &= -\frac{1}{2}\frac{\partial u^A\left(x^{A*}-\frac{1}{2}h, y^*+h\right)}{\partial x^A} + \frac{\partial u^A\left(x^{A*}-\frac{1}{2}h, y^*+h\right)}{\partial y} \\
\frac{dF^B(h)}{\partial h} &= -\frac{1}{2}\frac{\partial u^B\left(x^{B*}-\frac{1}{2}h, y^*+h\right)}{\partial x^B} + \frac{\partial u^B\left(x^{B*}-\frac{1}{2}h, y^*+h\right)}{\partial y}
\end{aligned}$$

Evaluateingat $h = 0$ we learn what "a small increase" in the level of the public good implies.

$$\begin{aligned}
\left.\frac{dF^A(h)}{\partial h}\right|_{h=0} &= -\frac{1}{2}\frac{\partial u^A\left(x^{A*}h, y^*\right)}{\partial x^A} + \frac{\partial u^A\left(x^{A*}, y^*\right)}{\partial y} \\
&= \frac{1}{2}\frac{\partial u^A\left(x^{A*}h, y^*\right)}{\partial x^A} \underbrace{- \frac{\partial u^A\left(x^{A*}h, y^*\right)}{\partial x^A} + \frac{\partial u^A\left(x^{A*}, y^*\right)}{\partial y}}_{=0 \text{ by FOC for Nash equilibrium}} \\
&= \frac{1}{2}\frac{\partial u^A\left(x^{A*}h, y^*\right)}{\partial x^A} > 0 \\
\frac{dF^B(h)}{\partial h} &= -\frac{1}{2}\frac{\partial u^B\left(x^{B*}, y^*\right)}{\partial x^B} + \frac{\partial u^B\left(x^{B*}, y^*\right)}{\partial y} \\
&= \frac{1}{2}\frac{\partial u^B\left(x^{B*}, y^*\right)}{\partial x^B} > 0.
\end{aligned}$$

Hence, both A and B are made better off if h is sufficiently small, implying that the decentralized equilibrium is suboptimal.

4.4.2 Discussion: Free Riding

The source of the inefficiency in the private provision of a public good is often referred to as a *free-rider problem*. What this means is that consumers benefit from contributions by others, but, by the egoistic nature of their preferences, they do not take into account the fact that their own provision makes other agents happier. To remedy such a situation some sort of intervention in the market place is needed. We will discuss details of such remedies later on, but, for now, lets simply observe that a market failure doesn't necessarily imply that the government should directly provide the good in question. Tax and subsidy instruments will usually work as well (at least in our models).

You may be interested in knowing that there is currently a very active debate in economics about how relevant this free-riding problem is in the real world. One area in which this has attracted particular attention is for charitable giving. Here, the natural assumption about preferences seems to be that agents are truly concerned with the welfare of other, less fortunate, human beings. But then, a dollar contributed by me should be as good as a dollar contributed by any other agent, so charitable giving is naturally modelled as private provision of a public good much like the analysis above (in fact, in the literature, the model you've just seen have been used a lot). There are more subtle issues debated in the literature, but some people have argued that with lots of agents the equilibrium outcome should be very little giving. In the real world on the other hand, people seem to contribute "a lot".

A somewhat more scientific approach to the issue is to run experiments. There have been thousands of experiments run with roughly the following theme. Set up a simple game in the lab where agents are given, say, 10 dollars. Give the agents a choice on how much to keep and how much to put in a common pot. Now, the reason to even contemplate to the common pot is that all the money that is put there earns some interest, before it is divided equally among all the agents in the experiment. For example, with 2 agents in the experiment, one example is to give each agent $\frac{3}{4}$ of all the money that is contributed. Hence, if both agents contribute all the 10 dollars to the common pot, they walk home with $\frac{3}{4} \times 20 = \$15$, whereas if they contribute nothing they only get \$10. The problem is of course that if I am a nice guy and put my \$10 into the common pot and my opponent keeps his or her money, I walk home with \$7.50 and the opponent gets \$17.50. That is, the logic of free riding dictates that both agents should contribute nothing if they cannot coordinate their decisions somehow!

The point of these experiments is that, while free riding is indeed observed in any experiment, there seems to be less free riding than the model with purely self-interested agents would predict.

4.4.3 A Closed Form Example

Suppose that

$$u^A(x,y) = u^B(x,y) = xy,$$

which is a special case of Cobb-Douglas Preferences with equal weights on the two goods. Also suppose that $e^A = e^B = 1$

A Nash equilibrium is then a pair $\left(y^{A*}, y^{B*}\right)$ where

$$\begin{aligned} y^{A*} \text{ solves } & \max_{y^A} \left(1 - y^A\right)\left(y^A + y^{B*}\right) \\ y^{B*} \text{ solves } & \max_{y^B} \left(1 - y^B\right)\left(y^{A*} + y^B\right) \end{aligned}$$

The first order condition for y^{A*} and y^{B*} espectively is then that

$$\begin{aligned} -\left(y^{A*} + y^{B*}\right) + 1 - y^{A*} &= 0 \\ -\left(y^{A*} + y^{B*}\right) + 1 - y^{B*} &= 0 \end{aligned} \tag{4}$$

Inspection tells us that

$$1 - y^{A*} = \left(y^{A*} + y^{B*}\right) = 1 - y^{B*}$$

which implies that $y^{A*} = y^{B*}$. In words, since preferences are symmetric we get a symmetric equilibrium too! Hence, we have that

$$\begin{aligned} 1 - y^{A*} &= \left(y^{A*} + y^{B*}\right) = 2y^{A*} \\ &\Rightarrow \\ y^{A*} &= y^{B*} = \frac{1}{3} \end{aligned}$$

Another way to approach the example is to note that the equations in (1) defines As optimal response to the contribution by B and Bs optimal response to the contribution by A. That is, let the "best reply functions" be given by

$$\begin{aligned} r^A\left(y^B\right) &\equiv \frac{1 - y^B}{2} \\ r^B\left(y^A\right) &\equiv \frac{1 - y^A}{2}, \end{aligned}$$

which are obtained by rearranging the first order conditions in (1). The reason we write y^B instead of y^{B*} in the first function is that we now are defining the best reply to any choice by the other player as opposed to be best reply to the equilibrium contribution. A Nash equilibrium is then a solution to

$$
\begin{aligned}
y^{A*} &= r^A\left(y^{B*}\right) \\
y^{B*} &= r^B\left(y^{B*}\right)
\end{aligned}
$$

and you should make sure that this gives exactly the same conditions as when we solved out for the Nash equilibrium above. However, the advantage with the best reply functions is that we can draw them.

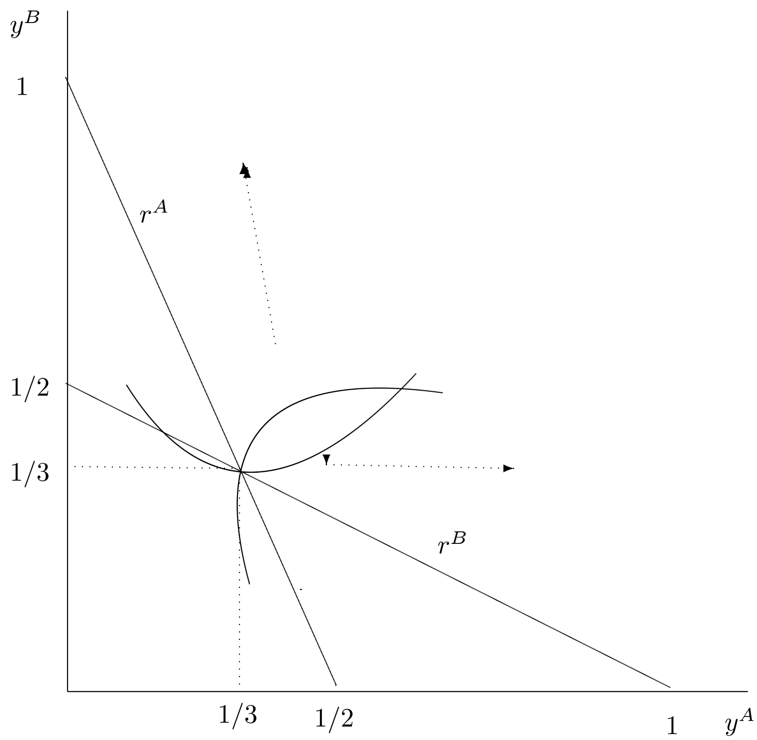

Figure 1: Best Replies and the Equilibrium

The graph is very similar to the graph of a Cournot duopoly with linear demands. However, there arealso some differences:

- In the Cournot duopoly model there is "overproduction" (from the point of view of maximizing industry profits). Here we have "underprovision". In some sense this is clearly just a matter of how we define our variables, but in terms of the picture, each player gets happier the more the other agent contributes (whereas in the Cournot model, the firms get happier the less the other firm produces.

- Here in the Public good example, consider a curve in $\left(y^A, y^B\right)$ −space such that A is indifferent, that is a curve solving

$$\left(1-y^A\right)\left(y^A+y^B\right)=K.$$

It is useful to observe that if we start at a point $\left(y^A, y^B\right)$ where A is best responding, that is where $y^A = r^A\left(y^B\right)$, then this point must be where the curve $\left(1-y^A\right)\left(y^A+y^B\right) = K$ is at its minimum level of y^B. The inuitive reason is that if A would increase or decrease his contribution relative the best reply, then (keeping y^B constant) A is at a lower utility level. Hence, to keep A indifferent when moving A to either side of the best reply it is necessary to increase y^B, implying that the indifference curve has the shape indicated in Figure 1.

- Making a symmetric argument for agent B we see that there is a region o the northeast of the equilibrium where both agents are made strictly better off. That is, thee is underprovision of the public good in equilibrium.

4.5 Pareto Optimality

Returning to the slightly more general formulation, we have that the set of Pareto optimal allocations are those solving

$$\begin{aligned} & \max u^A\left(x^A, y\right) \\ \text{s.t. } u^B\left(x^B, y\right) & \geq \overline{u} \\ x^A + x^B + y & \leq e^A + e^B \end{aligned}$$

This is a constrained optimization problem which cannot be reduced to an unconstrained problem. Hence, it is necessary to use Lagrangian methods to solve it. However, what is easy to understand is that (just like in the Edgeworth box) there will be many Pareto optima associated with different divisions of the resources. Formally, this shows up as a dependence of the solution on $\overline{u}$, which should be thought of as the target utility level for one of the agents.

The Optimality conditions to the Pareto problem can be obtained by forming the Lagrangian

$$L(\cdot) = u^A\left(x^A, y\right) + \lambda\left[u^B\left(x^B, y\right) - \overline{u}\right] + \gamma\left[e^A + e^B - x^A - x^B - y\right]$$

The first order conditions are

$$\begin{aligned} \frac{\partial u^A\left(x^A, y\right)}{\partial x^A} - \gamma &= 0 \\ \lambda\frac{\partial u^B\left(x^B, y\right)}{\partial x^B} - \gamma &= 0 \\ \frac{\partial u^A\left(x^A, y\right)}{\partial y} + \lambda\frac{\partial u^B\left(x^B, y\right)}{\partial y} - \gamma &= 0 \end{aligned}$$

Combining we have

$$\begin{aligned} \frac{\partial u^A\left(x^A, y\right)}{\partial y} + \lambda\frac{\partial u^B\left(x^B, y\right)}{\partial y} &= \frac{\partial u^A\left(x^A, y\right)}{\partial y} + \gamma\frac{\frac{\partial u^B\left(x^B, y\right)}{\partial y}}{\frac{\partial u^B\left(x^B, y\right)}{\partial y}} = \gamma \\ &\Rightarrow \\ \frac{\partial u^A\left(x^A, y\right)}{\partial y} + \frac{\partial u^A\left(x^A, y\right)}{\partial x^A}\frac{\frac{\partial u^B\left(x^B, y\right)}{\partial y}}{\frac{\partial u^B\left(x^B, y\right)}{\partial y}} &= \frac{\partial u^A\left(x^A, y\right)}{\partial x^A} \\ &\Rightarrow \\ MRS_A + MRS_B &= \frac{\frac{\partial u^A\left(x^A, y\right)}{\partial y}}{\frac{\partial u^A\left(x^A, y\right)}{\partial x^A}} + \frac{\frac{\partial u^B\left(x^B, y\right)}{\partial y}}{\frac{\partial u^B\left(x^B, y\right)}{\partial y}} = 1 \end{aligned}$$

To interpret, note that

- In equilibrium the consumers turn private goods to public goods up to the point where the benefit from getting an extra unit of the public good is equalized to the cost of giving up a unit of the private good. This ignores the positive externality that the contribution by A has on B and vice versa.

- In a Pareto optimum we have to equalize the units of private goods A is willing to give up for a unit of the public good PLUS the units B is willing to give up for a unit of the public good with the cost in terms of the private goods in order to equalize social benefits with social costs.

4.5.1 Illustration: The symmetric Pareto Optimum

One particular Pareto optimum can be obtained by maximizing

$$\max u^A\left(x^A, y\right) + u^B\left(x^B, y\right)$$
$$\text{s.t. } x^A + x^B + y \leq e^A + e^B.$$

If you "solve" this problem for general preferences (check!) you will only get the $MRS_A + MRS_B = 1$. However, if $u^A(\cdot) = u^B(\cdot) = u(\cdot)$ and $e^A = e^B = e$, then $x^A = x^B = x$ at the optimum, so we can solve

$$\max_{x,y} u(x, y)$$
$$2x + y \leq 2e$$

or

$$\max_y u\left(e - \frac{1}{2}y, y\right)$$

For concreteness, consider the example with $u(x, y) = xy$ and $e^A = e^B = e = 1$. Then

$$\max_y u\left(e - \frac{1}{2}y, y\right) = \max_y \left(1 - \frac{1}{2}y\right) y$$

solving this problem we get $y^* = 1$ and $x^* = \frac{1}{2}$, instead of $y = \frac{2}{3}$ and $x = \frac{2}{3}$ which was the equilibrium allocation.