5 Other Provision Mechanisms for Public Goods

5.1 Lindahl Equilibria

We have already seen that the "competitive" voluntary provision model in general implies that the resulting allocation is inefficient. There is however a much studied "market institution" called the *Lindahl equilibrium* model that in principle could achieve efficiency. The idea is to think of the amount purchased by each agent as a distinct commodity and have each agent to face a *personalized price* p^i and to have these prices chosen so that all agents agrees on the level of the public good. A Lindahl Equilibrium is then defined in the spirit of a standard competitive equilibrium as a vector of prices and an *expanded* allocation so that 1) each consumer is willing to buy "his share" of the public good at his personalized price, 2) firms are willing to supply the level of public good if paid the sum of all personalized prices$\times$the quantity supplied, 3) markets clear.

For ease of exposition we continue to consider the example with two agents, A and B.

Definition 1 *A Lindahl Equilibrium is a vector* $p^* = \left(p^{A*}, p^{B*}\right)$ *and an allocation* $\left(x^{*A}, x^{B*}, y^*\right)$ *such that*

a. $p^{A*} + p^{B*} = 1.$

The interpretation of p^{J*} *is that this is the personalized price (tax) that agent* i *pays for the public good in terms of of the private good.*

b. *Each consumer maximizes utility, that is* $\left(x^{J*}, y^*\right)$ *solves*

$$\begin{aligned} \max_{x,y} u^J(x,y) \\ s.t.\ e^J - x - p^{J*}y \geq 0 \end{aligned}$$

*Notice that each consumer must optimally purchase the **same** level of the the public good.*

c. *Market clears*

$$x^{A*} + x^{B*} + y^* \leq e^A + e^B$$

Consider the utility maximization problem for agent i, which after substituting the constraint becomes

$$\max_y u^J\left(e^J - p^{J*}y, y\right).$$

The first order condition for an interior solution is thus

$$\begin{aligned} \frac{\partial u^J\left(x^{J*}, y^*\right)}{\partial x} p^{J*} &= \frac{\partial u^J\left(x^{J*}, y^*\right)}{\partial y} \text{ for each agent } J, \qquad \text{(a')} \\ p^{J*} &= \frac{\frac{\partial u^J\left(x^{J*}, y^*\right)}{\partial y}}{\frac{\partial u^J\left(x^{J*}, y^*\right)}{\partial x}} \end{aligned}$$

Summing over $J = A, B$ we get

$$\frac{\frac{\partial u^A\left(x^{A*}, y^*\right)}{\partial y}}{\frac{\partial u^A\left(x^{A*}, y^*\right)}{\partial x}} + \frac{\frac{\partial u^B\left(x^{B*}, y^*\right)}{\partial y}}{\frac{\partial u^B\left(x^{B*}, y^*\right)}{\partial x}} = p^{A*} + p^{B*} = 1,$$

which is the condition for a Pareto optimal allocation.

The idea can be explained as follows:

- instead of a public good, we create a consumer A "public" good and a consumer B "public" good. The agents demand these personalized goods and the equilibrium condition that requires that A and B must agree on the same level of the public good may be thought of as a condition stating that y^A and y^B are jointly produced.
- the short version is that the commodity space is enlarged so that the public good is "privatized".

5.2 A Concrete Example of a Lindahl Equilibrium

To make the example below simple, consider the case with 3 agents A, B, C. Assume that the utility function is

$$u^J\left(x^J, y\right) = \ln x^J + \alpha^J \ln y$$

for $J = A, B, C$. Also assume that each agent has a endowment of the private good $e^J = 1$ and no public good. The only difference compared with the definition of a Lindahl equilibrium for two agents is that there must be a Lindahl price for each of the three agents. That is, the problem for agent J is then to solve

$$\begin{aligned} \max \ln x^J + \alpha^J \ln y \quad (1) \\ \text{s.t } x^J + p^J y \;\; & \leq \;\; 1 \end{aligned}$$

and $p^A + p^B + p^C = 1$ in order for the Lindahl prices to cover for the costs of provision. Eliminating x^J the problem for agent J is

$$\max_y \ln \left(1 - p^J y\right) + \alpha^J \ln y.$$

The first order condition is

$$-\frac{p^J}{1 - p^J y} + \frac{\alpha^J}{y} \text{ for } J = A, B, C$$

Simplifying we get

$$\begin{aligned} \Big/\text{multiply with } \frac{y}{1 - p^J y}\Big/ \; p^J y \;\; & = \;\; \alpha^J \left(1 - p^J y\right) \\ /\text{add } \alpha^J p^J y \text{ and rearrange}/ \;\; & \Rightarrow \;\; p^J y = \frac{\alpha^J}{1 + \alpha^J} \\ /\text{sum over conditions for } A, B, C/ \;\; & \Rightarrow \\ \left[p^A + p^B + p^C\right] y \;\; & = \;\; y = \frac{\alpha^A}{1 + \alpha^A} + \frac{\alpha^B}{1 + \alpha^B} + \frac{\alpha^C}{1 + \alpha^C} \end{aligned}$$

Plugging back into the first line gives the Lindahl prices

$$p^{J*} = \frac{\frac{\alpha^J}{1+\alpha^J}}{\frac{\alpha^A}{1+\alpha^A} + \frac{\alpha^B}{1+\alpha^B} + \frac{\alpha^C}{1+\alpha^C}}$$

5.3 Is Lindahl Equilibrium a Reasonable Market Mechanism?

We will now look at a simple example that illustrates the incentives to mis-report preferences if the planner/policy maker has to elicit information from the agents in order to compute the Lindahl equilibrium. Consider the example above and assume that the true preference parameters are $\alpha^J = 1$ for $J = A, B, C$ The Lindahl equilibrium is then $p^{J*} = \frac{1}{3}, x^{J*} = \frac{1}{2}$ and $y^* = \frac{3}{2}$,so the equilibrium utility level or agent J is

$$\ln x^{J*} + \ln y^* = \ln \left(\frac{1}{2}\right) + \ln \left(\frac{3}{2}\right) = \ln 3 - \ln 4$$

Now make the following thought experiment. Suppose agents B and C report truthfully that their type as $\alpha^B = \alpha^C = 1$, but that Mr. A lies and claims that $\alpha^A = 0$. Furthermore, the "auctioneer" or planning agency that computes the Lindahl price believes all agents and implements the corresponding Lindahl equilibrium, with is $p^A = 0$, $p^B = p^B = \frac{1}{2}$, $x^1 = 1, x^2 = x^3 = \frac{1}{2}$ and $y = 1$. The utility of Mr. 1 would then be

$$2\ln(1) = 0 > \ln 3 - \ln 4$$

implying that lying is better.

The example is special, but the logic is perfectly general. If agents have to report preferences they will take into consideration that under-reporting means a lower price, so a "free-riding" problem similar to the basic voluntary provision model comes back into the analysis.

5.4 Threshold Public Goods

Lets consider the example where

$$\begin{aligned} U^A(x,y) &= U^B(x,y) = x + \ln y \\ e^A &= e^B = 2. \end{aligned}$$

Suppose that some planner wants to implement the symmetric Pareto optimum. That is,

$$\begin{aligned} \max U^A(x,y) + U^B(x,y) &= 2\max\{x + \ln y\} \\ \text{s.t. } 2x + y &= 4 \end{aligned}$$

This can be written as

$$\max\left\{\frac{1}{2}y - 2 + \ln y\right\}$$

and the solution is

$$\left(x^{A*}, x^{B*}, y^*\right) = (1,1,2).$$

Now consider a mechanism where it is decided in advance that the provision level is given by $y^* = 2$, but where agents need to voluntary contribute towards the public good. That is, let y^A, y^B be the voluntary contributions of agents A and B and assume that;

1. If $y^A + y^B \geq 2$, then the public good is provided at level 2
2. If $y^A + y^B < 2$, then the public good will not be provided at all.
3. Also suppose that if $y^A + y^B > 2$, then the extra revenue is thrown into the ocean.

The question is. What is the Nash equilibrium of this game?

Proposition 1 *Any y^A, y^B such $y^A + y^B = 2$ is a Nash equilibrium of the threshold public good game.*

To see this, suppose that y^A is given. Then, the payoff for agent B is

$$\begin{aligned} 2 - y^B + \ln(0) &= -\infty \text{ if } y^B < 2 - y^A \\ 2 - y^B + \ln(2) \text{ if } y^B &= 2 - y^A \\ 2 - y^B + \ln(2) \text{ if } y^B &> 2 - y^A \end{aligned}$$

It is immediate that $y^B = 2 - y^A$ is the best response. By symmetry, the set of equilibria is as described in the claim.

The conclusion is that (in the example) all equilibria are efficient, but that there are lots of equilibria. In fact, it is not clear how one can view efficiency as the prediction of the model even though all equilibria are efficient...which equilibrium the agents are supposed to play is up for grabs. Also, the only reason that there are no inefficient equilibria is that ach agent would want to pay the whole cost of the public good if the other agent would not contribute (in the example this follows because $\ln(0) = -\infty$). This is obviously suspect (in particular in generalizations with many agents) and usually there are equilibria with no provision at all as well in this type of games.

5.5 Raffles

A somewhat bizarre fact is that many public goods are financed through lotteries. A common type of lottery is a "raffle", which means that the prize of winning is fixed, but that the number of lottery tickets that can be sold in principle is unlimited. In mathematical notation, there is a prize of value z and the probability of winning the prize for A is

$$\Pr\left[A \text{ wins}\right] = \frac{r^A}{r^A + r^B},$$

where r^J is the number of raffle tickets bought by J.

Now, the point of the raffle is that the profit is used to provide the public good, so that

$$y = r^A + r^B - z.$$

We maintain the assumption that the preferences are given by

$$U^A(x, y) = U^B(x, y) = x + \ln y,$$

but unlike the previous example we assume that $e^J > z$, without specifying any particular number (this is because we will let $z \to \infty$).

From the point of view of A and B this is a game just like the voluntary contribution game. That is, we seek a Nash equilibrium $\left(r^{A*}, r^{B*}\right)$ where r^{A*} solves

$$\begin{aligned}
& \max_{r^A} \left\{\Pr\left[A \text{ wins}\right] U^A\left(e^A + z - r^A, r^A + r^{B*} - z\right) + \Pr\left[B \text{ wins}\right] U^A\left(e^A - r^A, r^A + r^{B*} - z\right)\right\} \\
= & \max_{r^A} \left\{\frac{r^A}{r^A + r^B}\left[e^A + z - r^A + \ln\left(r^A + r^{B*} - z\right)\right] + \frac{r^B}{r^A + r^B}\left[e^A - r^A + \ln\left(r^A + r^{B*} - z\right)\right]\right\} \\
= & \max_{r^A} \left\{e^A - r^A + \frac{r^A}{r^A + r^B}\left[z + \ln\left(r^A + r^{B*} - z\right)\right] + \left[1 - \frac{r^A}{r^A + r^B}\right]\left[\ln\left(r^A + r^{B*} - z\right)\right]\right\} \\
= & \max_{r^A} \left\{e^A - r^A + \frac{r^A}{r^A + r^B} z + \left[\ln\left(r^A + r^{B*} - z\right)\right]\right\}
\end{aligned}$$

The first order condition is

$$-1 + \frac{r^B}{\left(r^A + r^B\right)^2} z + \frac{1}{r^A + r^{B*} - z} = 0$$

Obviously, we get the same condition for B so we may write the condition as

$$-1 + \frac{1}{2r} z + \frac{1}{2r - z} = 0$$

where $r = r^A = r^B$. We note that:

1. $2r - z = y$ is the provision of the public good. Hence, we have that

$$\frac{1}{y} = 1 - \frac{z}{2r} < 1$$

which means that we get more provision than in the basic voluntary contribution model, because the voluntary provision model is equivalent with setting $z = 0$.